Business Processes Modelling

MPB (6 cfu, 295AA)

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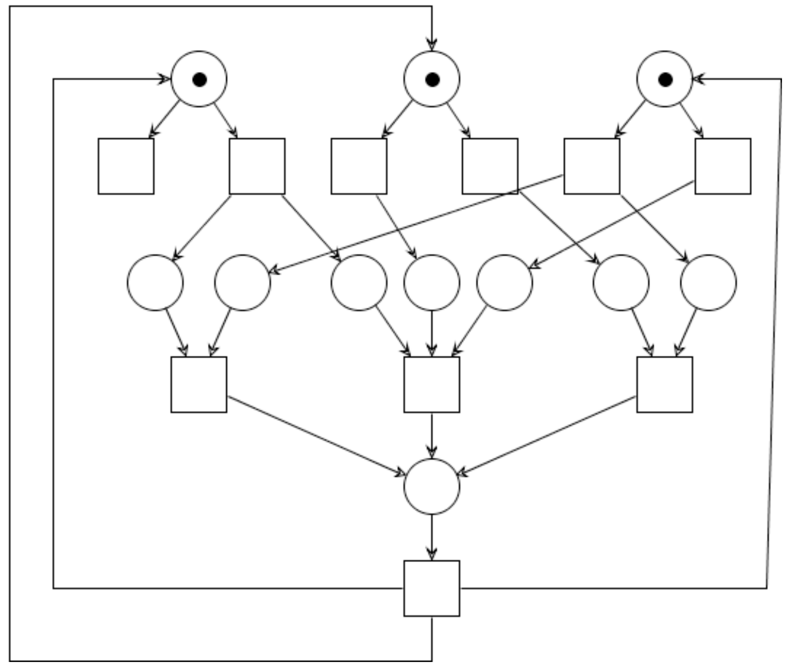
17 - Free-choice nets

Object



We study some “good” properties of free-choice nets

Free Choice Nets (book, optional reading)
https://www7.in.tum.de/~esparza/bookfc.html

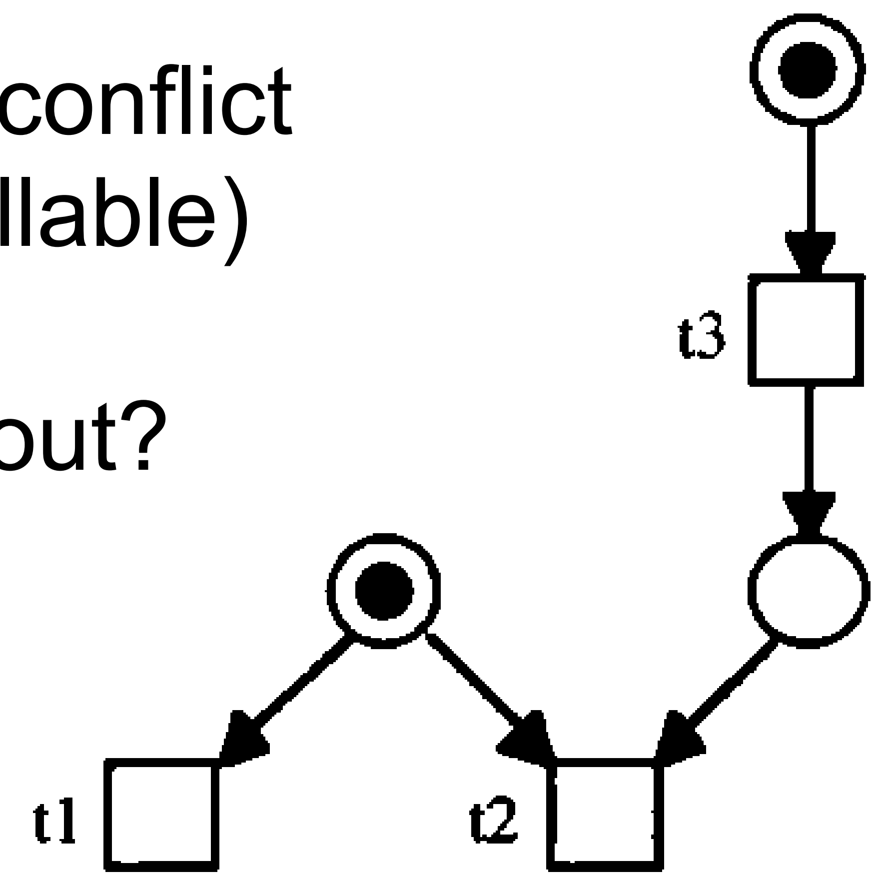
Interference of conflicts and synch

Typical situation:

initially t1 and t2 are not in conflict

but when t3 fires they are in conflict
(the firing of t3 is not controllable)

How to rule this situation out?



Free-choice nets

The aim is to avoid that a choice between transitions
is influenced by the rest of the system

Easiest way:
keep places with more than one output transition apart
from transitions with more than one input place

In other words, if (p,t) is an arc, then it means that
t is the only output transition of p (no conflict)
OR
p is the only input place of t (no synch)

Free-choice systems / nets

But we can study a slightly more general class of nets by requiring a weaker constraint

A Petri net is **free-choice** if
for any pair of transitions
their pre-sets are either disjoint or equal

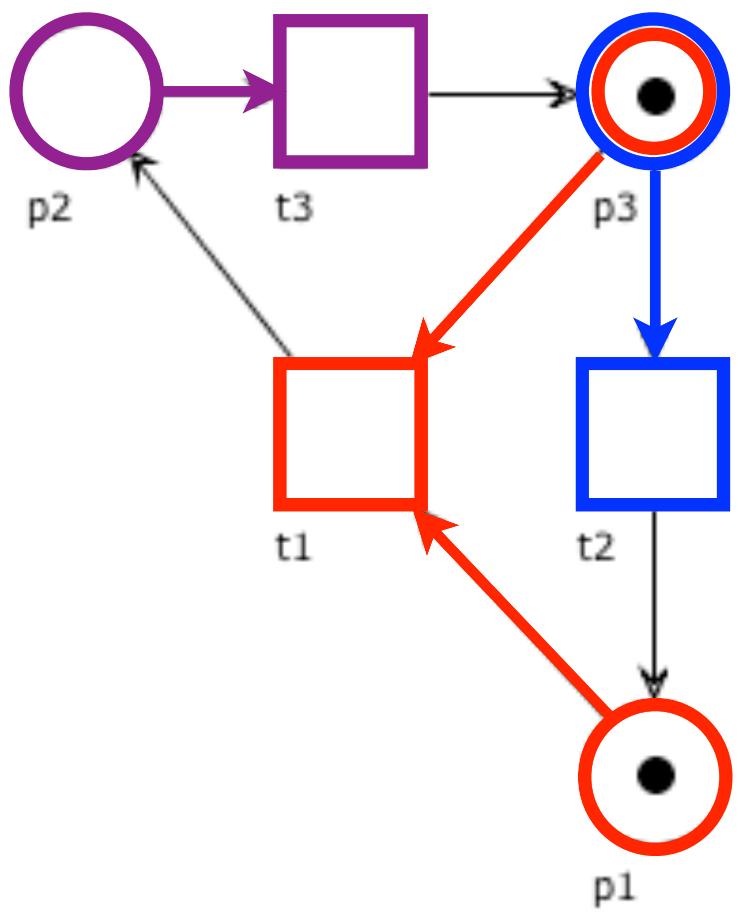
or, equivalently, if
for any pair of places
their post-sets are either disjoint or equal

A system (N,M_0) is **free-choice** if N is free-choice

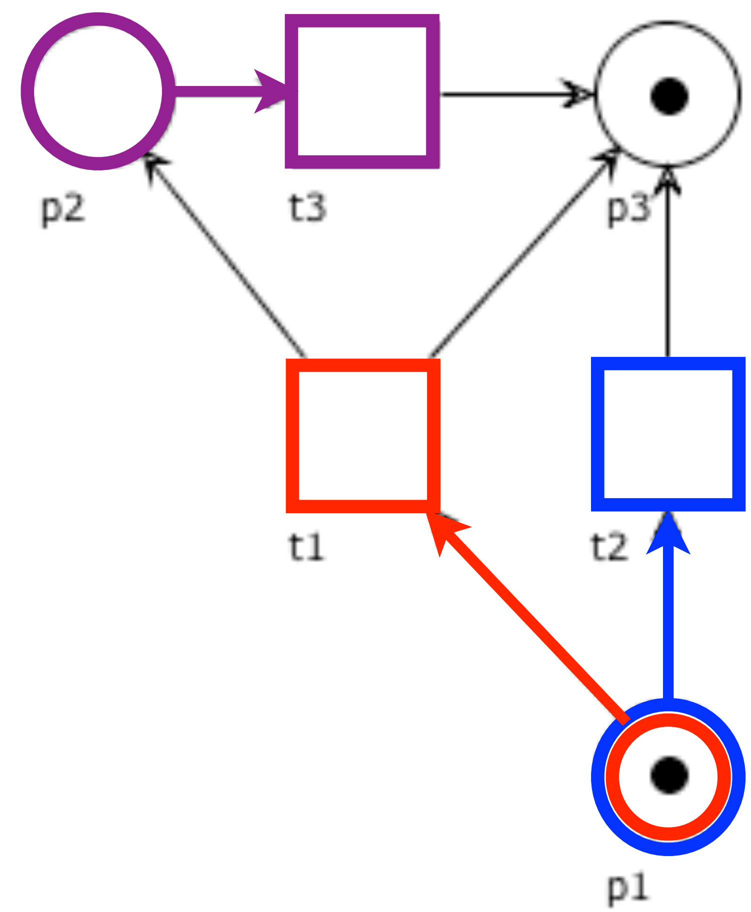
Example

$$\begin{aligned} \bullet t_1 &= \{\, p_1, p_3 \,\} \\ \bullet t_2 &= \{\, p_3 \,\} \\ \bullet t_1 &\neq \bullet t_2 \\ \bullet t_1 \cap \bullet t_2 &= \{\, p_3 \,\} \neq \emptyset \end{aligned}$$

$$\begin{aligned} \bullet t_1 &= \bullet t_2 \\ \bullet t_1 \cap \bullet t_3 &= \emptyset \\ \bullet t_2 \cap \bullet t_3 &= \emptyset \end{aligned}$$



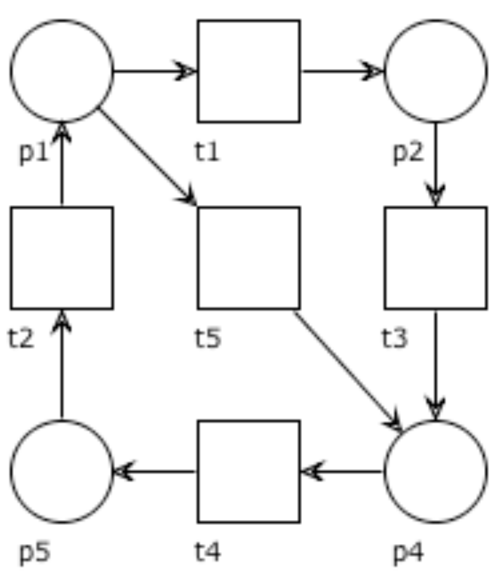
non free-choice

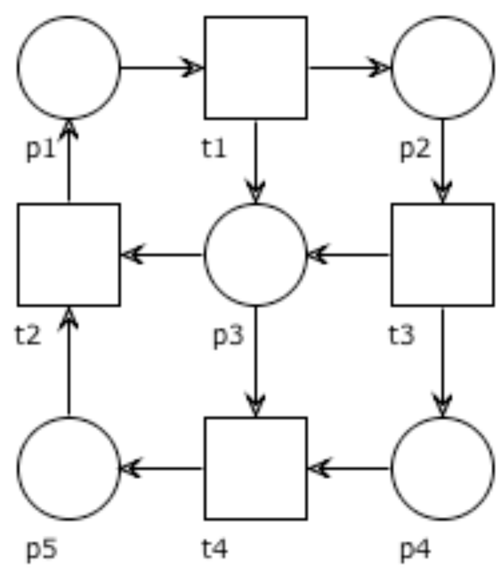


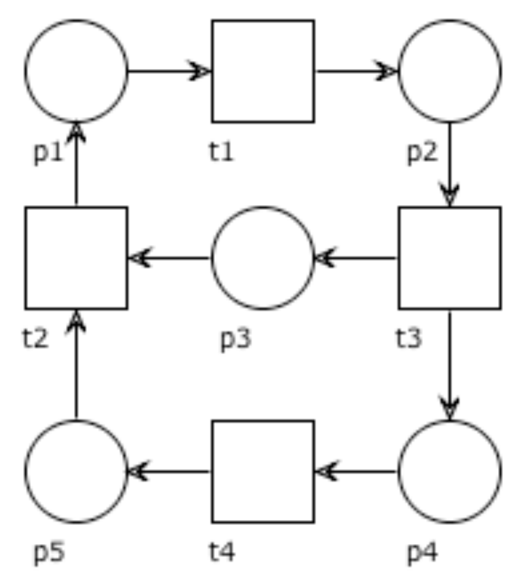
free-choice

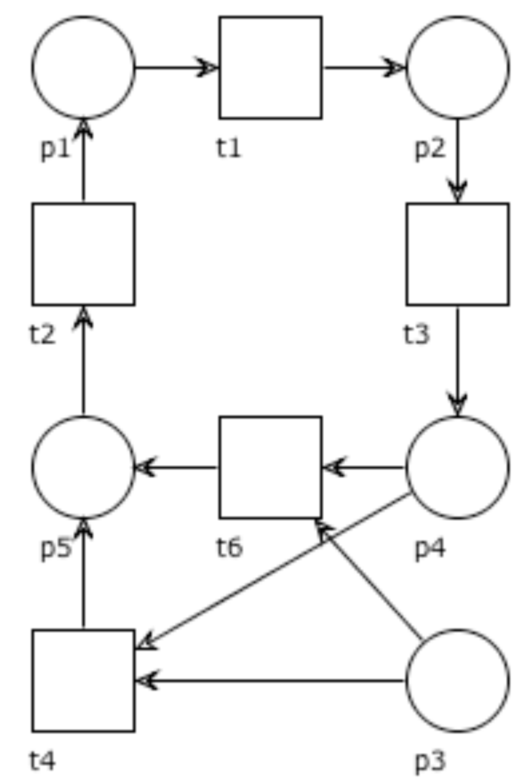
Question time

Is the net free-choice?





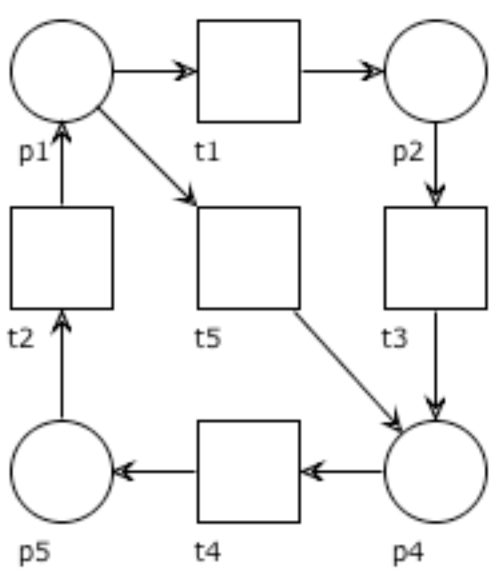


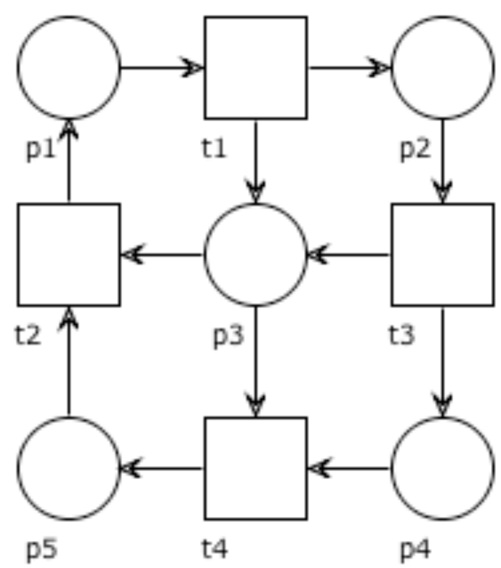


Question time

Is the net free-choice?

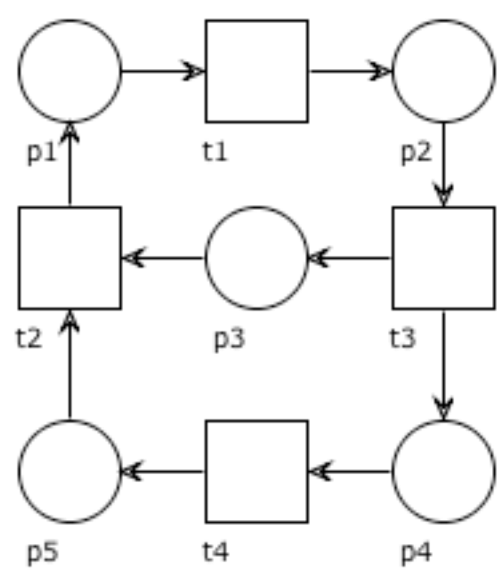
FC-net

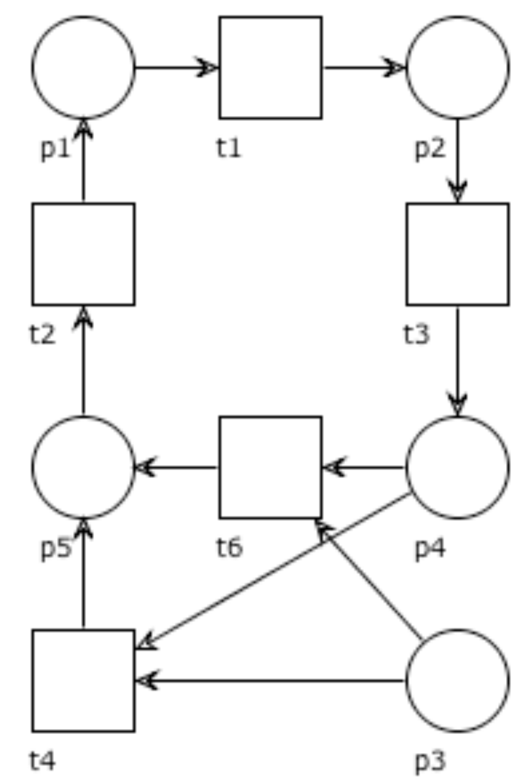




FC-net

FC-net





FC-net

Exercises

Prove that every S-net is free-choice

Let N be an S-net.
Take any two transitions t, s.
By definition of S-net their pre-sets are singletons:
either they coincide
or they are different and thus disjoint

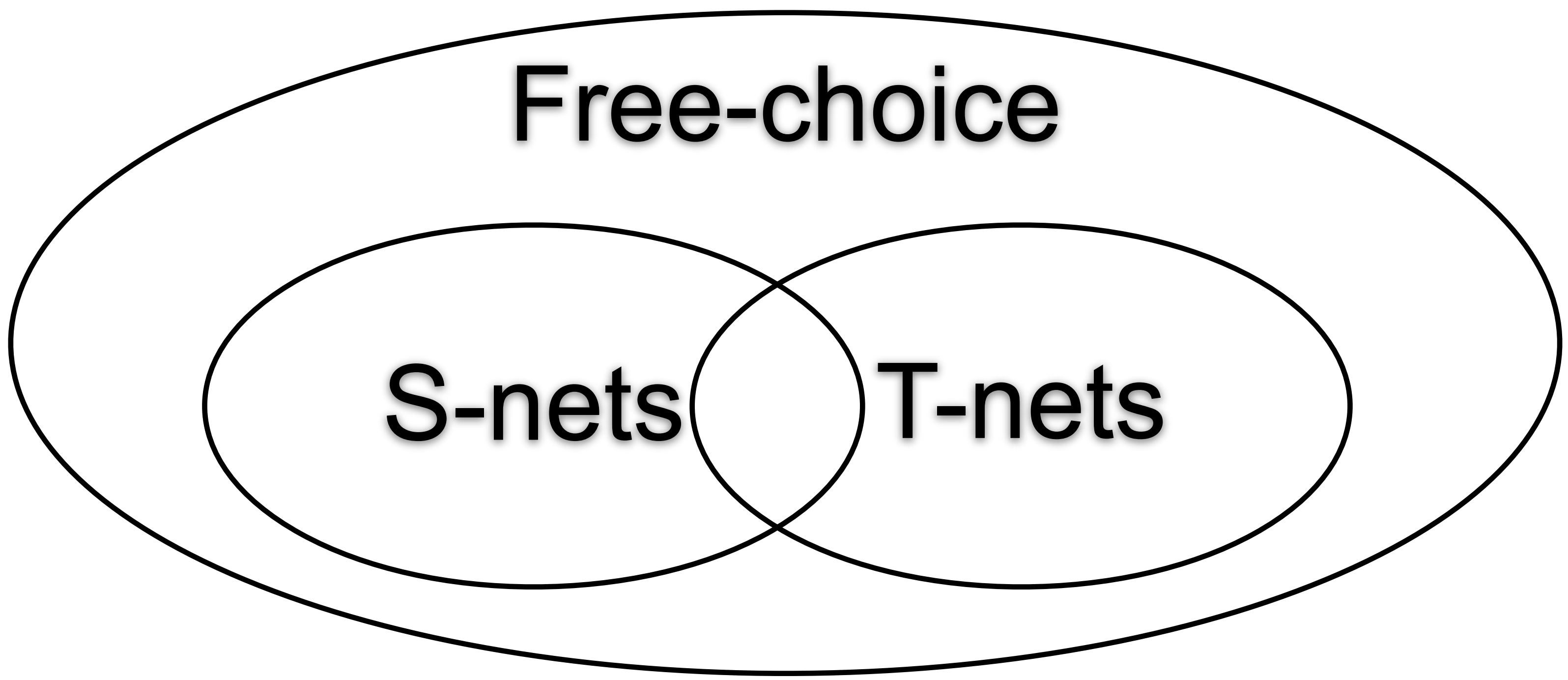
Exercises

Prove that every T-net is free-choice

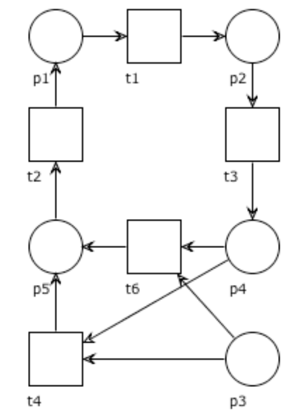
Let N be a T-net.
Take any two places p, q.
By definition of T-net their post-sets are singletons:
either they coincide
or they are different and thus disjoint

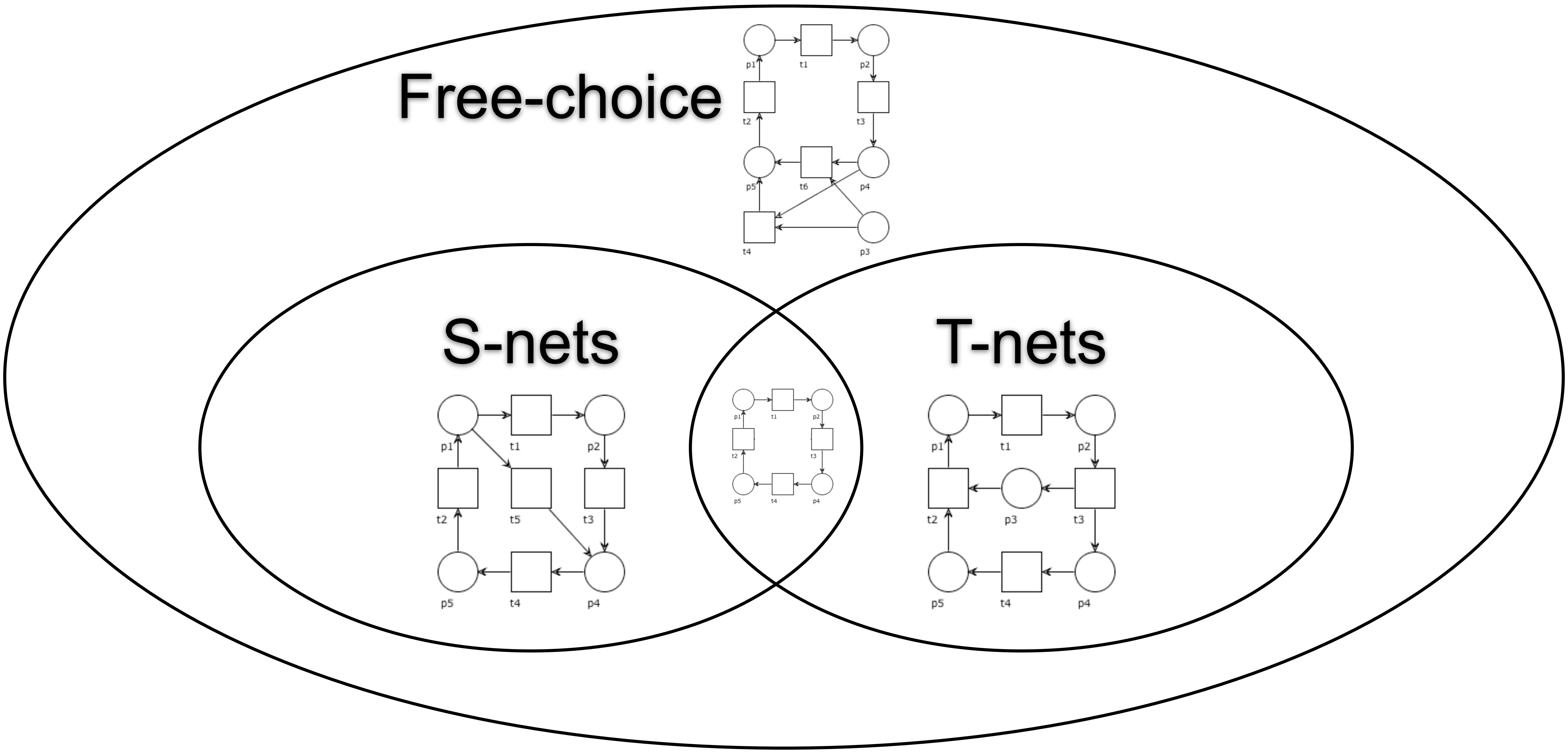
Exercises

Show a net for each area of the Eulero-Venn diagram below



Exercises





Fundamental property of free-choice nets

Proposition: Let (P, T, F, M_0) be free-choice.
If $M \xrightarrow{t}$ and $t \in p\bullet$, then $M \xrightarrow{t'}$ for every $t' \in p\bullet$.

The proof is trivial, by definition of free-choice net
($t, t' \in p\bullet$ implies $\bullet t = \bullet t'$)

Free-choice N*

Proposition: A workflow net N is free-choice
iff
N* is free-choice

N and N* differ only for the reset transition, whose pre-set (the final place o) is disjoint from the pre-set of any other transition

Liveness = Place liveness (in Free Choice systems)

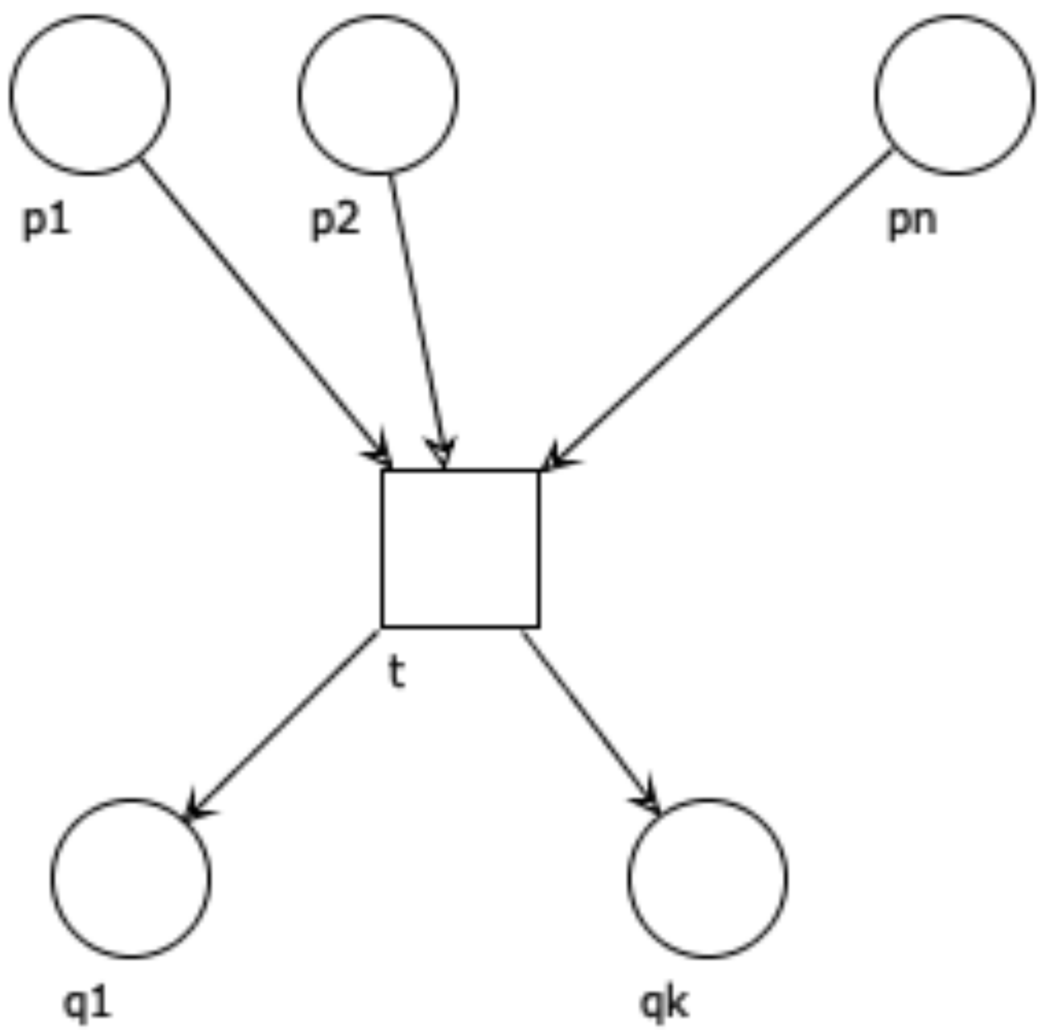
In any system:
liveness implies place-liveness
p dead implies any transition t in its pre/post-set is dead

It can be shown that
If a free-choice system is place-live, then it is live

Corollary:
A free-choice system is live **iff** it is place-live

FC Place-live implies FC Live (intuition)

From a reachable marking M we would like to enable t



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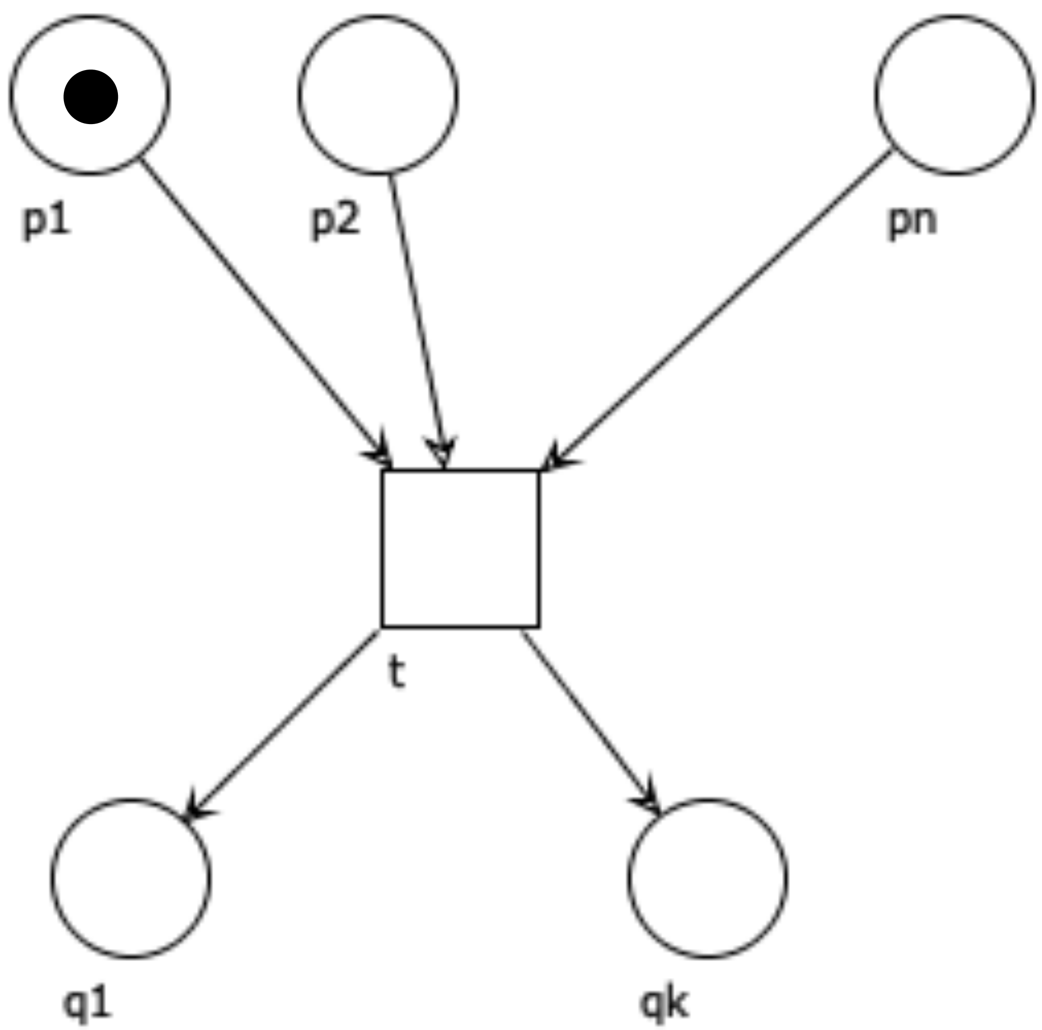

from M we can reach M_1 that marks p_1 (because place-live)

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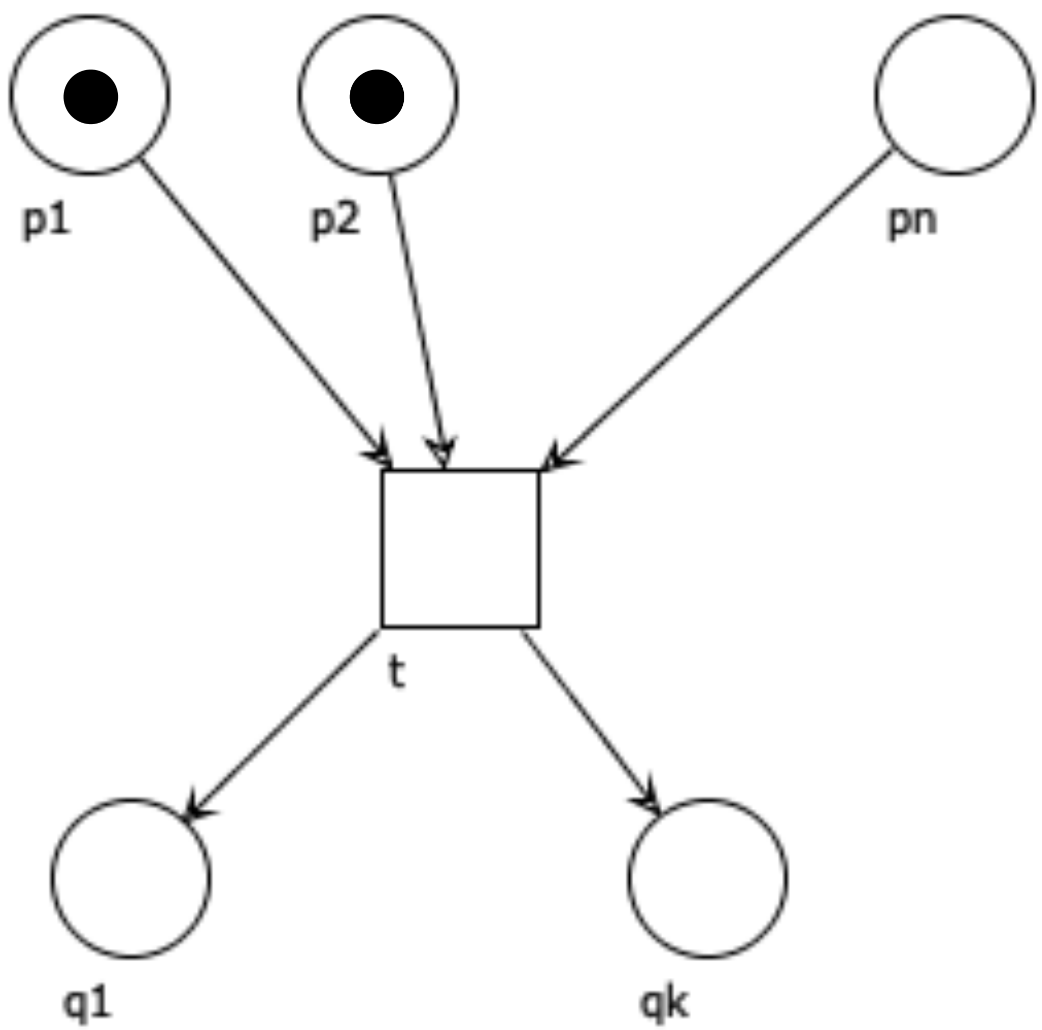


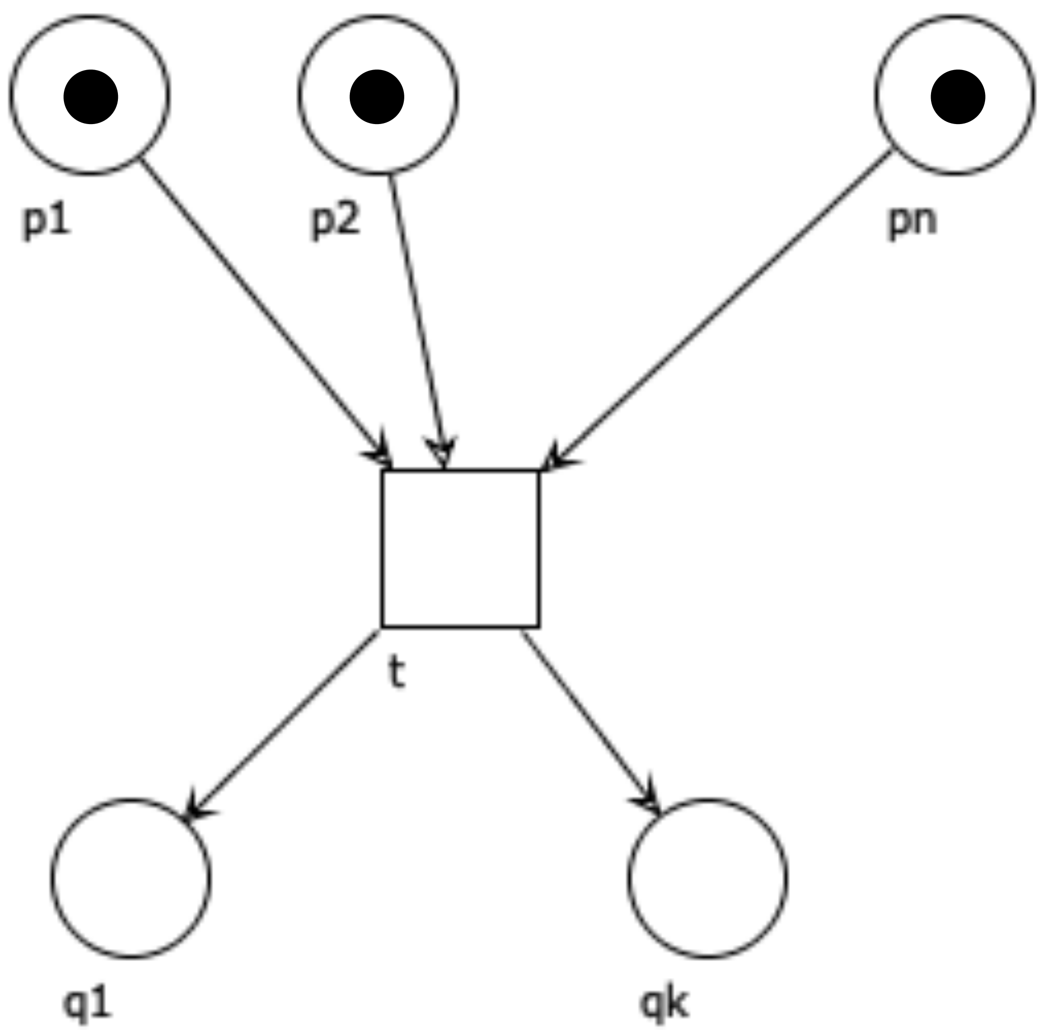
from M we can reach M_1 that marks p_1 (because place-live)

from M_1 we can reach M_2 that marks p_2 (because place-live)

Note: the token remains in p_1 (fundamental property of FC: if t' can remove a token from p_1, then t' has the same preset as t)

FC Place-live implies FC Live (intuition)

From a reachable marking M we would like to enable t



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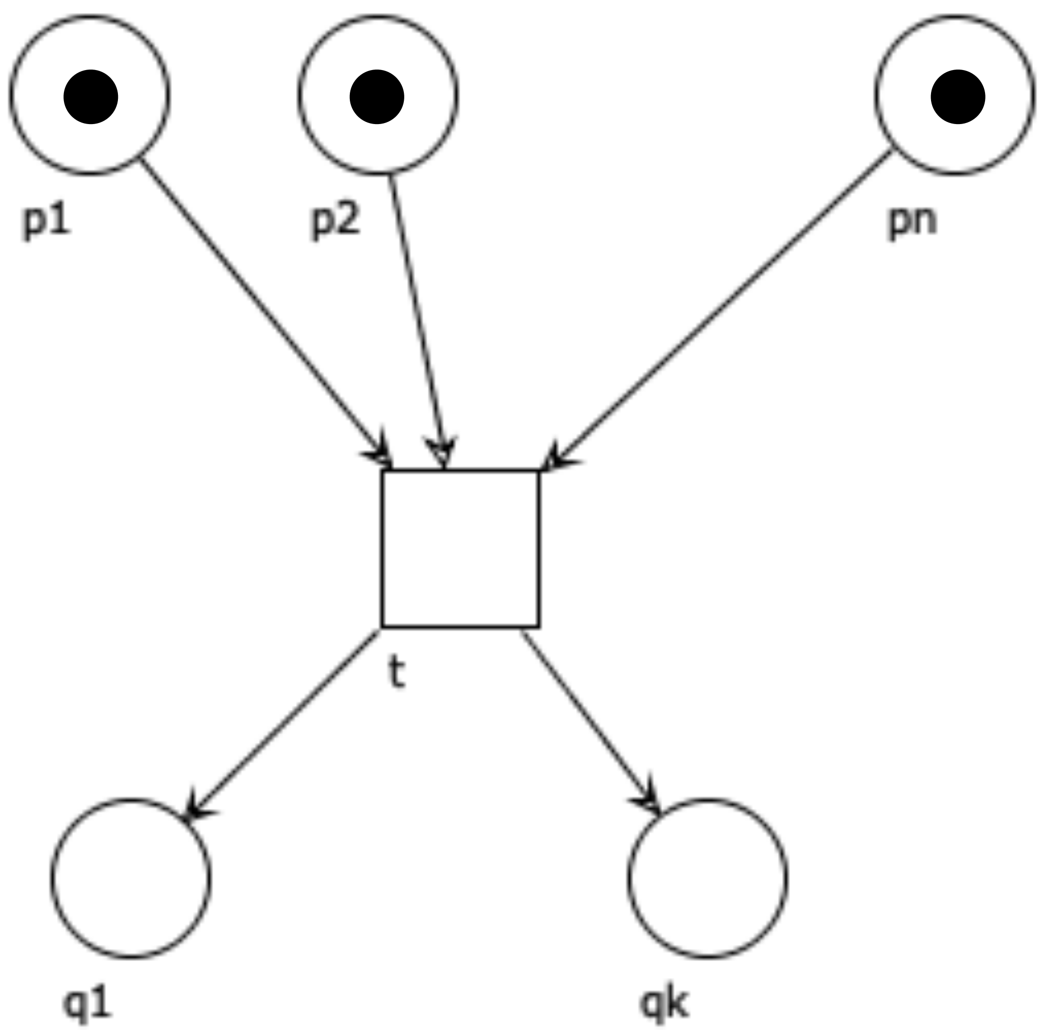
…

from M_{n-1} we can reach M_n that marks p_n (because place-live)

FC Place-live implies FC Live (intuition)

From a reachable marking M we would like to enable t



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…

from M_{n-1} we can reach M_n that marks p_n (because place-live)

from M we reach M_n that enables t !

Commoner's theorem (proof omitted)

Theorem:
A free-choice system is live
iff
every proper siphon includes an initially marked trap

What is a siphon?
What is a trap?

Is it computationally expensive
to show that a free-choice net is live?

A convenient concept: Stable set of markings

Stable set of markings

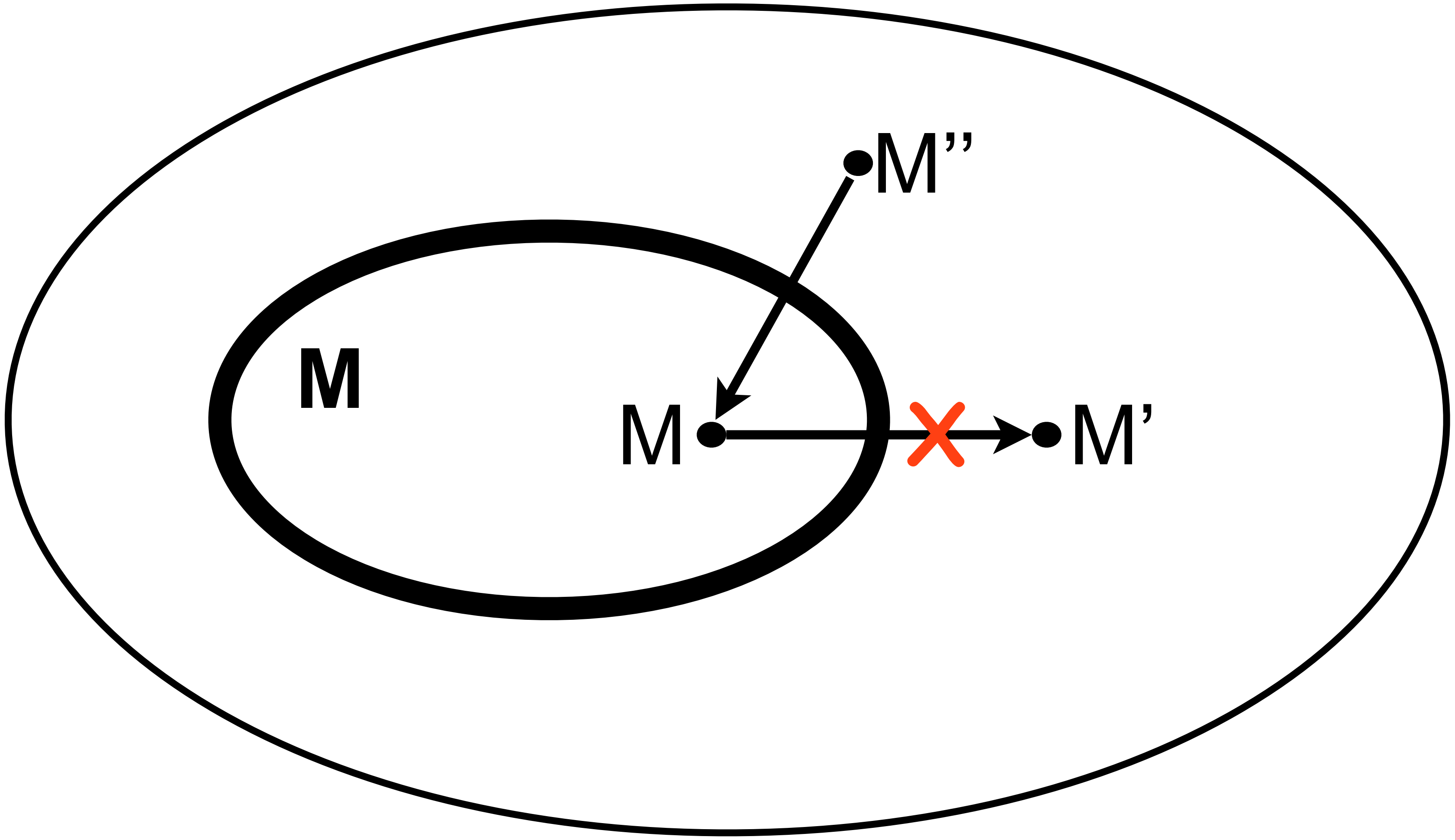
Definition: A set of markings $\mathbf{M}$ is called **stable** if

$$M \in \mathbf{M} \qquad \text{implies} \qquad [\, M \,\rangle \subseteq \mathbf{M}$$

(starting from any marking in the stable set **M**,
no marking outside **M** is reachable)

$[M_0\rangle$ is the least stable set that includes the marking M_0

Stable set of markings



(starting from any marking M in the stable set **M**, no marking M' outside **M** is reachable)

Stability check

$$\mathbf{M} \text{ is stable iff}$$
$$\forall M, t, M'. \, (M \in \mathbf{M} \;\wedge\; M \xrightarrow{t} M' \text{ implies } M' \in \mathbf{M})$$

Example

$$\forall M, t, M'. (M \in \mathbf{M} \ \wedge \ M \xrightarrow{t} M' \text{ implies } M' \in \mathbf{M})$$

Given a net system:

empty marking

Is the singleton set { $\mathbf{0}$ } a stable set?

Is the set of all markings a stable set?

Is the set of live markings a stable set?

Is the set of deadlock markings a stable set?

Is the set { $M \mid \mathbf{I} \cdot M = \mathbf{I} \cdot M_0$ } a stable set?

Example

$$\forall M, t, M'.\,(M \in \mathbf{M} \;\wedge\; M \xrightarrow{t} M' \text{ implies } M' \in \mathbf{M})$$

Given a net system:

empty marking

Is the singleton set { $\mathbf{0}$ } a stable set?

YES: no firing is possible

Is the set of all markings a stable set?

Is the set of live markings a stable set?

Is the set of deadlock markings a stable set?

Is the set { $M \mid \mathbf{I} \cdot M = \mathbf{I} \cdot M_0$ } a stable set?

Example

$$\forall M, t, M'. \left(M \in \mathbf{M} \ \wedge \ M \xrightarrow{t} M' \text{ implies } M' \in \mathbf{M}\right)$$

Given a net system:

Is the singleton set { $\mathbf{0}$ } a stable set?

YES

Is the set of all markings a stable set?

YES: it is not possible to leave the set of all markings

Is the set of live markings a stable set?

Is the set of deadlock markings a stable set?

Is the set { M | $\mathbf{I} \cdot M = \mathbf{I} \cdot M_0$ } a stable set?

Example

$$\forall M, t, M'. \, (M \in \mathbf{M} \;\wedge\; M \xrightarrow{t} M' \text{ implies } M' \in \mathbf{M})$$

Given a net system:

Is the singleton set { $\mathbf{0}$ } a stable set?

YES

Is the set of all markings a stable set?

YES

Is the set of live markings a stable set?

YES: liveness is an invariant

Is the set of deadlock markings a stable set?

Is the set $\{ M \mid \mathbf{I} \cdot M = \mathbf{I} \cdot M_0 \}$ a stable set?

Example

$$\forall M, t, M'. (M \in \mathbf{M} \ \wedge \ M \xrightarrow{t} M' \text{ implies } M' \in \mathbf{M})$$

Given a net system:

Is the singleton set { $\mathbf{0}$ } a stable set?

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YES

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YES

Is the set of deadlock markings a stable set?

YES: no firing is possible

Is the set { $M \mid \mathbf{I} \cdot M = \mathbf{I} \cdot M_0$ } a stable set?

Example

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Given a net system:

Is the singleton set { $\mathbf{0}$ } a stable set?

YES

Is the set of all markings a stable set?

YES

Is the set of live markings a stable set?

YES

Is the set of deadlock markings a stable set?

YES: no firing is possible

Is the set { $M \mid \mathbf{I} \cdot M = \mathbf{I} \cdot M_0$ } a stable set?

YES: fundamental property of invariants

Exercises

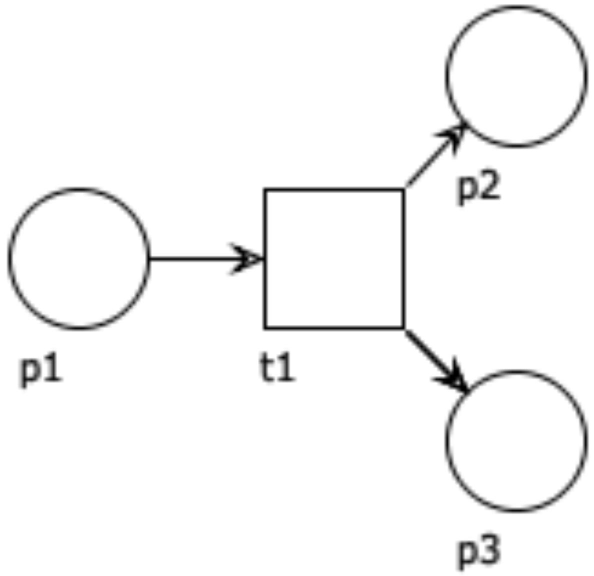
Given a net (P,T,F):

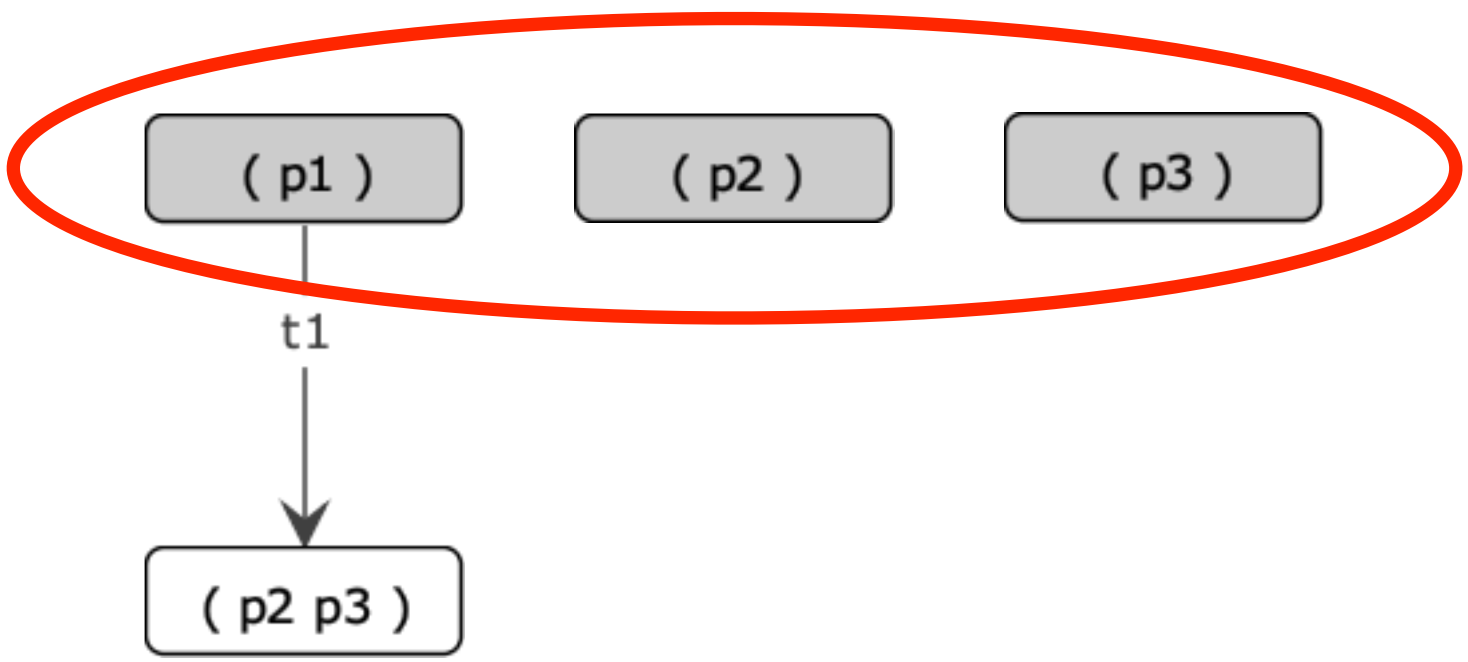
Show that the set $\{ M \mid M(P)=1 \}$ is not necessarily stable.

Exercises

Given a net (P,T,F):

Show that the set { M | M(P)=1 } is not necessarily stable.





What is a siphon?

Siphons, intuitively

A set of places R is a siphon if

all transitions that can produce tokens in the places of R

$$\bullet R \subseteq R \bullet$$

require some place in R to be marked

Therefore:

if no token is present in R,

then no token will ever be produced in R

Proper siphon

Definition:

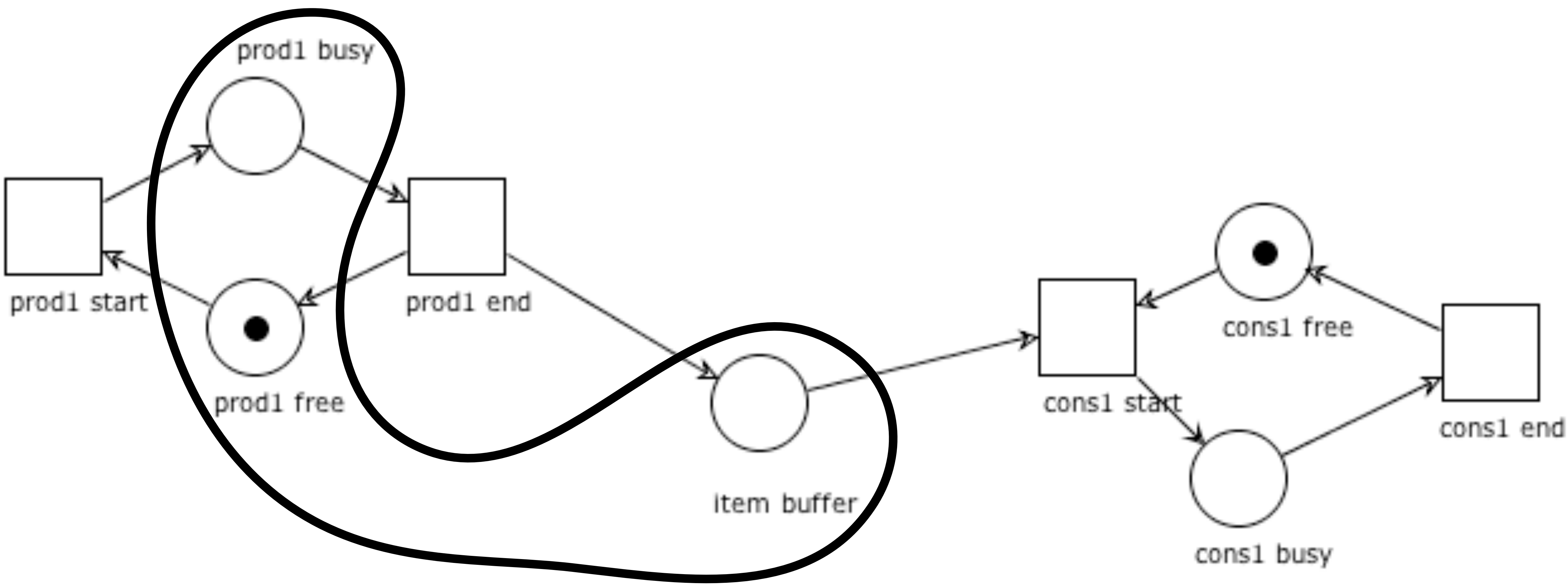
A set of places R is a **siphon** if $\bullet R \subseteq R\bullet$

It is a **proper siphon** if $R \neq \emptyset$

$\bullet R \subseteq R \bullet$

Siphon check: example

Is R = { prod1busy, prod1free, itembuffer} a siphon?



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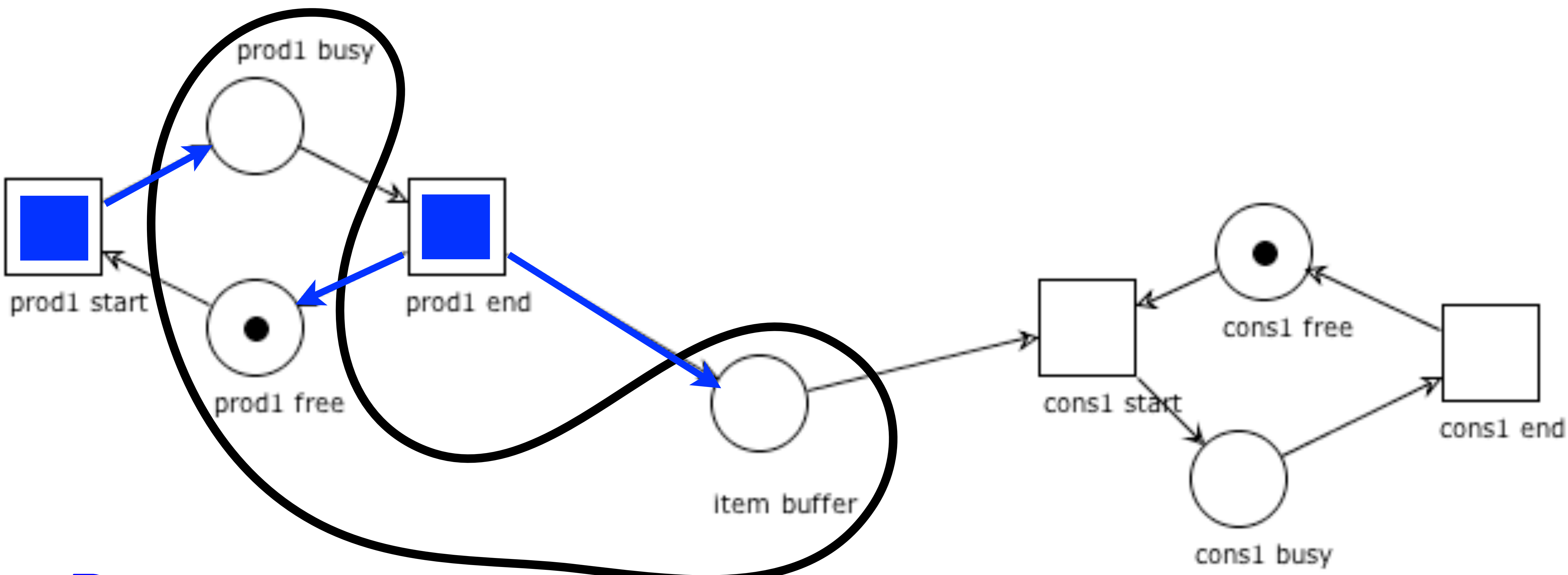

$\bullet R$

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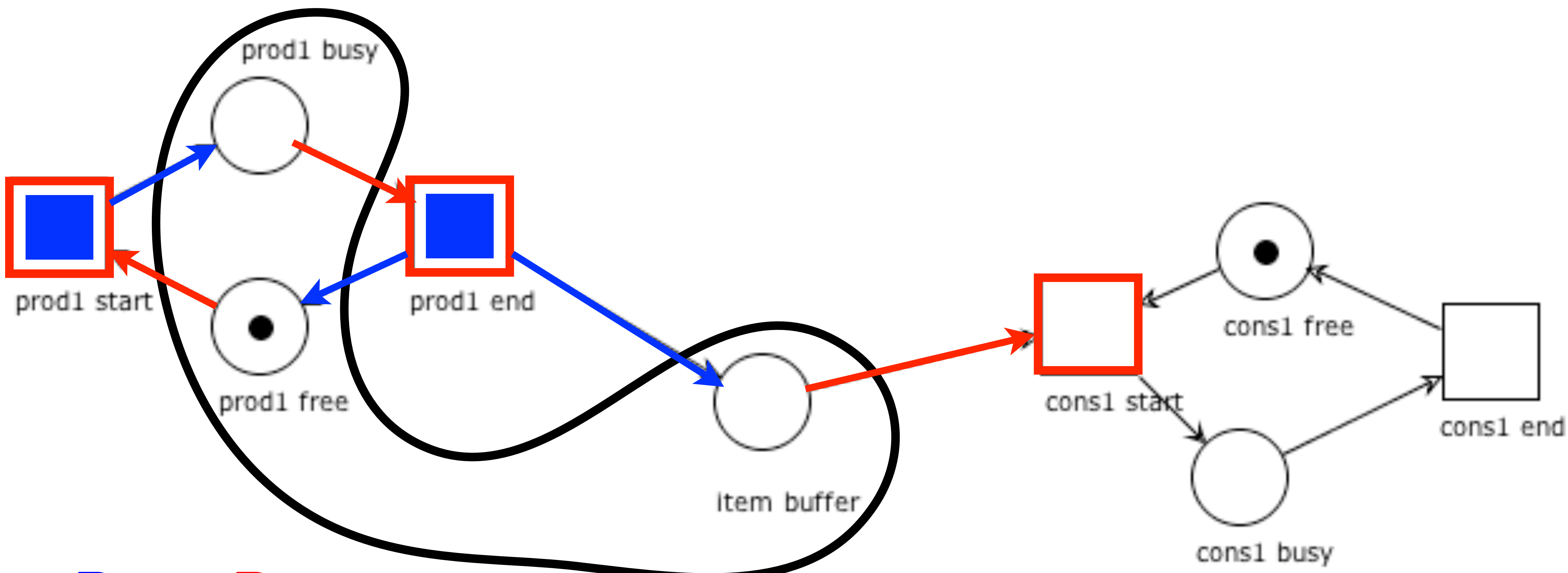

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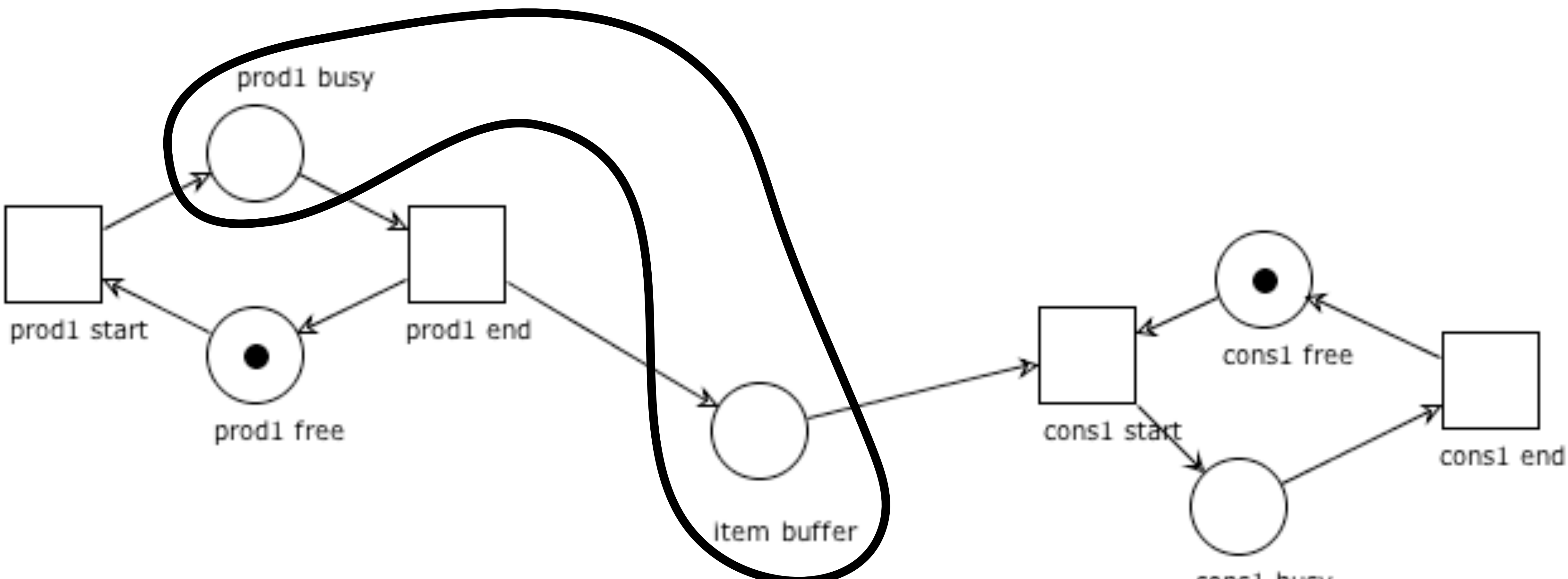

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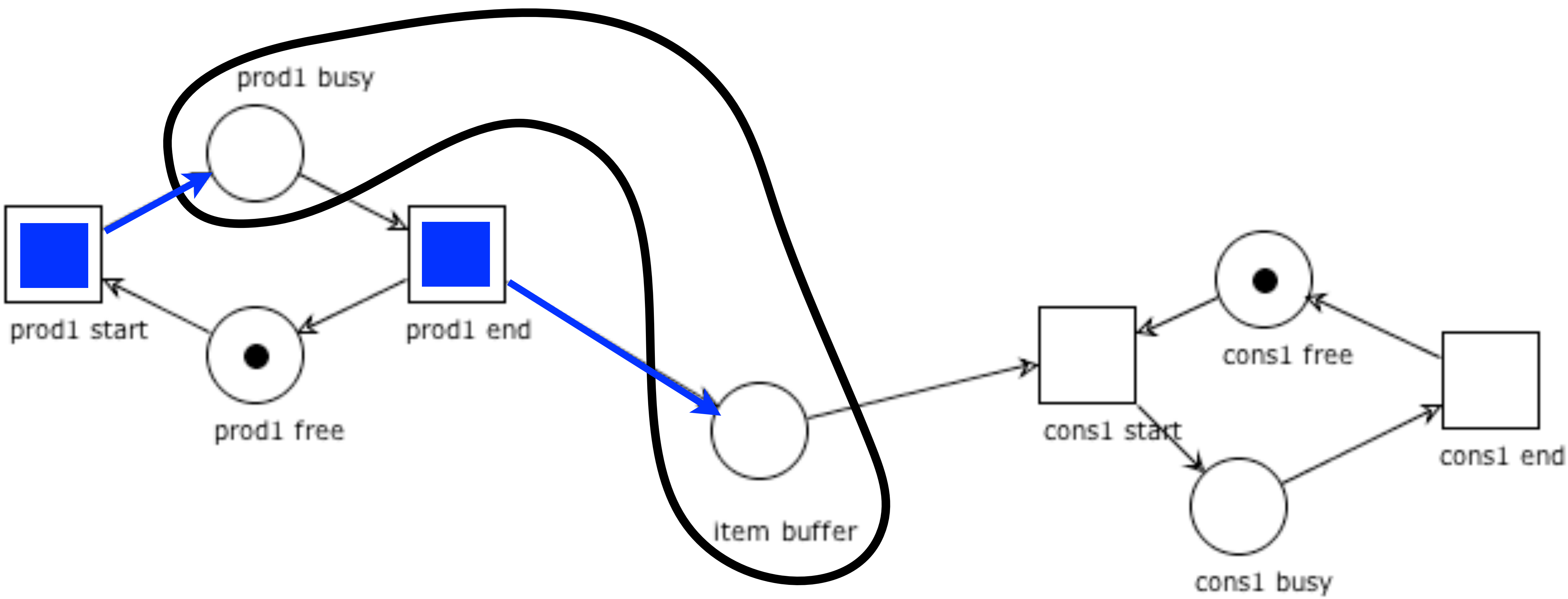

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Siphon check: example

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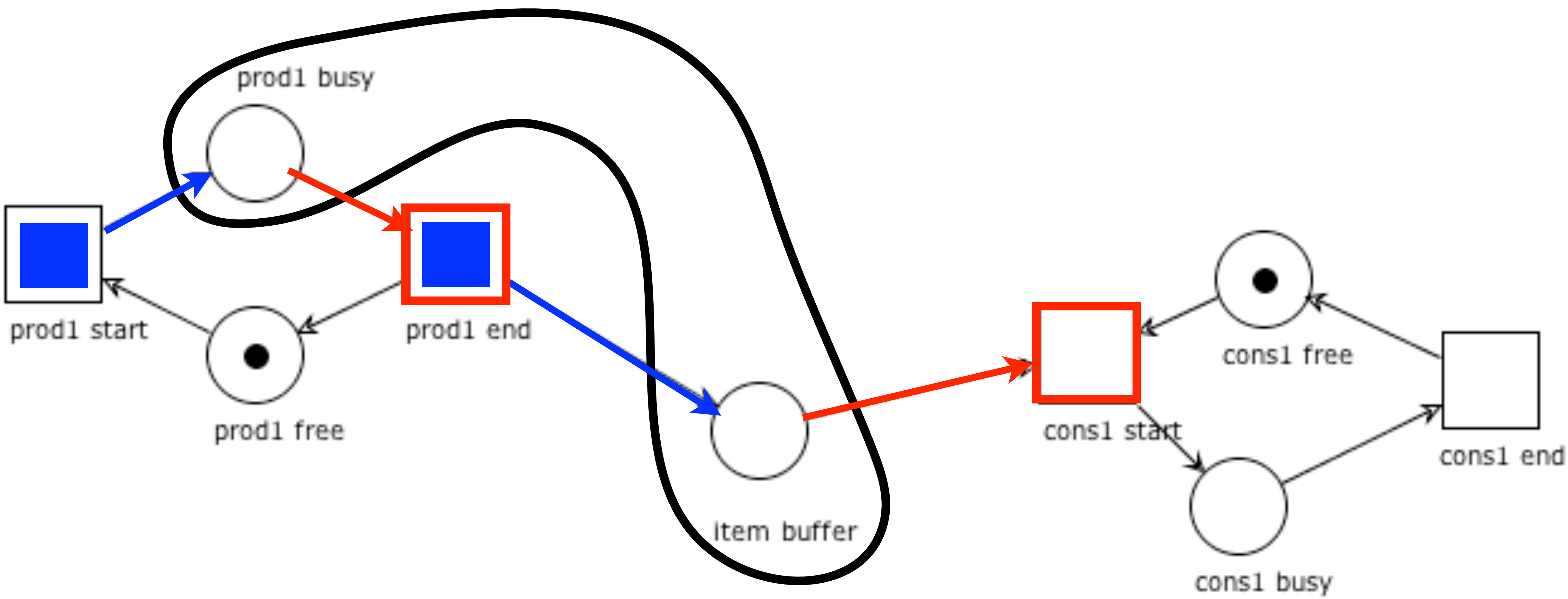


$\bullet R \not\subseteq R\bullet$

Fundamental property of siphons

Proposition: Unmarked siphons remain unmarked

Take a siphon R.
We just need to prove that the set of markings
M = { M | M(R)=0 }
is stable, which is immediate by definition of siphon

Corollary:
If a siphon R is marked at some reachable marking M, then it was initially marked at M_0

Siphons and liveness

Prop.: If a system is live any proper siphon R is marked

Take $p \in R$ and let $t \in \bullet p \cup p \bullet$

Since the system is live, then there are $M, M' \in [\, M_0 \,\rangle$ such that

$$M \stackrel{t}{\longrightarrow} M'$$

Therefore p is marked at either M or M'
Therefore R is marked at either M or M'
Therefore R was initially marked (at M_0)

Corollary: If a system has an unmarked proper siphon then it is not live

Commoner's theorem (proof omitted)

Theorem:
A free-choice system is live
iff
every proper siphon includes an initially marked trap
$\bullet R \subseteq R \bullet$

What is a trap?

Is it computationally expensive
to show that a free-choice net is live?

What is a trap?

Traps, intuitively

A set of places R is a trap if

all transitions that can consume tokens from R

$$\bullet R \supseteq R \bullet$$

produce some token in some place of R

Therefore:
if some token is present in R,
then it is never possible for R to become empty

Proper trap

Definition:

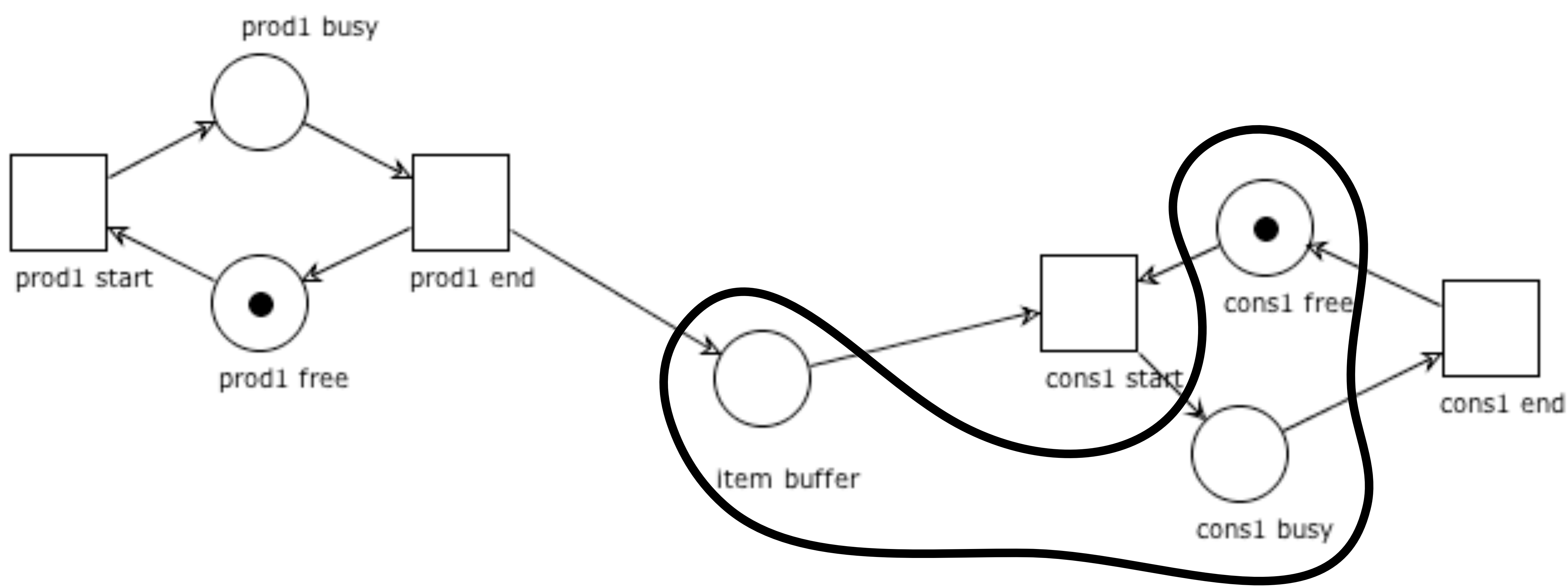
A set of places R is a **trap** if $\bullet R \supseteq R\bullet$

It is a **proper trap** if $R \neq \emptyset$

$\bullet R \supseteq R \bullet$

Trap check: example

Is R = { itembuffer, cons1busy, cons1free} a trap?



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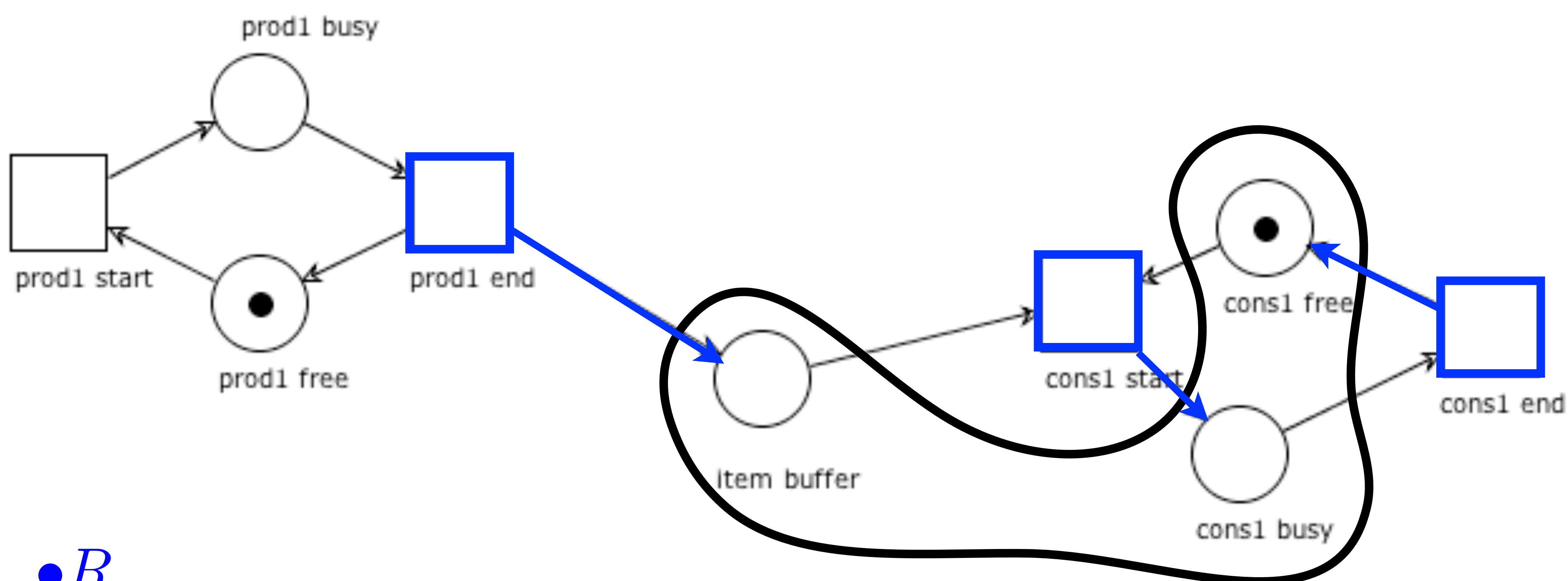

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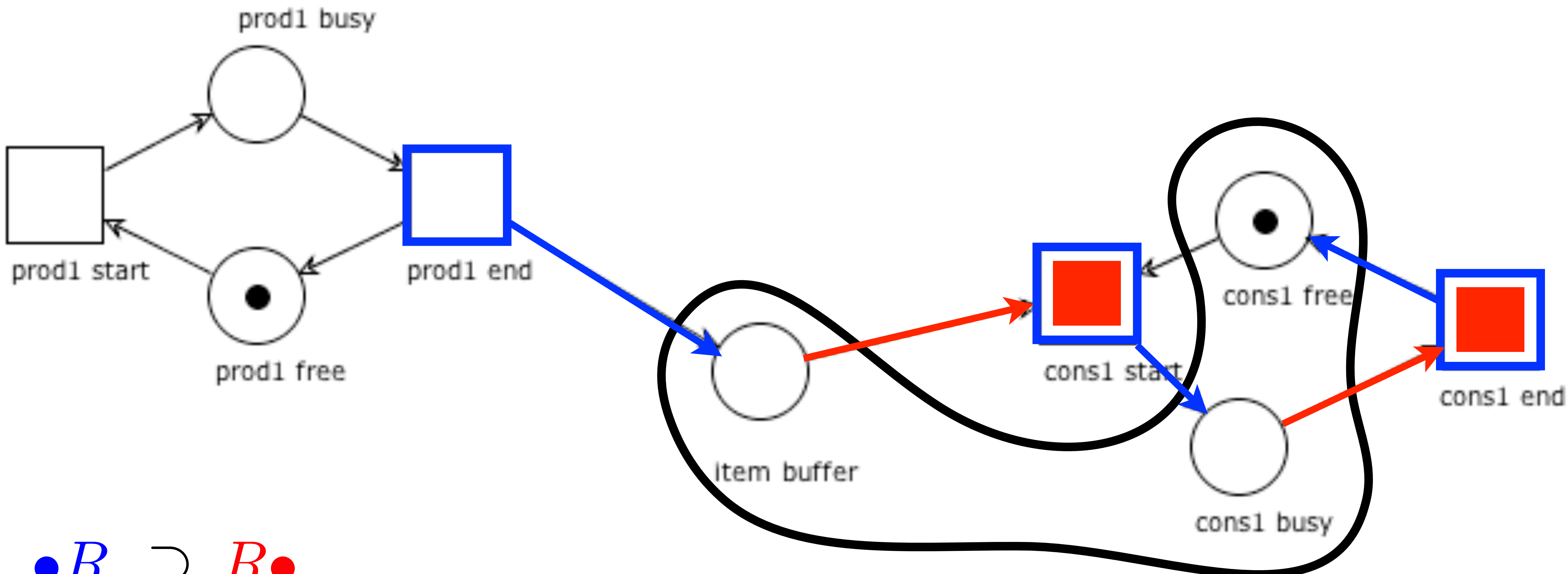

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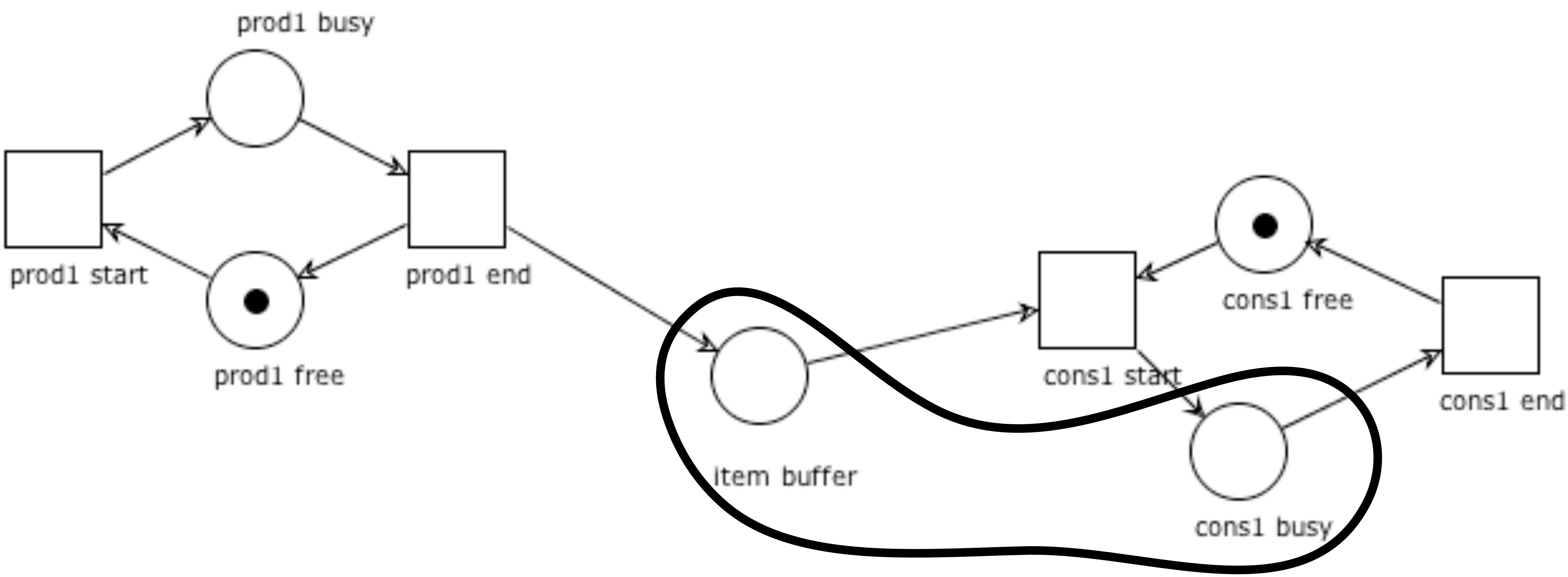

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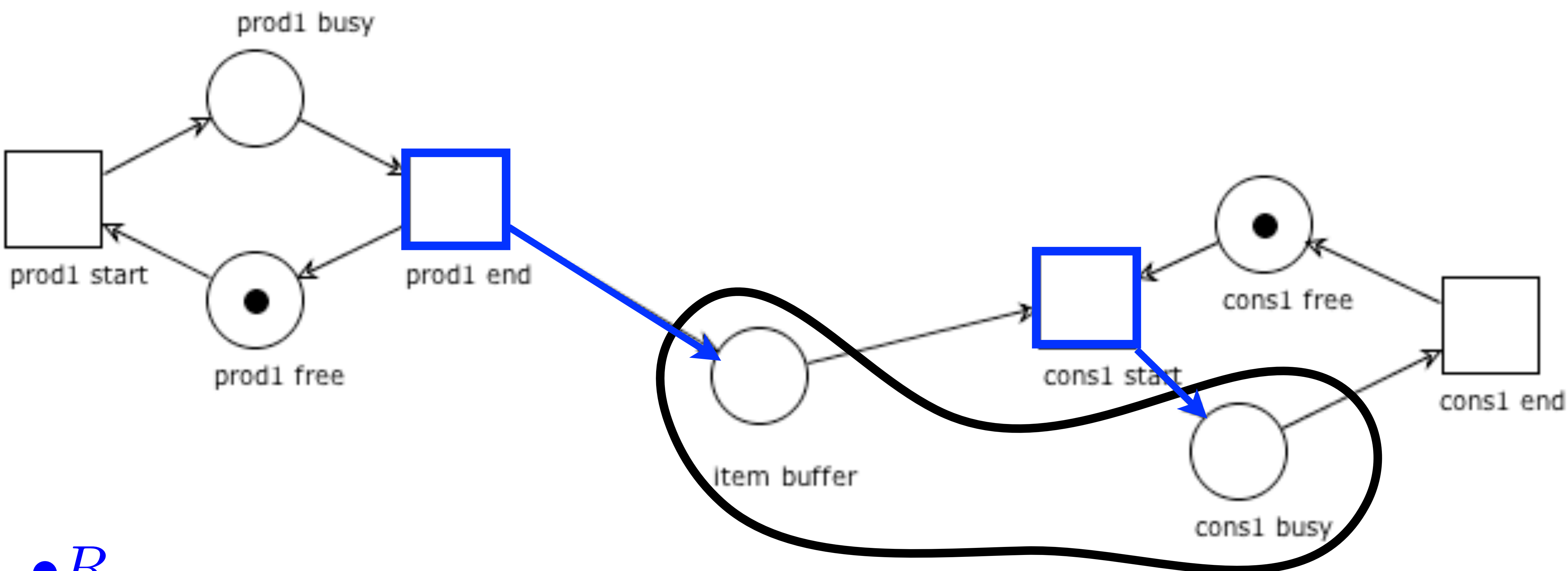

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Trap check: example

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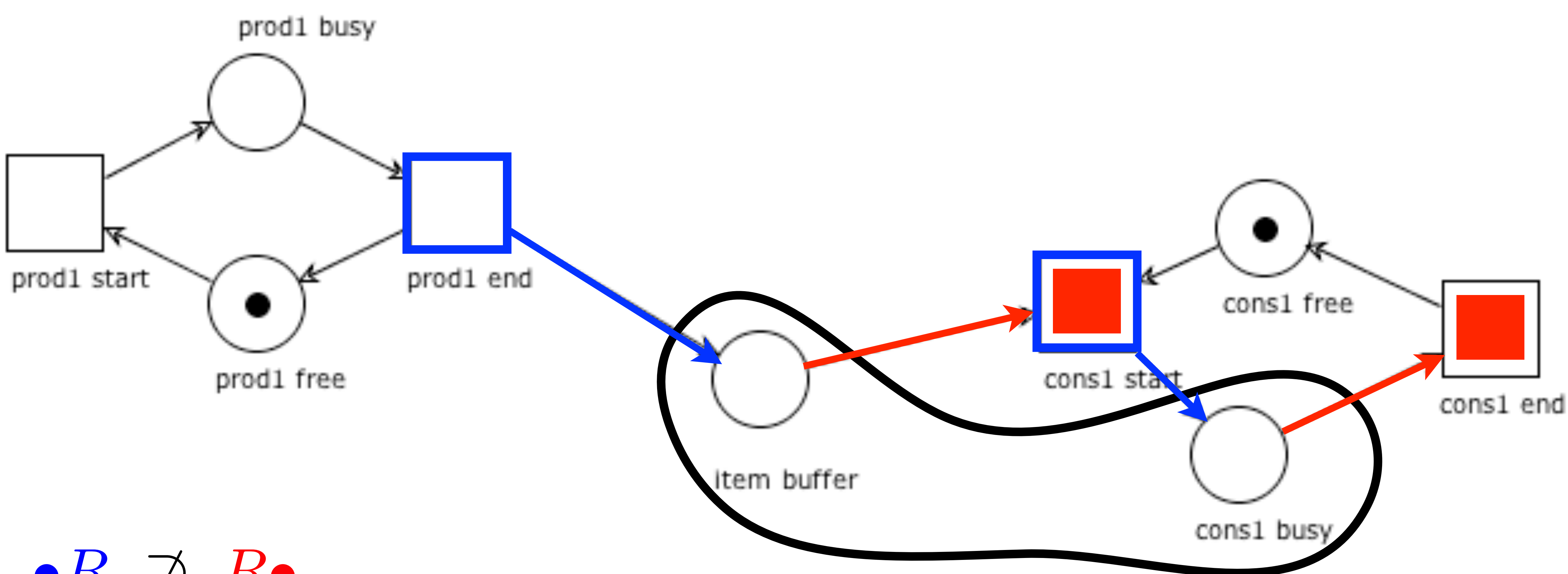

$\bullet R \not\supseteq R \bullet$

Fundamental property of traps

Proposition: Marked traps remain marked

Take a trap R.
We just need to prove that the set of markings
M = { M | M(R)>0 }
is stable, which is immediate by definition of trap

Corollary:
If a trap R is unmarked at some reachable marking M, then it was initially unmarked at M_0

Traps are closed under union

Lemma. The union of traps is a trap

Let X_1, X_2 be traps.
From $X_1\bullet \subseteq \bullet X_1$ and $X_2\bullet \subseteq \bullet X_2$ we have:

$$(X_1 \cup X_2)\bullet = X_1 \bullet \cup X_2\bullet \subseteq \bullet X_1 \cup \bullet X_2 = \bullet(X_1 \cup X_2)$$

Commoner's theorem (proof omitted)

Theorem:
A free-choice system is live
iff
every proper siphon includes an initially marked trap

$\bullet R \subseteq R \bullet$ $\bullet R \supseteq R \bullet$

Is it computationally expensive
to show that a free-choice net is live?

Note

It is easy to observe that every siphon includes a
(possibly empty) unique maximal trap
with respect to set inclusion
(the union of traps is a trap)

Moreover, a siphon includes a marked trap
iff
its maximal trap is marked

Commoner's theorem

Theorem:
A free-choice system is live
iff
every proper siphon includes an initially marked trap
iff
every proper siphon includes a marked maximal trap

Theorem:
A free-choice system is non-live
iff
there is a proper siphon whose maximal trap is unmarked

Complexity issues 1:
Is it hard to show that a free-choice net is live?

Commoner's theorem

Theorem:
A free-choice system is non-live
iff
there is a proper siphon whose maximal trap is unmarked

We show that
the non-liveness problem for free-choice systems
is NP-complete

A non-deterministic algorithm for non-liveness

1. guess a set of places R
   [polynomial time, non-deterministic step]

2. check if R is a siphon ($\bullet R \subseteq R\bullet$)
   [polynomial time]

3. if R is a siphon, compute the maximal trap $Q \subseteq R$
   [complexity?]

4. if $M_0(Q)=0$, then answer "non-live", otherwise "live"
   [polynomial time]

A polynomial algorithm for maximal trap in a siphon

$\bullet R \subseteq R \bullet$ $\bullet Q \supseteq Q \bullet$

3. if R is a siphon, compute the maximal trap Q ⊆ R

Input: A net $N = (P, T, F)$ and $R \subseteq P$
Output: $Q \subseteq R$ maximal trap in R

$Q := R$
while $(\exists p \in Q,\ \exists t \in p\bullet,\ t \notin \bullet Q)$
$\quad Q := Q \setminus \{p\}$
return Q

A polynomial non-det. algorithm for non-liveness

1. guess a set of places R
   [polynomial time, non-deterministic step]

2. check if R is a siphon ($\bullet R \subseteq R\bullet$)
   [polynomial time]

3. if R is a siphon, compute the maximal trap $Q \subseteq R$
   [polynomial time]

4. if $M_0(Q)=0$, then answer “non-live”, otherwise "live"
   [polynomial time]

Non-liveness for f.c. nets is in NP

The non-liveness problem for free-choice systems is in NP

Is the same problem in P?

The corresponding deterministic algorithm cannot make the guess in step 1

It has to explore all possible subsets of places
$2^{|P|}$ cases!

NP-completeness

We next sketch the proof of the reduction to non-liveness in a free-choice net of the CNF-SAT problem

(SATisfiability problem for propositional formulas in Conjunctive Normal Form)

CNF-SAT is an NP-complete problem

CNF-SAT decision problem

Variables: $x_1, x_2, ..., x_n$

Literals: $x_1, \bar{x}_1, x_2, \bar{x}_2, ..., x_n, \bar{x}_n$

Clause: disjunction of literals

Formula: conjunction of clauses

Example: $\phi = (x_1 \vee \bar{x_3}) \wedge (x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_2 \vee \bar{x}_3)$

Is there an assignment of boolean values to the variables such that $\phi = true$?

The free-choice net of a formula

Given a formula ϕ, the idea is
to construct a free-choice system (P,T,F,M_0)
and show that

the formula ϕ is satisfiable
iff
(P,T,F,M_0) is not live

The free-choice net of a formula

Given a formula ϕ, the idea is

to construct a free-choice system (P,T,F,M_0)
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the formula ϕ is not satisfiable

iff

(P,T,F,M_0) is live

CNF-SAT formulas

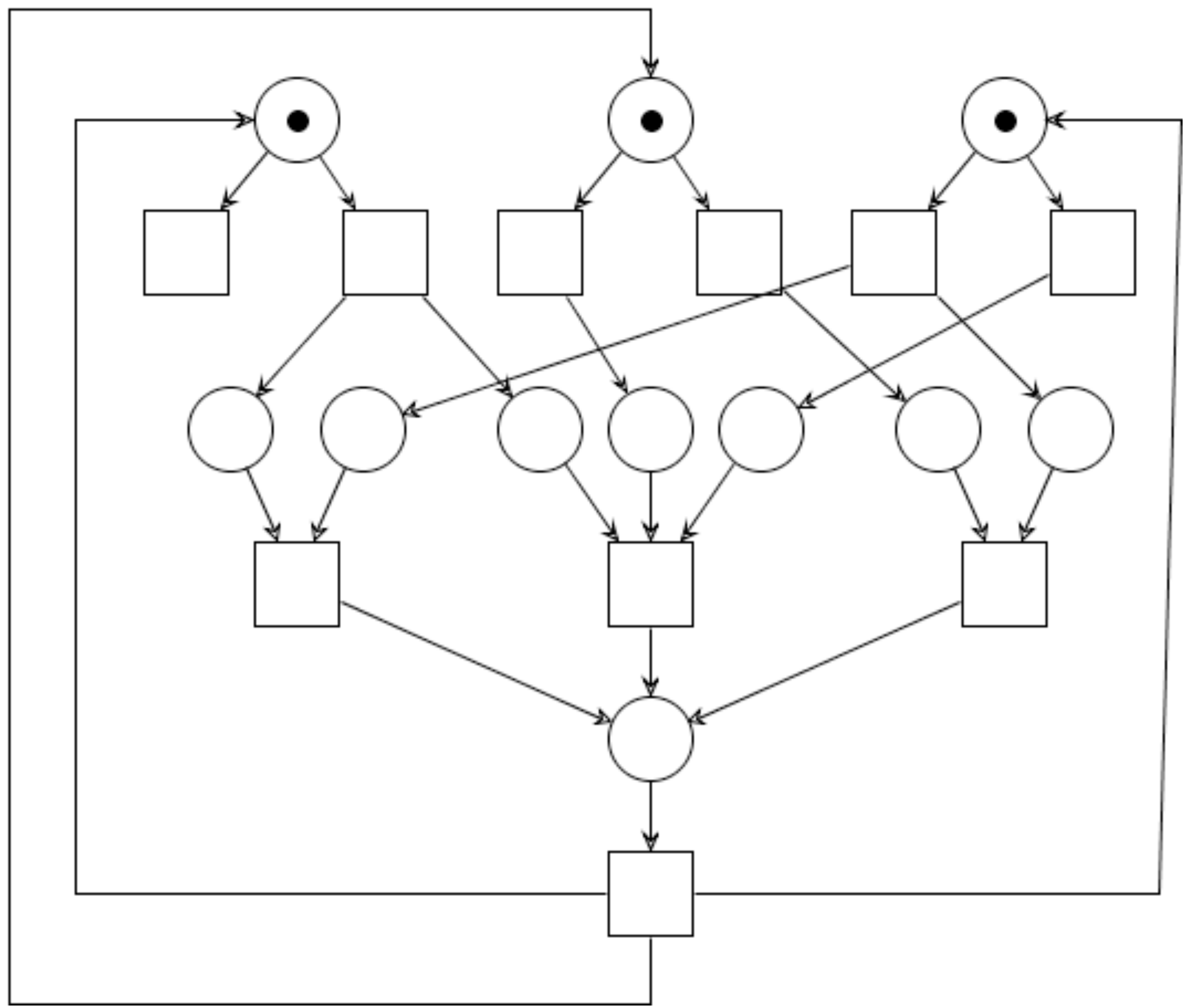
Is there an assignment of boolean values to the variables such that $\phi = true$?

Is there an assignment of boolean values to the variables such that $\neg\phi = false$?

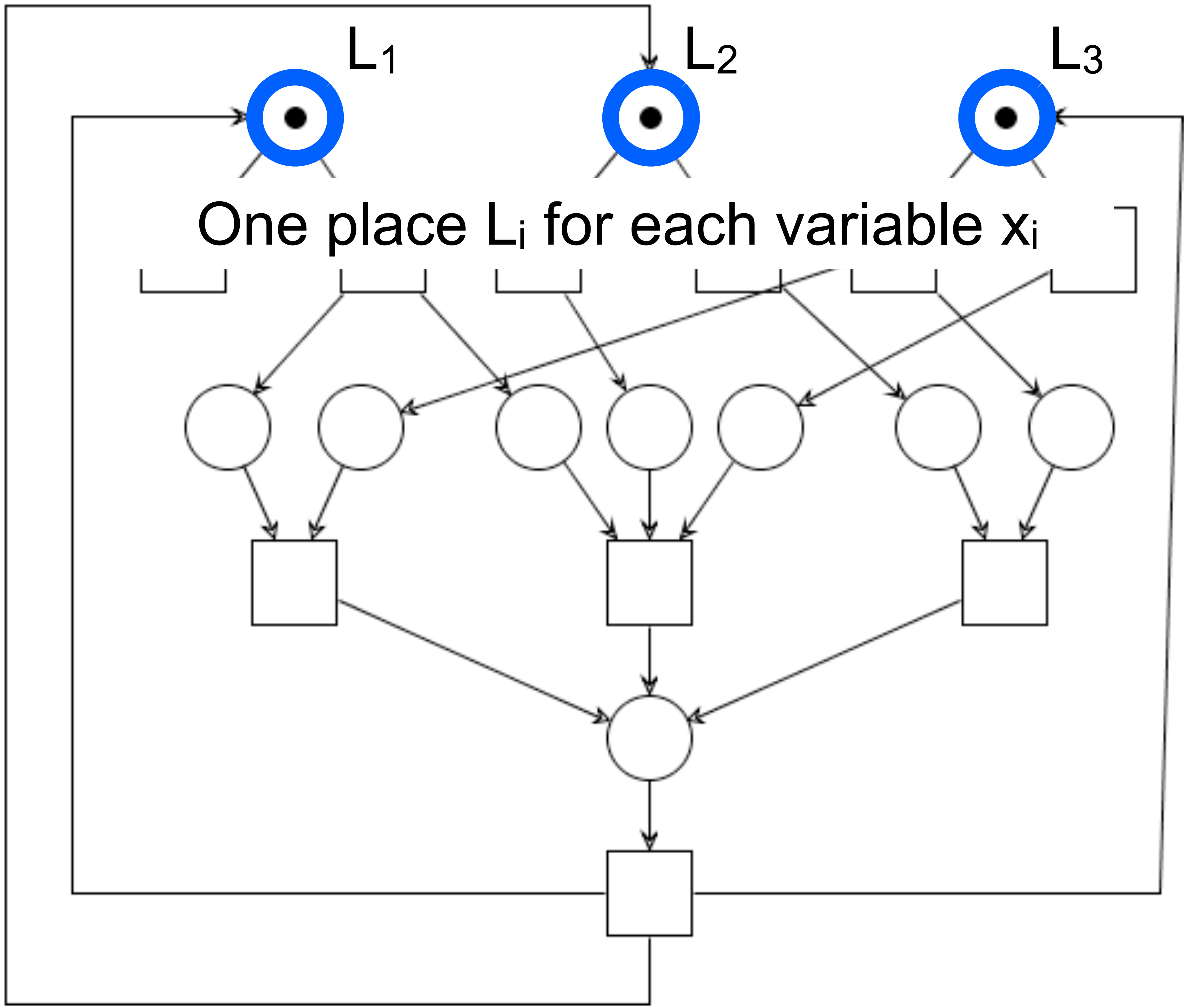
$$\phi = (x_1 \vee \overline{x}_3) \wedge (x_1 \vee \overline{x}_2 \vee x_3) \wedge (x_2 \vee \overline{x}_3)$$

$$\neg\phi = (\overline{x}_1 \wedge x_3) \vee (\overline{x}_1 \wedge x_2 \wedge \overline{x}_3) \vee (\overline{x}_2 \wedge x_3)$$

$$\neg\phi = (\overline{x}_1 \wedge x_3) \vee (\overline{x}_1 \wedge x_2 \wedge \overline{x}_3) \vee (\overline{x}_2 \wedge x_3)$$



$$\neg\phi = (\overline{x}_1 \wedge x_3) \vee (\overline{x}_1 \wedge x_2 \wedge \overline{x}_3) \vee (\overline{x}_2 \wedge x_3)$$



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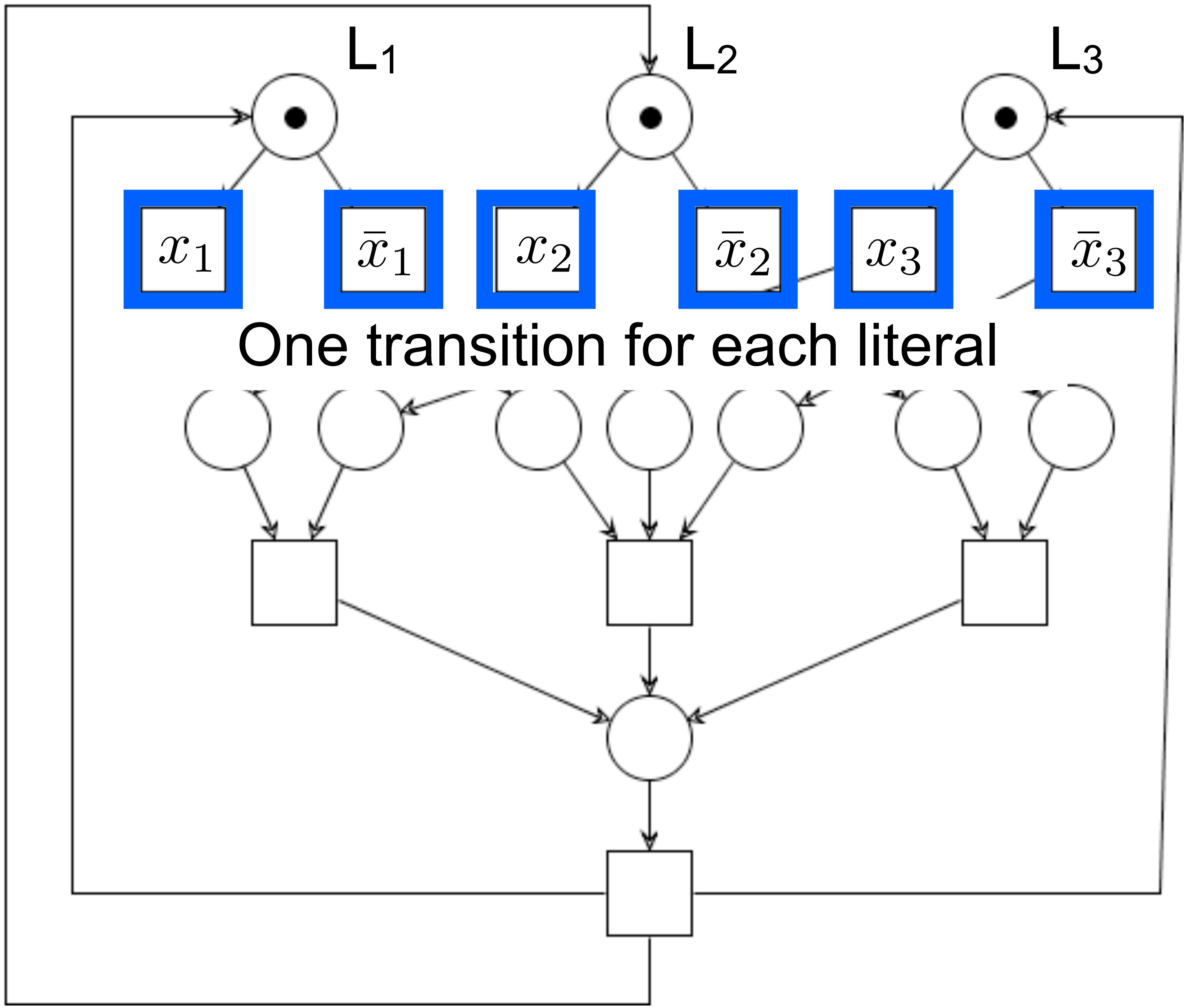

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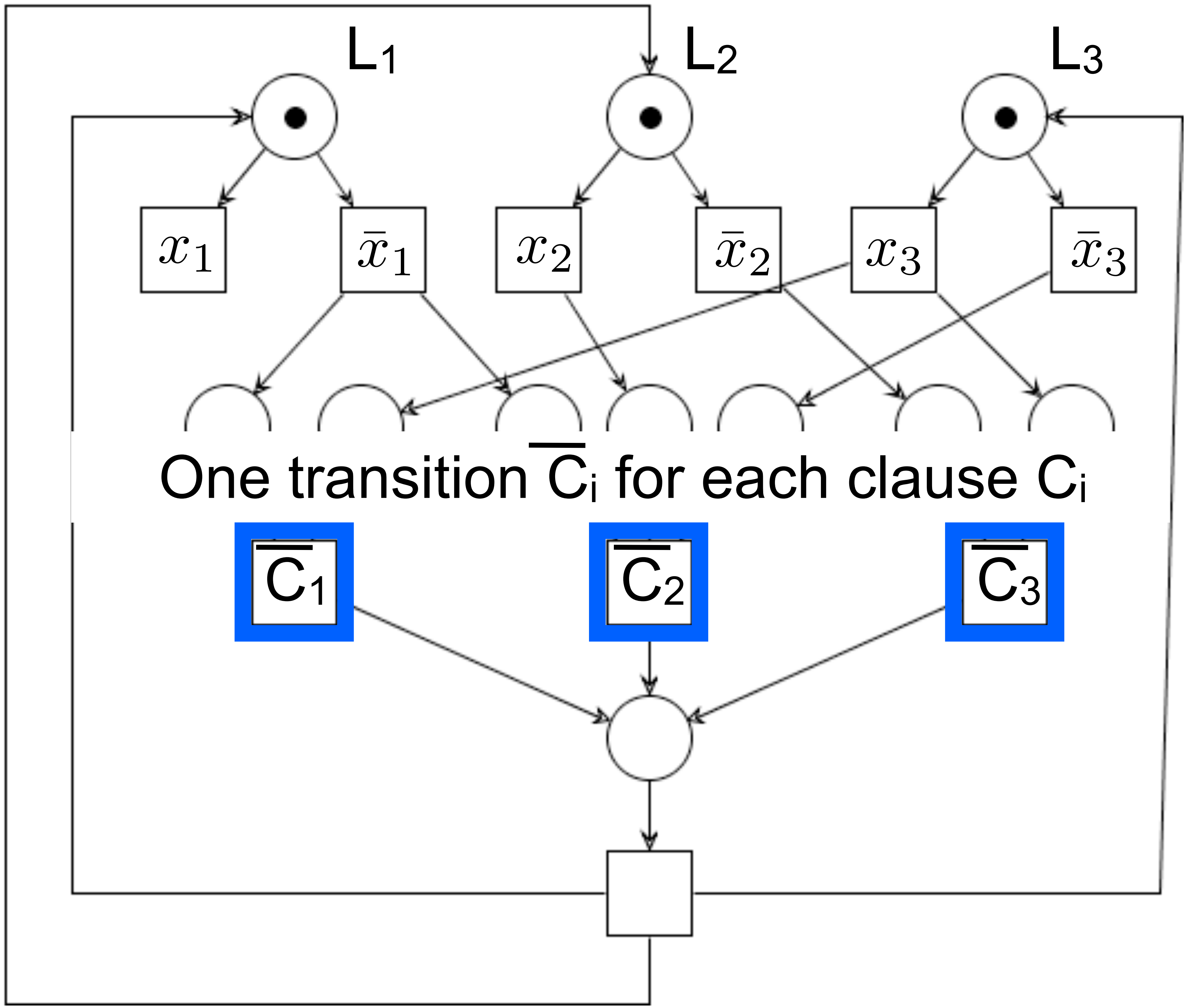

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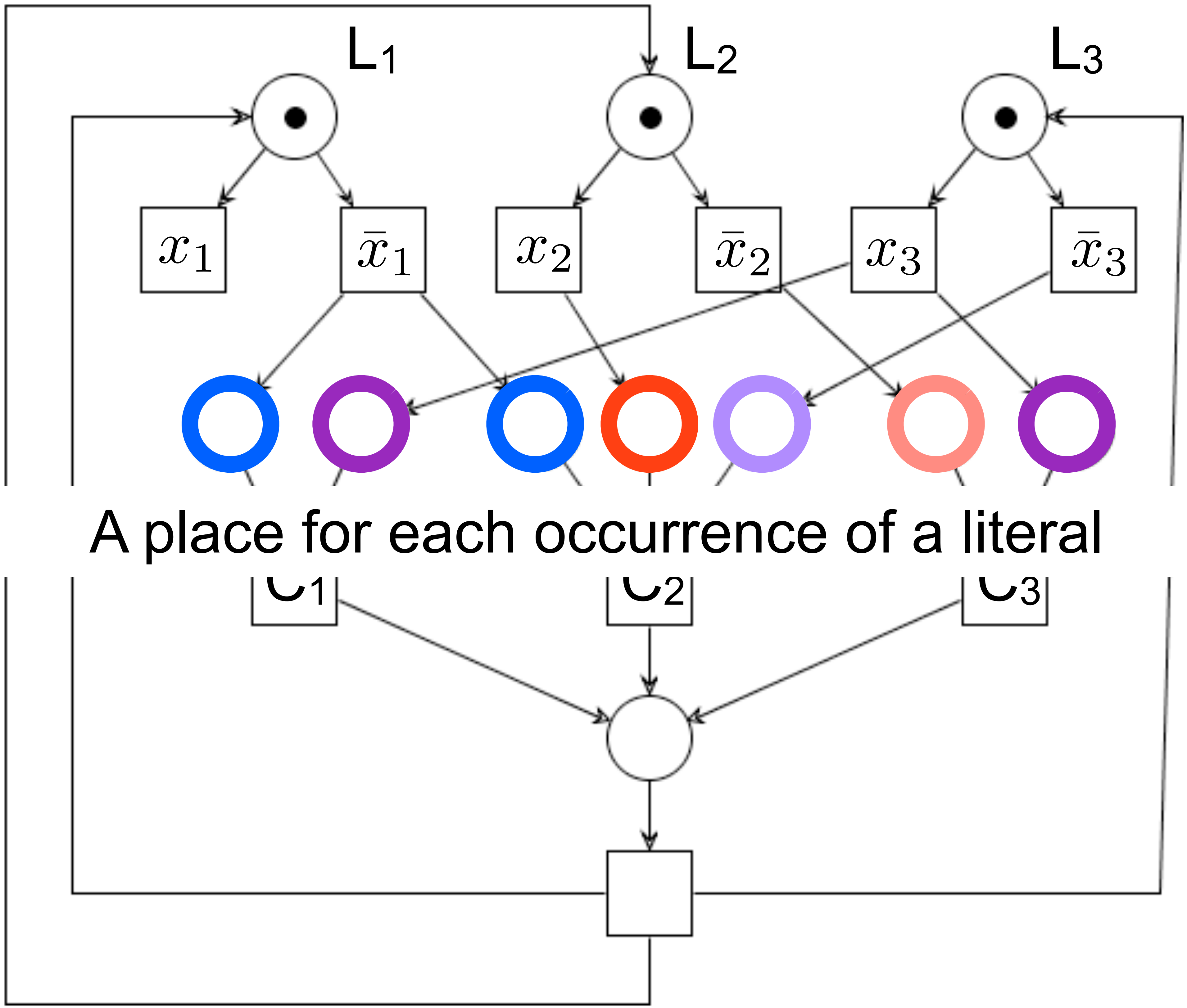
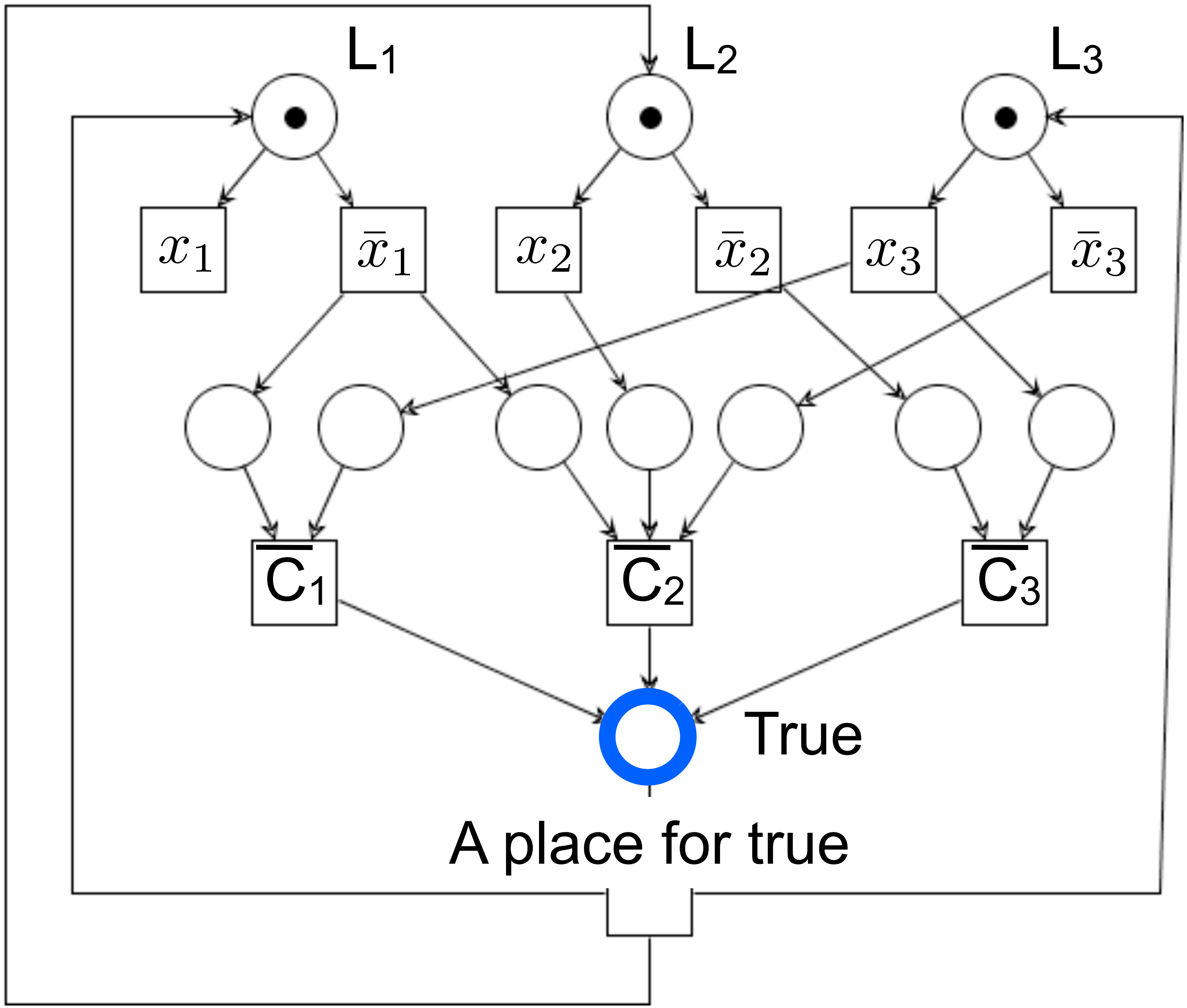


A place for each occurrence of a literal

$$\neg\phi = (\overline{x}_1 \wedge x_3) \vee (\overline{x}_1 \wedge x_2 \wedge \overline{x}_3) \vee (\overline{x}_2 \wedge x_3)$$



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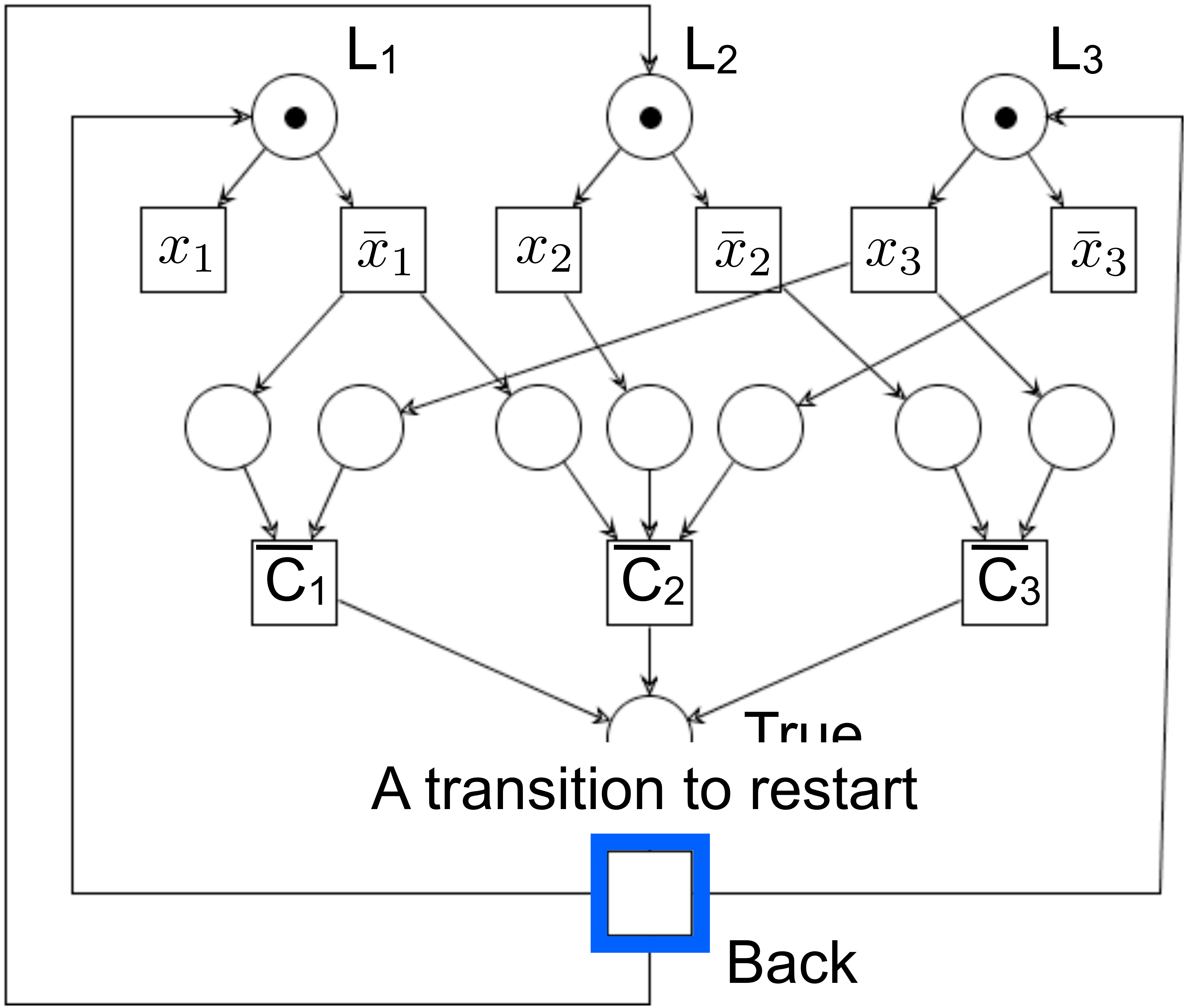

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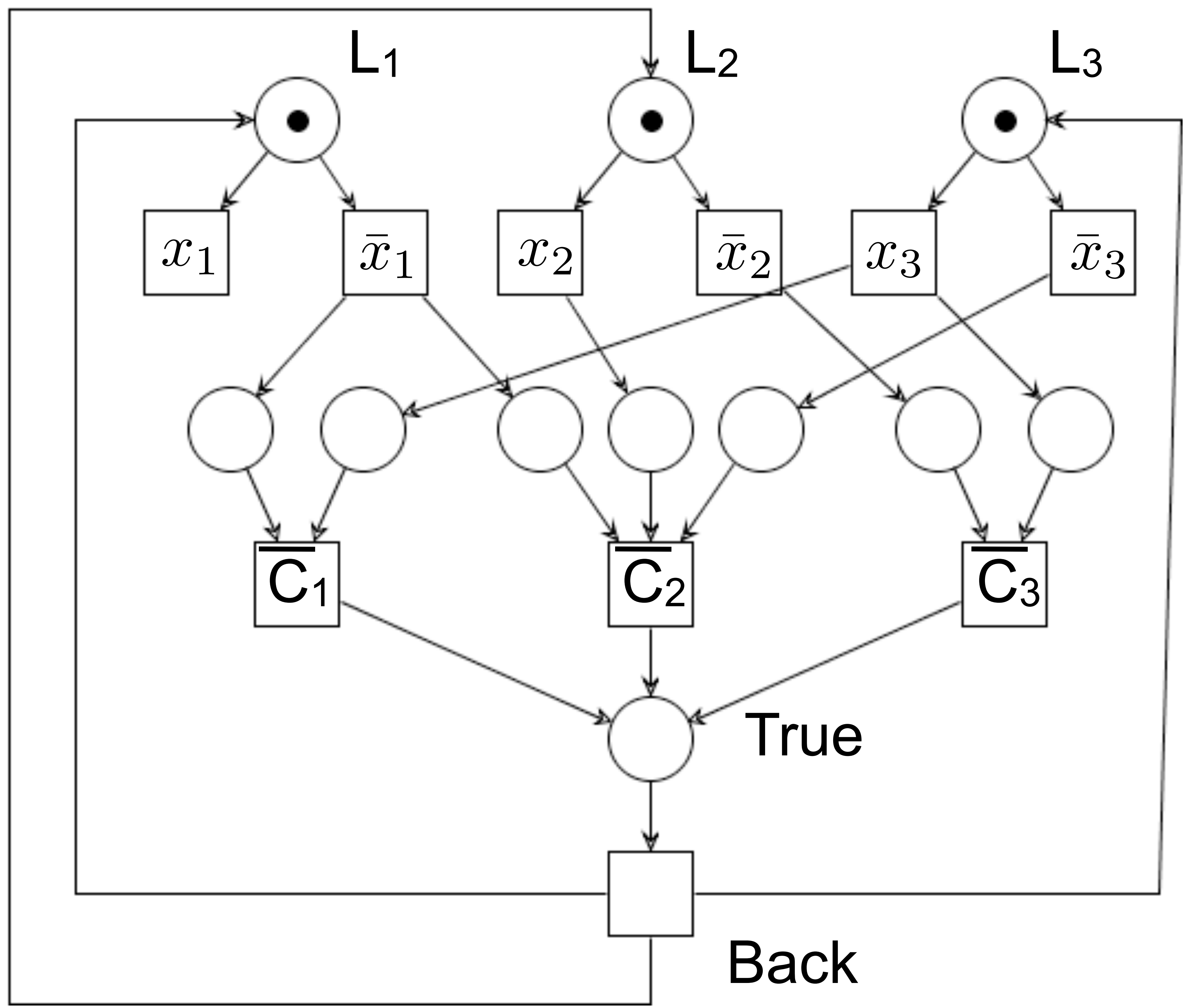

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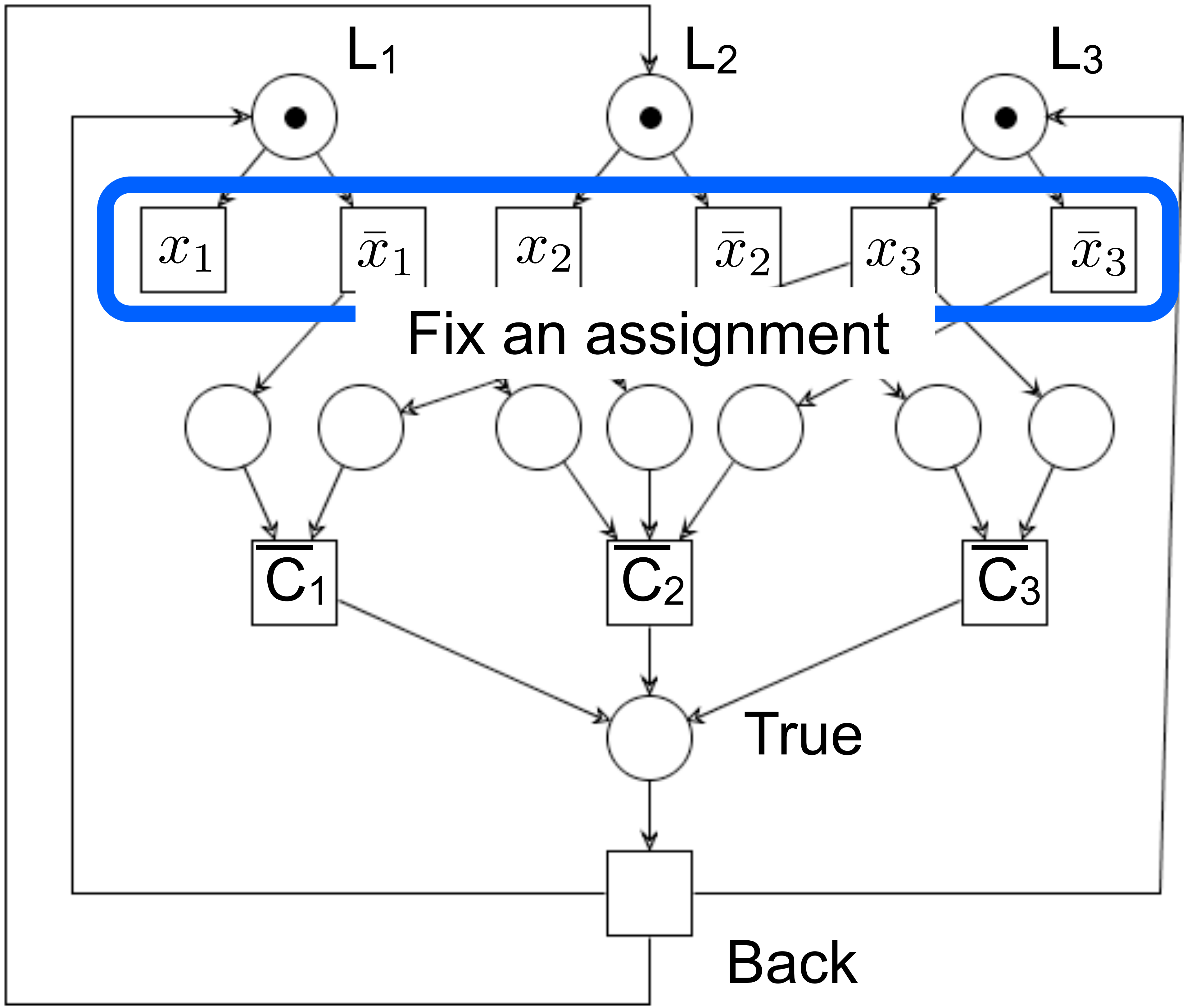

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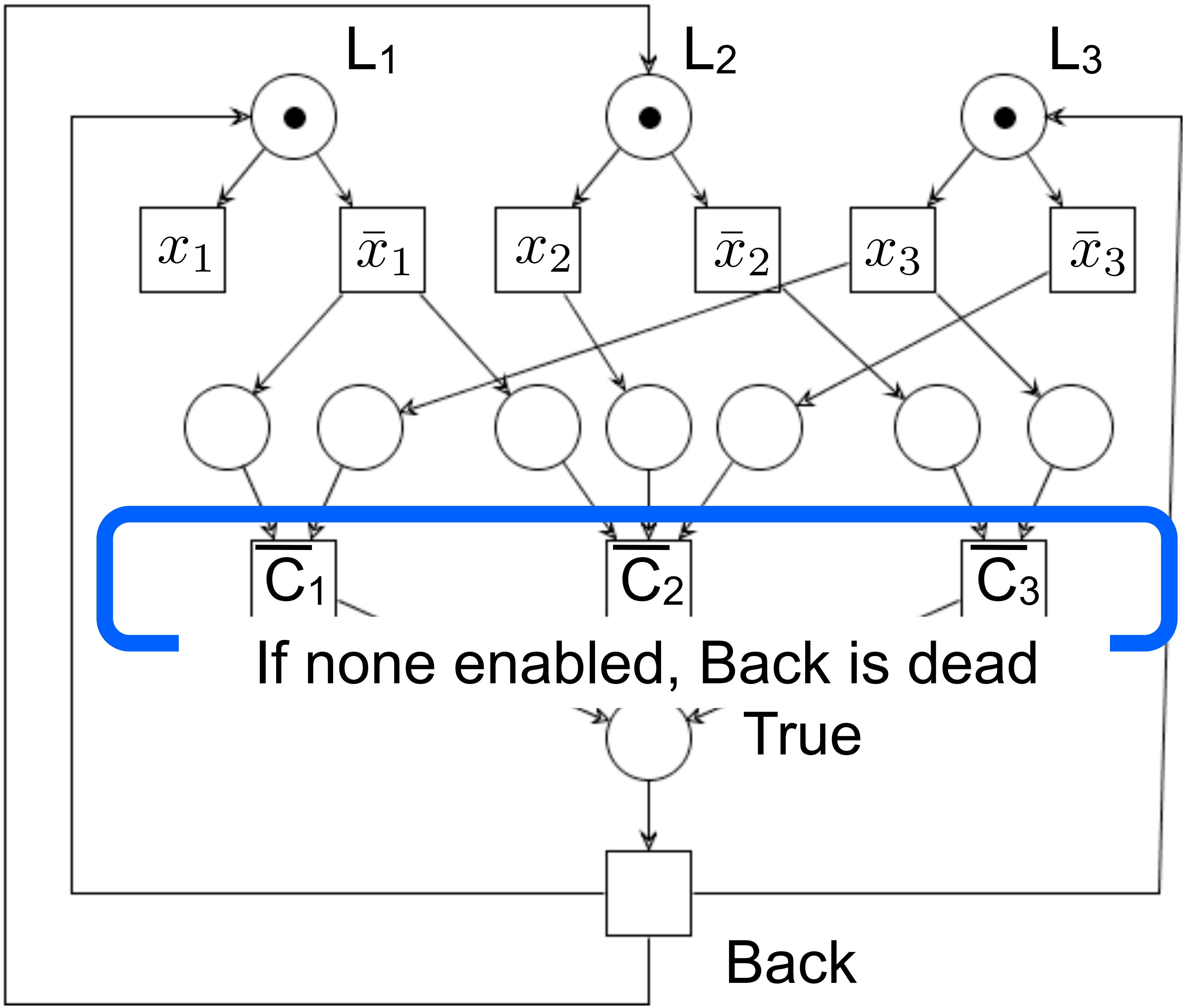

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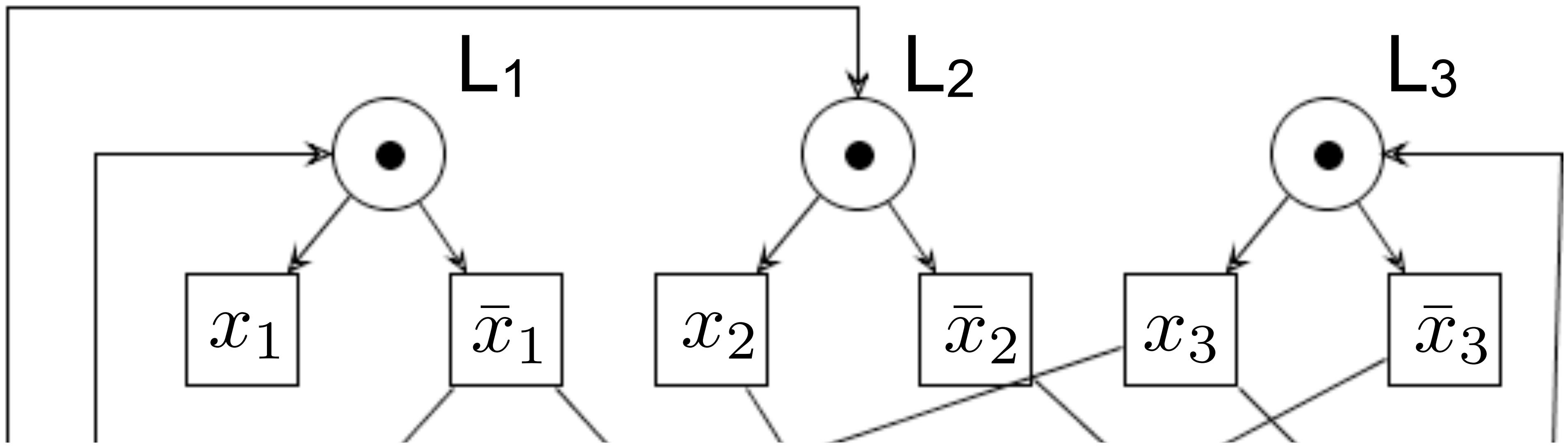
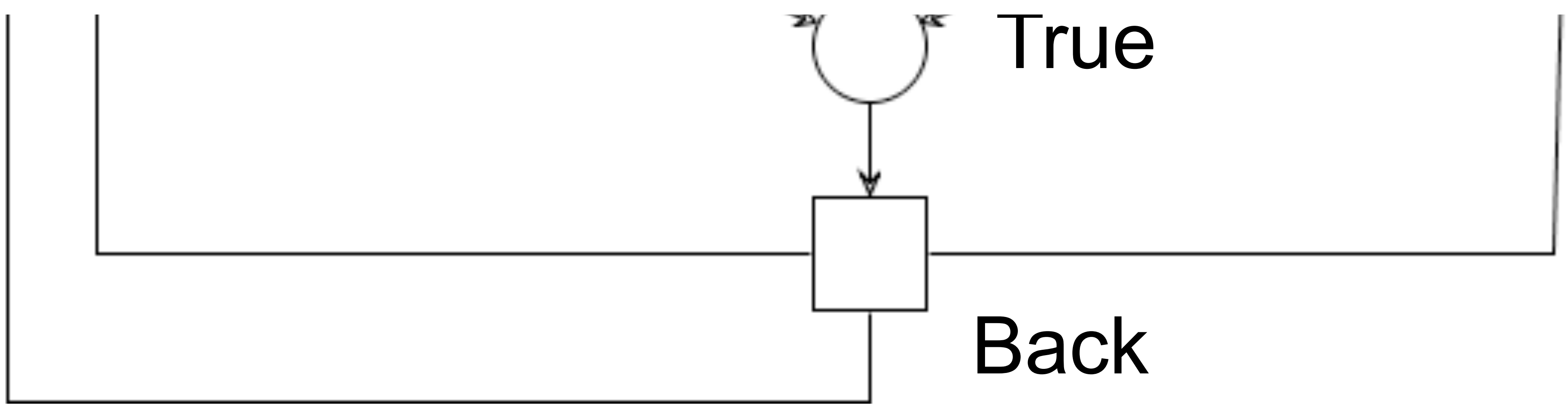


If ϕ is satisfiable, then the net is not live

If the net is not live, then ϕ is satisfiable



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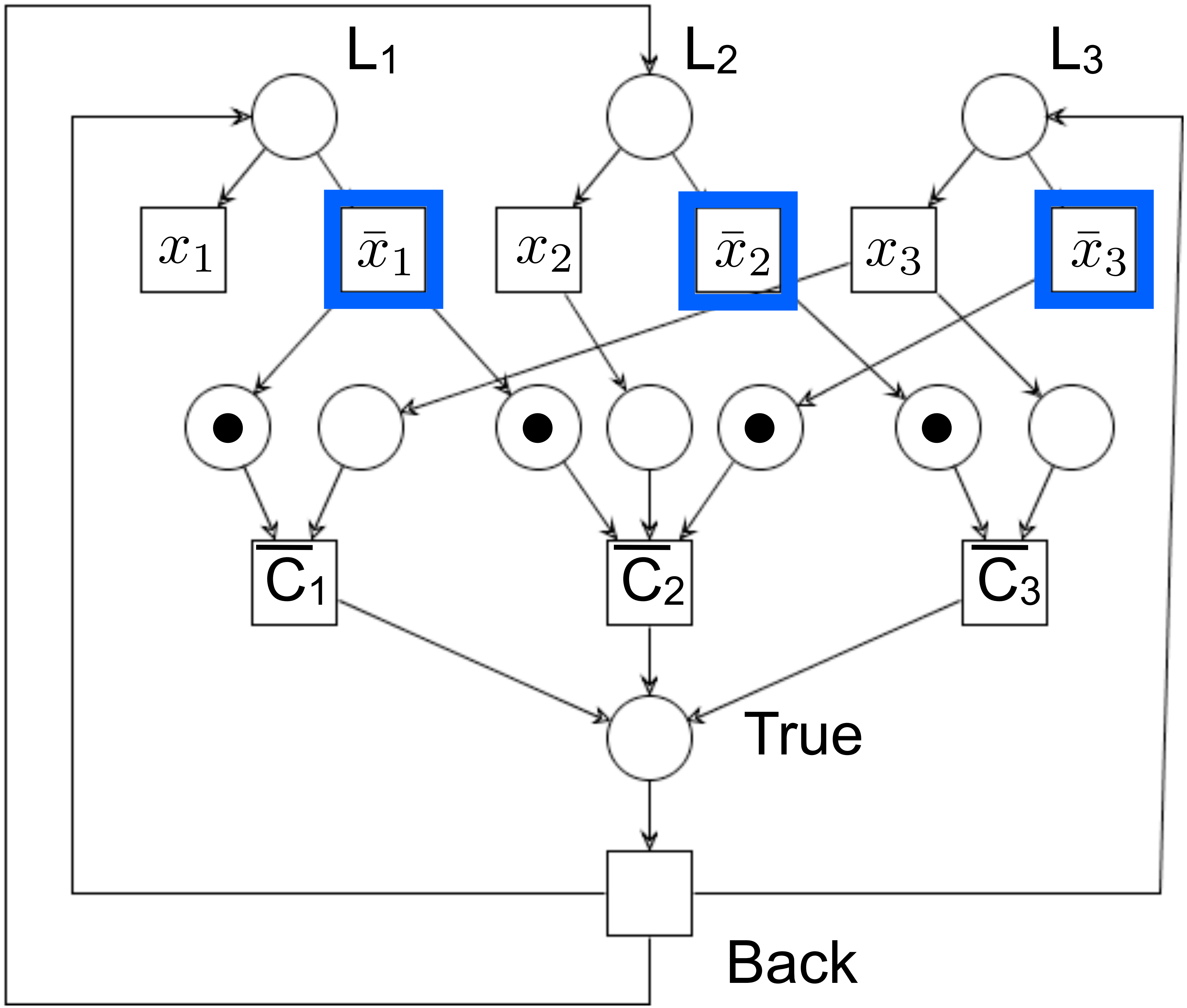


Main consequence

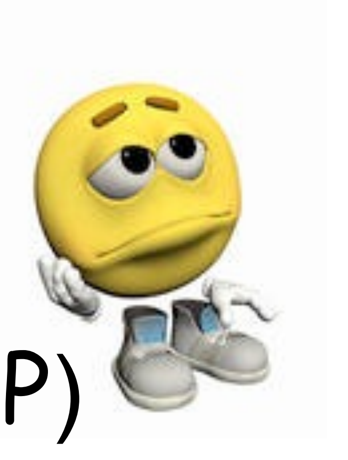
Given a formula ϕ,

the formula ϕ is satisfiable

iff

its corresponding free-choice system (P,T,F,M_0) is not live

No deterministic polynomial algorithm to decide liveness of a free-choice system is currently available



(unless P=NP)

Liveness and boundedness?

Rank Theorem (main result, proof omitted)

Theorem:

A free-choice system (P,T,F,M_0) is live and bounded

iff

1. it has at least one place and one transition
2. it is connected
3. M_0 marks every proper siphon
4. it has a positive S-invariant
5. it has a positive T-invariant
6. $\mathrm{rank}(N) = |C_N| - 1$

(where C_N is the set of clusters)

Coming next

What is a cluster?

Is it computationally expensive
to show that a free-choice net is live and bounded?

What is a cluster?

Cluster

Let x be the node of a net $N = (P, T, F)$ (not necessarily free-choice)

Definition:
The **cluster** of x, written $[x]$, is the least set s.t.

1. $x \in [x]$

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(if a place p is in the cluster, then all transitions in the post-set of p are in the cluster)

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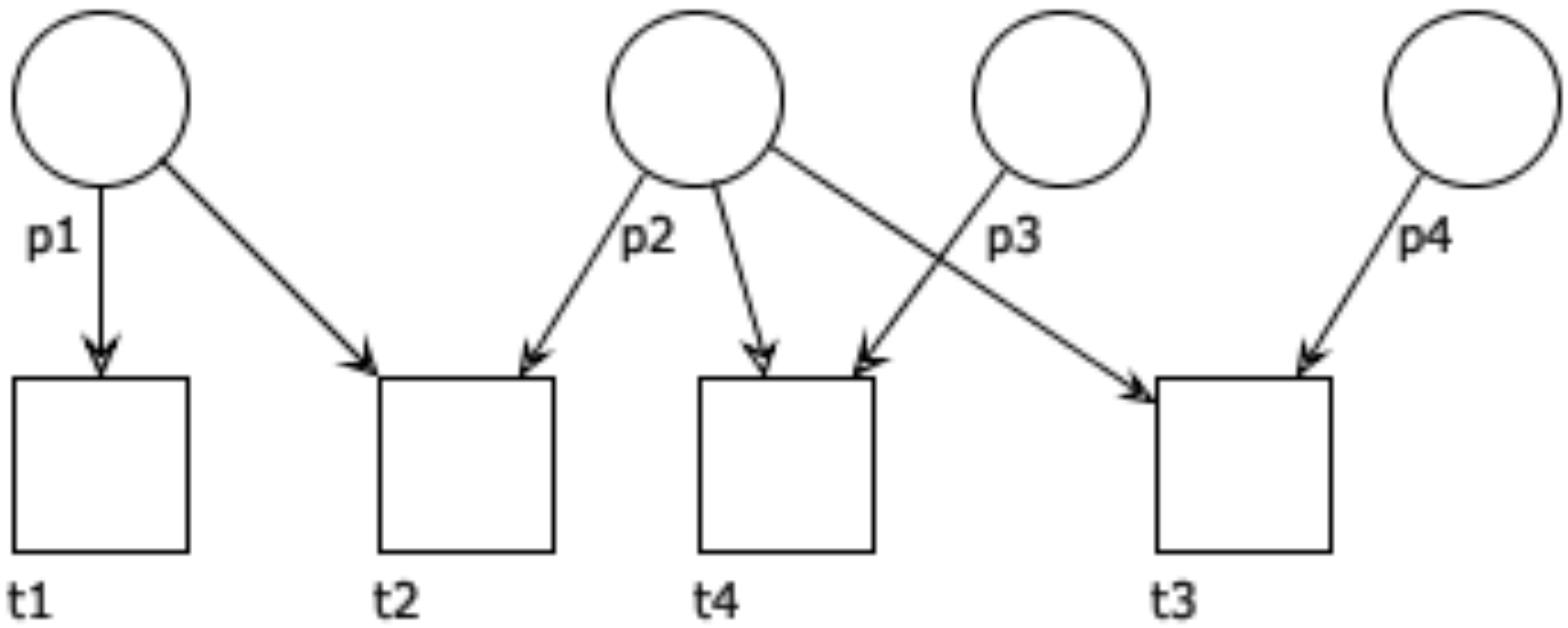
1. $x \in [x]$
2. if $p \in [x] \cap P$ then $p\bullet \subseteq [x]$

   (if a place p is in the cluster, then all transitions in the post-set of p are in the cluster)
3. if $t \in [x] \cap T$ then $\bullet t \subseteq [x]$

   (if a transition t is in the cluster, then all places in the pre-set of t are in the cluster)

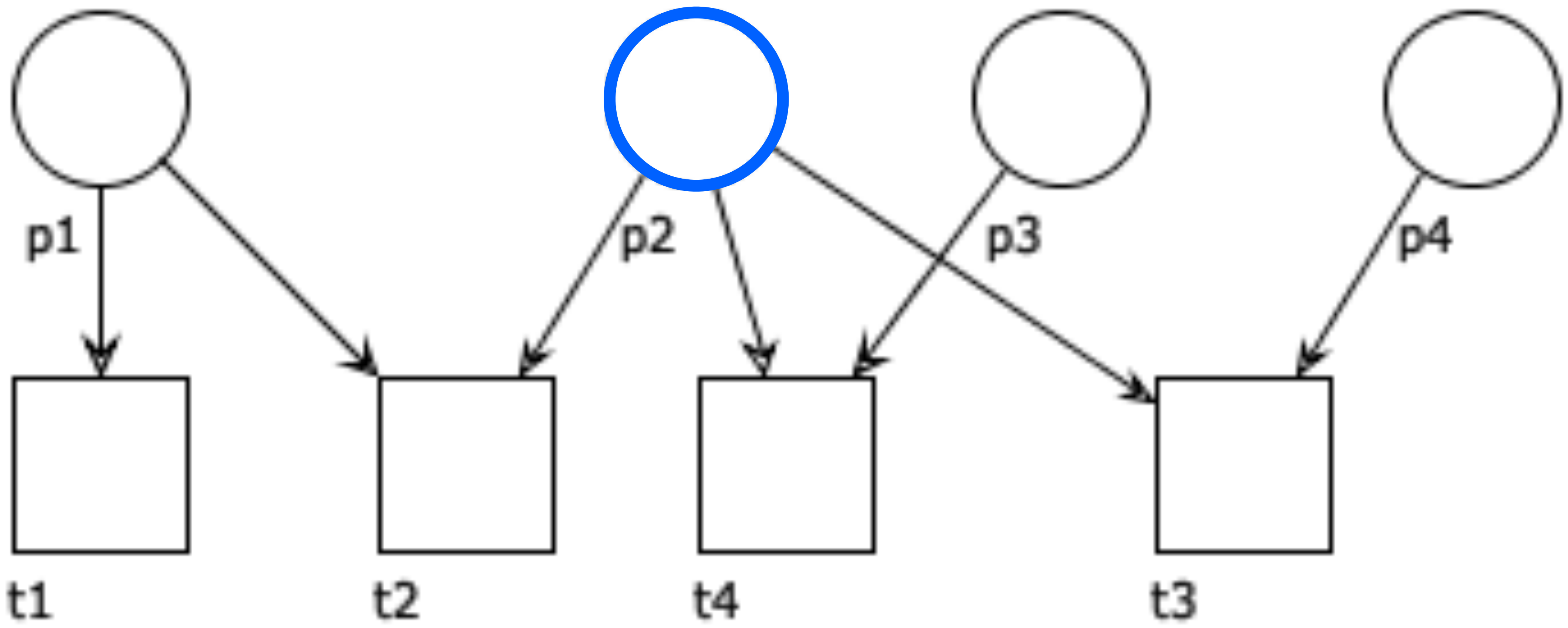
Cluster: intuition

[p2] = ?



Cluster: intuition

[p2] = { p2 ,… }



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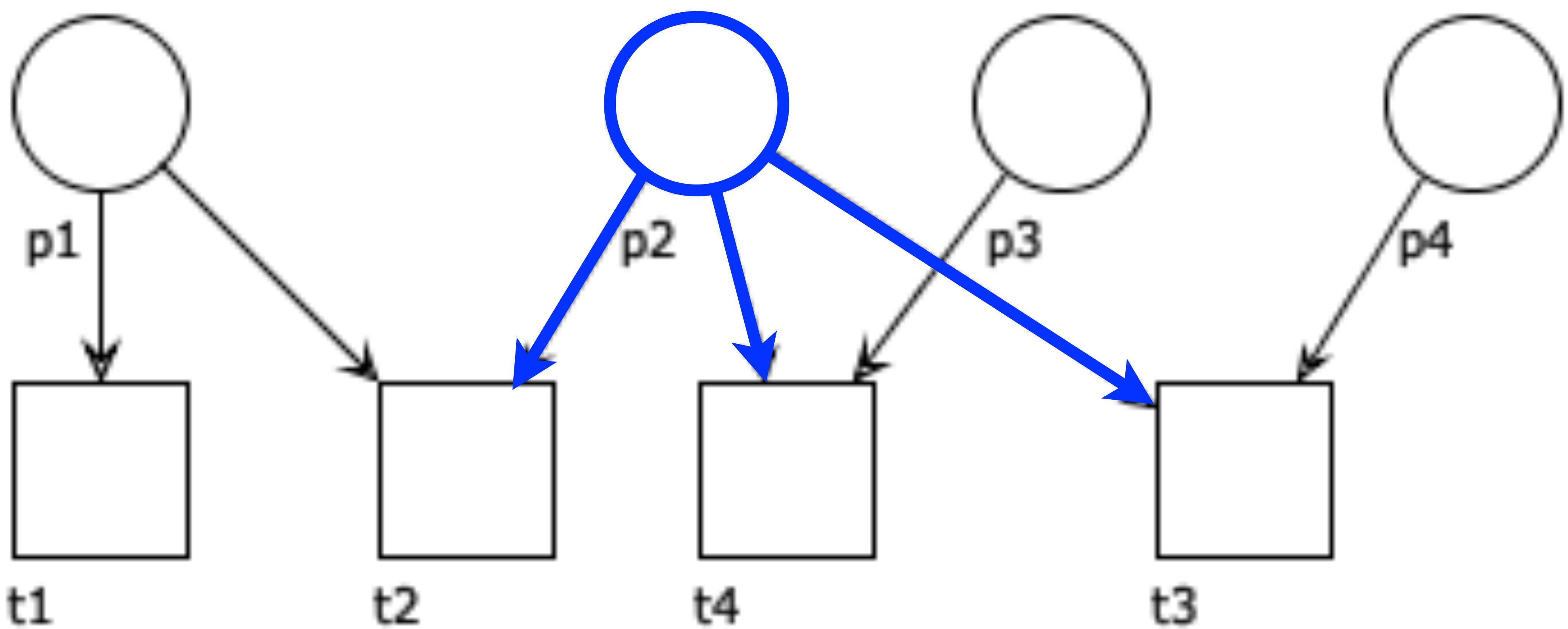
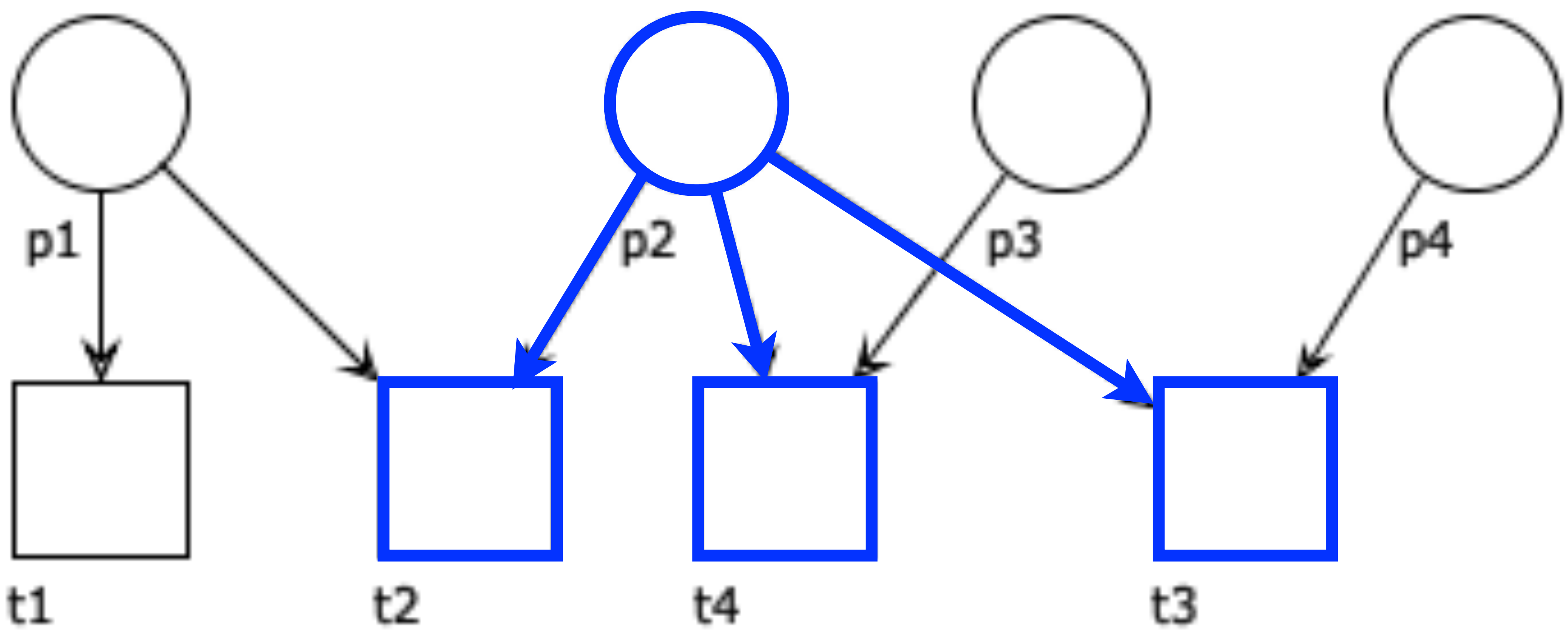


Cluster: intuition

[p2] = { p2 , t2, t4, t3,… }

(if a place p is in the cluster, then all transitions in the post-set of p are in the cluster)



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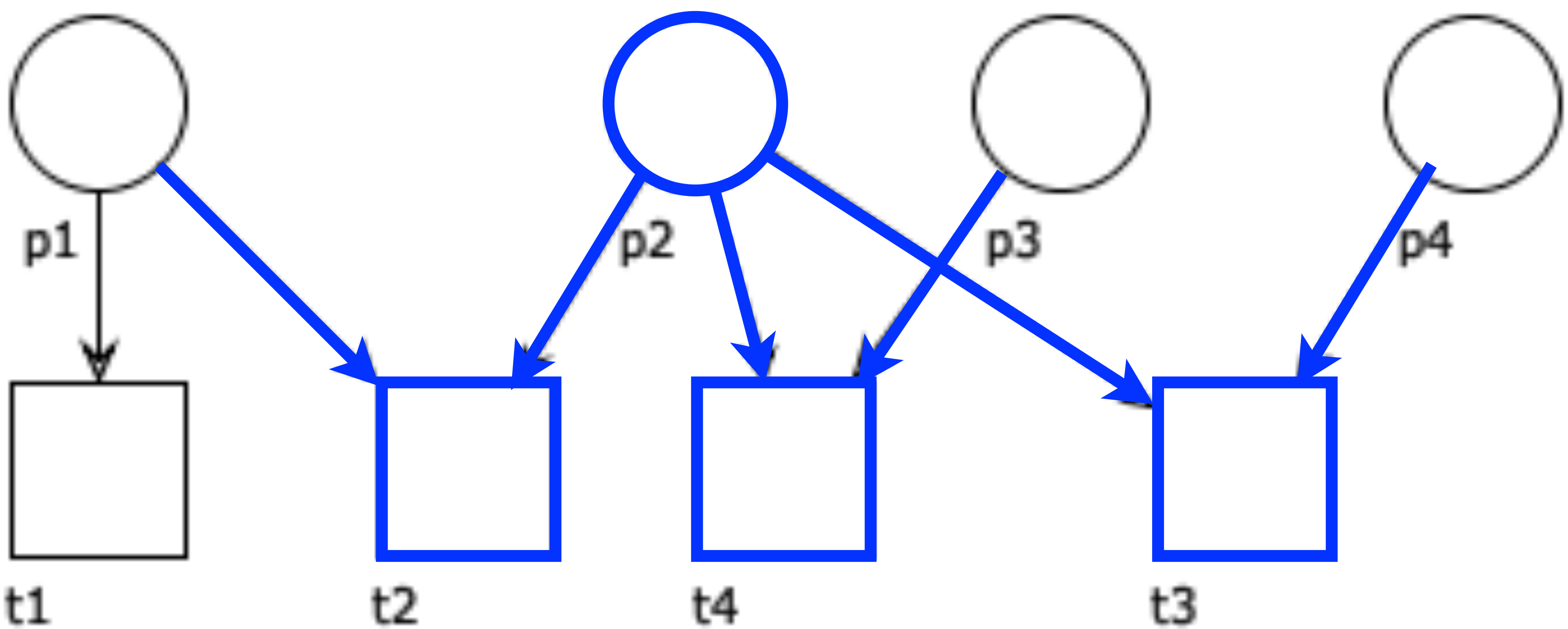

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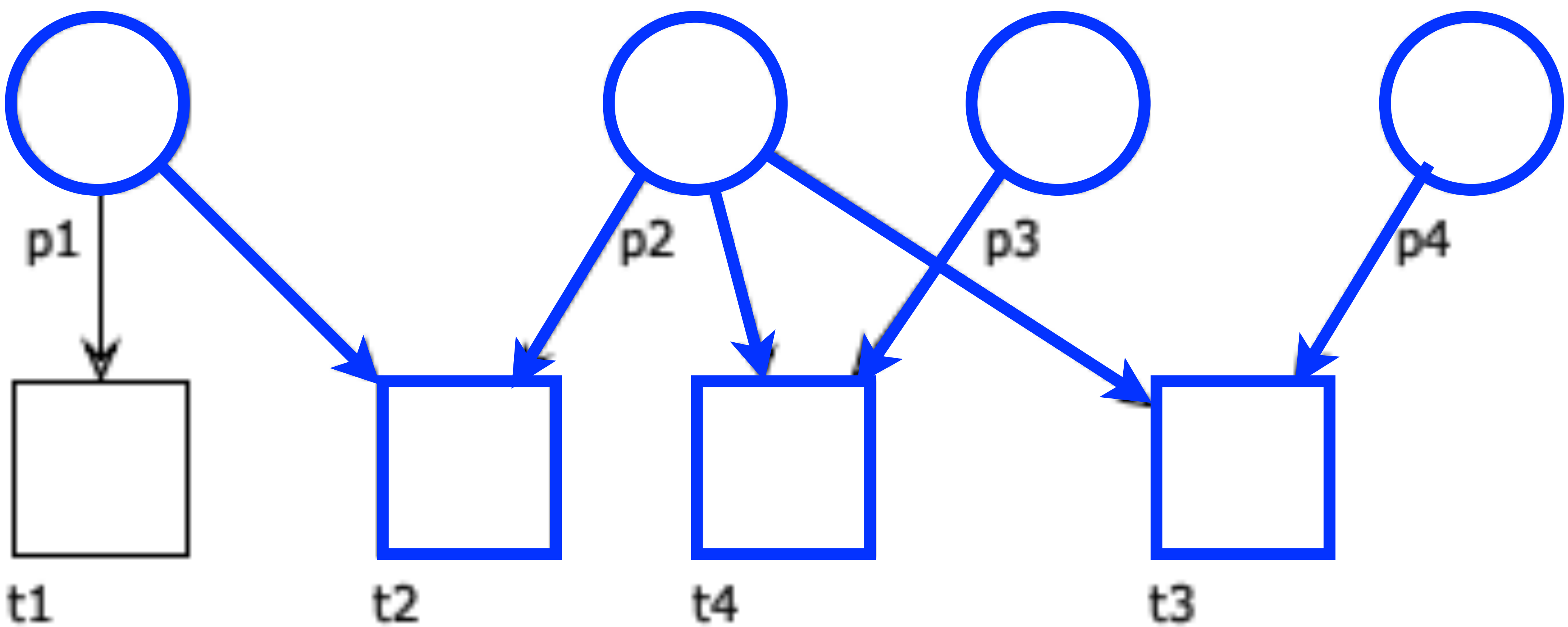

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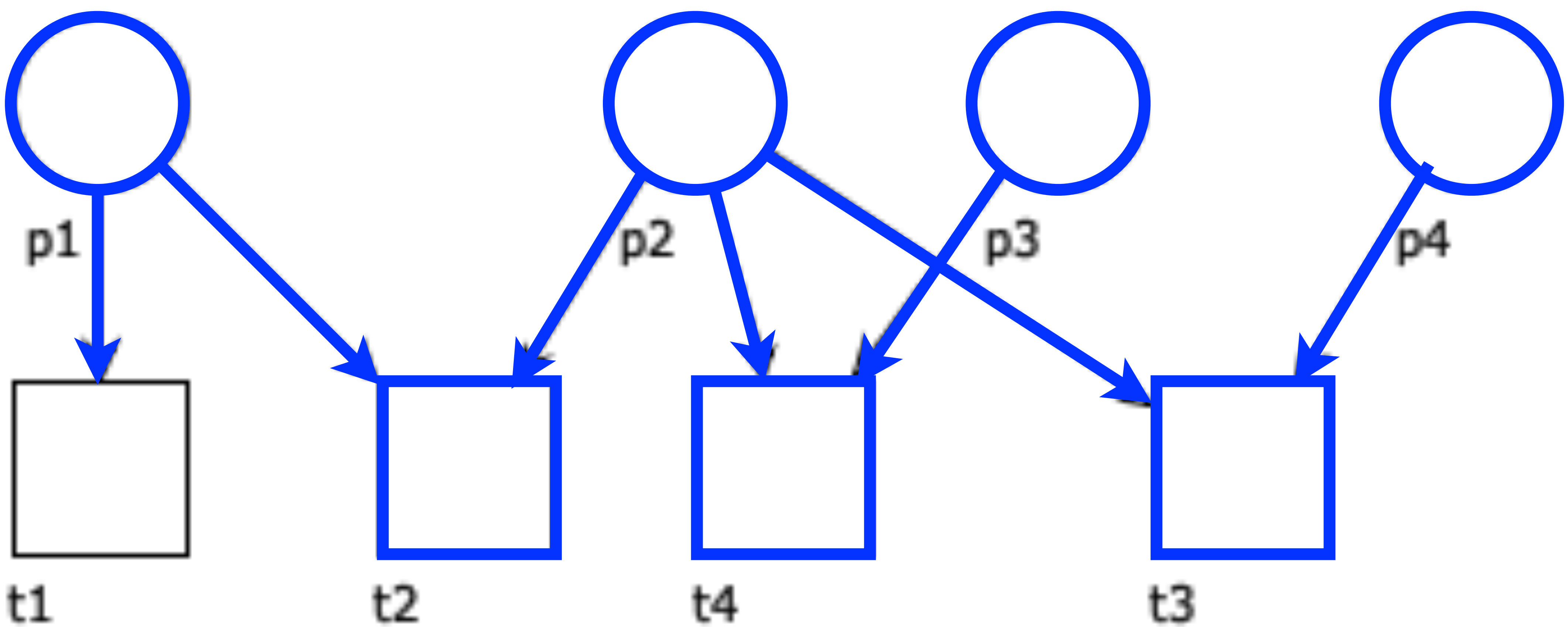
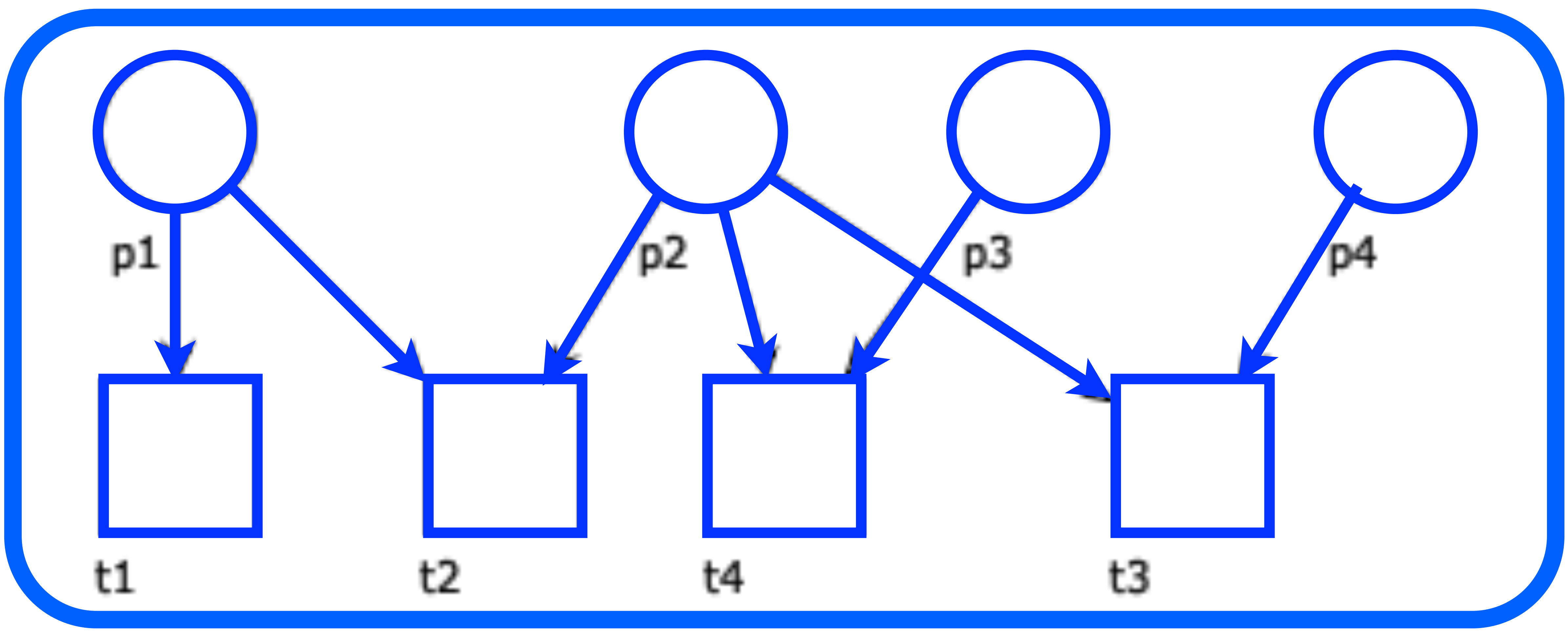


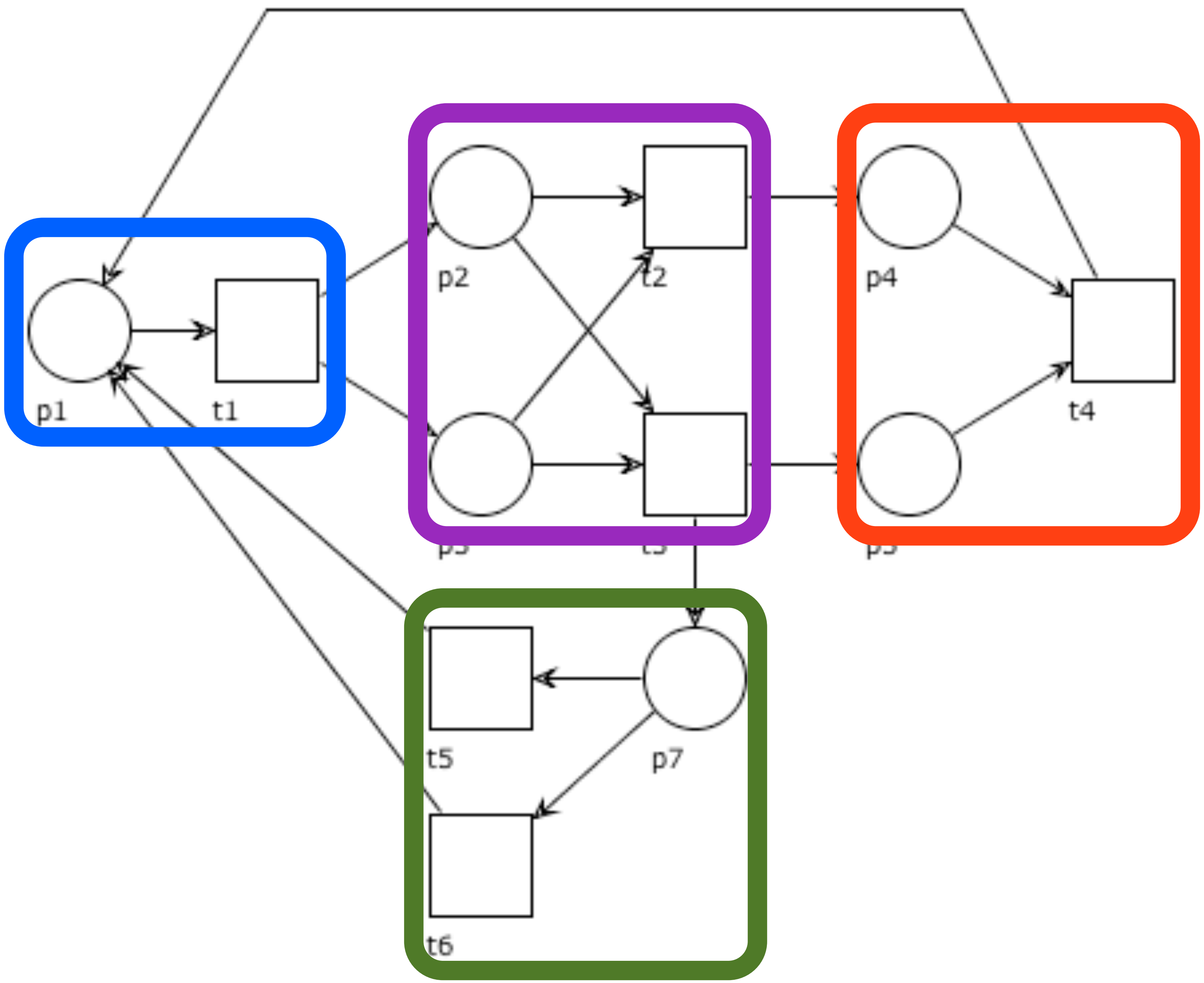
Cluster: intuition

[p2] = { p2 , t2, t4, t3, p1, p3, p4, t1 }

(if a place p is in the cluster, then all transitions in the post-set of p are in the cluster)

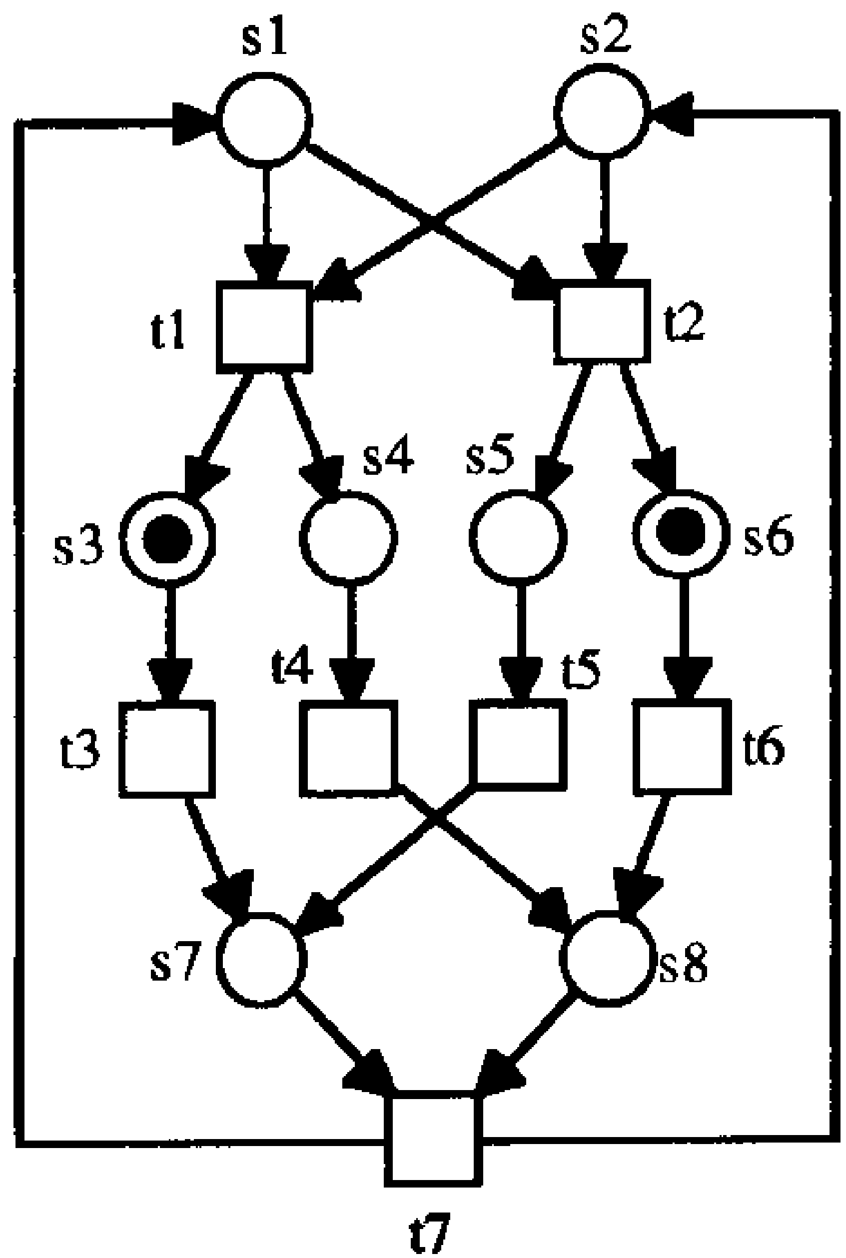


Clusters: example



Exercise

Draw all clusters in the free-choice net below



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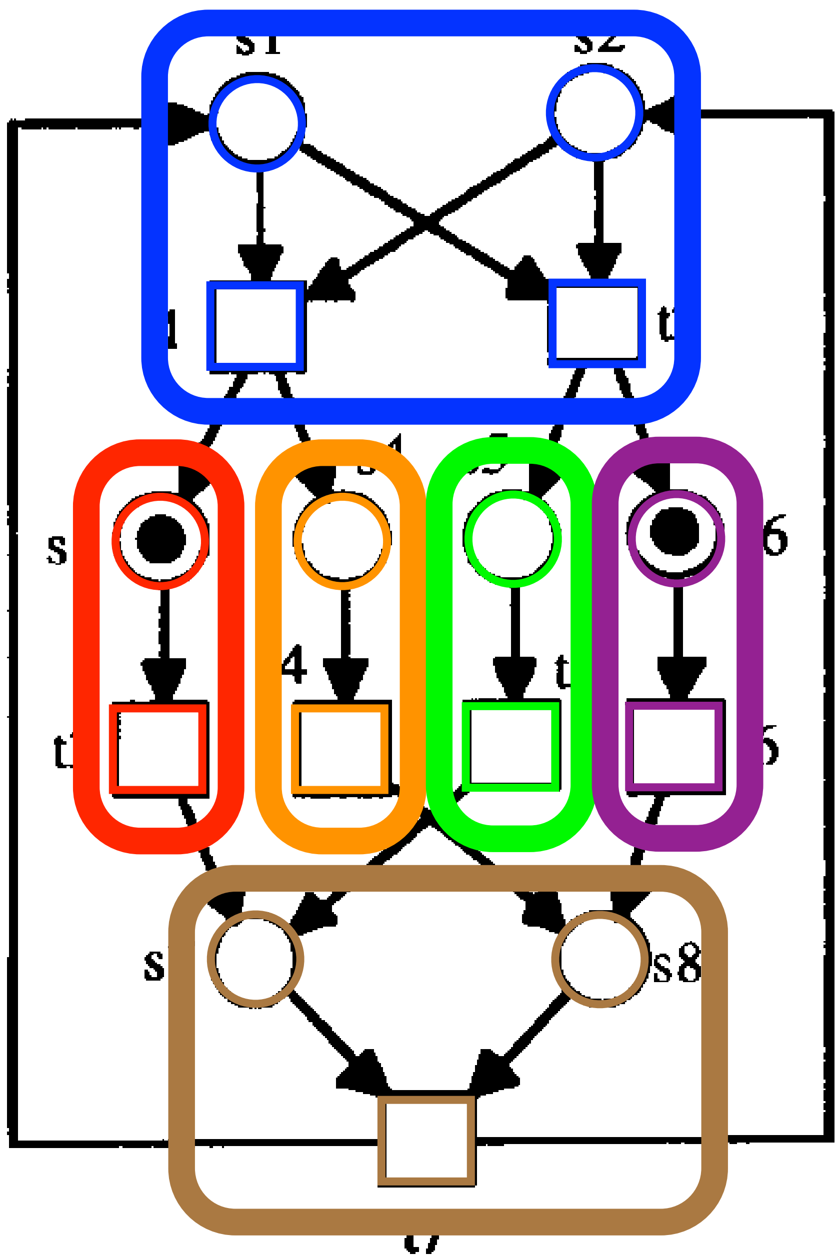


Clusters and Rank Theorem

Theorem:

A free-choice system (P,T,F,M_0) is live and bounded

iff

1. it has at least one place and one transition
2. it is connected
3. M0 marks every proper siphon
4. it has a positive S-invariant
5. it has a positive T-invariant
6. $\mathrm{rank}(N) = |C_N| - 1$

(where C_N is the set of clusters)

Complexity issues 2: Is it hard to show that a free-choice net is live and bounded?

Rank Theorem: a polynomial decision algorithm

Theorem:

A free-choice system (P,T,F,M_0) is live and bounded

iff

1. it has at least one place and one transition polynomial
2. it is connected polynomial
3. M_0 marks every proper siphon
4. it has a positive S-invariant polynomial
5. it has a positive T-invariant polynomial
6. rank(N) = $|C_N|$ - 1 polynomial

(where C_N is the set of clusters)

A polynomial algorithm for maximal unmarked siphon

3. M_0 marks every proper siphon $\boxed{\text{polynomial}}$

Input: A net $N = (P, T, F, M_0)$, $\boxed{R = \{\, p \mid M_0(p) = 0 \,\}}$
Output: $Q \subseteq R$ maximal unmarked siphon

$(\bullet Q \subseteq Q \bullet)$

$Q := R$
while $(\exists p \in Q,\ \exists t \in \bullet p,\ t \notin Q\bullet)$
$\quad Q := Q \setminus \{p\}$
return Q If Q is empty then M_0 marks every proper siphon

Main consequence

The problem to decide
if a free-choice system is live and bounded
can be solved in polynomial time
(thanks to the Rank Theorem)

Recap: free-choice nets

f.c. net: place liveness <**=>** liveness

f.c. net: non-live => exists a proper siphon R and $M \in [M_0\rangle$
such that $M(R)=0$

f.c. net: every siphon contains a marked trap <=> live

f.c. net: bounded and live <=> 6 conditions in Rank Theorem

Exercise

Draw the free choice net corresponding to the formula

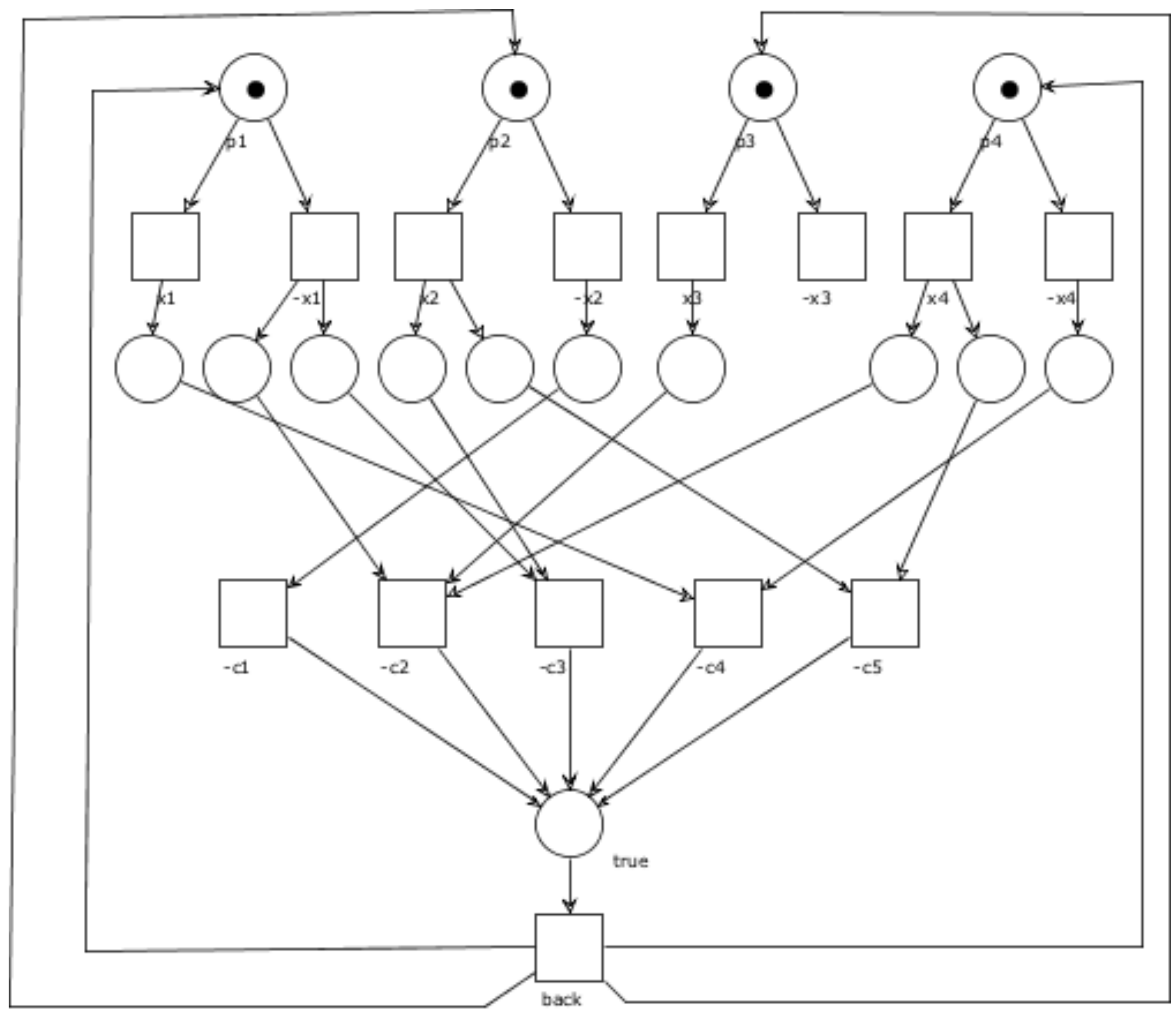
$$x_2 \;\wedge\; (x_1 \vee \overline{x}_3 \vee \overline{x}_4) \;\wedge\; (x_1 \vee \overline{x}_2) \;\wedge\; (\overline{x}_1 \vee x_4) \;\wedge\; (\overline{x}_2 \vee \overline{x}_4)$$

Is it satisfiable?

Exercise

$$x_2 \wedge (x_1 \vee \overline{x}_3 \vee \overline{x}_4) \wedge (x_1 \vee \overline{x}_2) \wedge (\overline{x}_1 \vee x_4) \wedge (\overline{x}_2 \vee \overline{x}_4)$$

$$\overline{x}_2 \vee (\overline{x}_1 \wedge x_3 \wedge x_4) \vee (\overline{x}_1 \wedge x_2) \vee (x_1 \wedge \overline{x}_4) \vee (x_2 \wedge x_4)$$



Not satisfiable (the net is live)

Exercise

$$x_2 \wedge (x_1 \vee \overline{x}_3 \vee \overline{x}_4) \wedge (x_1 \vee \overline{x}_2) \wedge (\overline{x}_1 \vee x_4) \wedge (\overline{x}_2 \vee \overline{x}_4)$$

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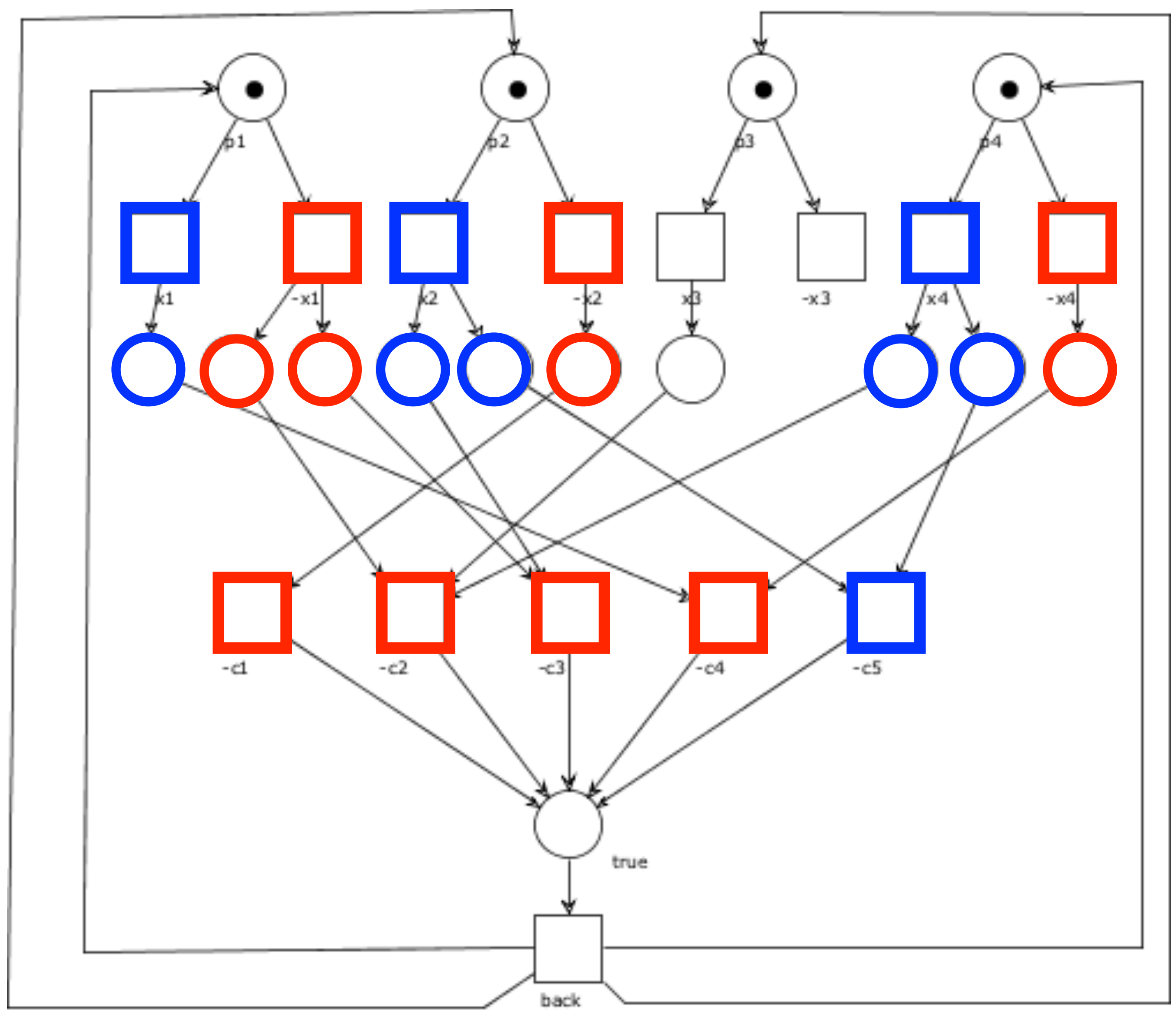


Not satisfiable (the net is live)

$\bullet R \subseteq R \bullet$

siphon

empty siphons remain empty

Exercise

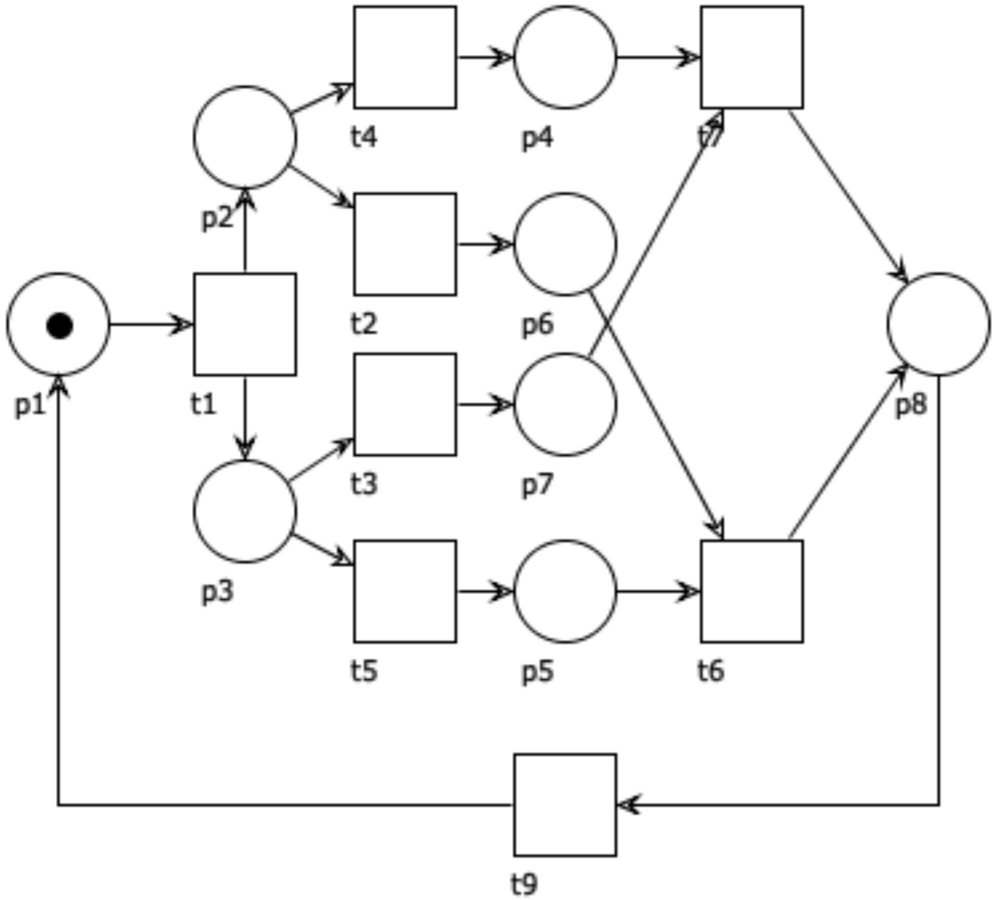
$\bullet Q \supseteq Q \bullet$

trap

marked traps remain marked

The system below is free-choice and non-live:
find a proper siphon that does not include a marked trap

Hint: take
$R=\{p_1,p_2,p_3,p_4,p_5,p_8\}$
and show that:
it is a siphon and
it contains no trap



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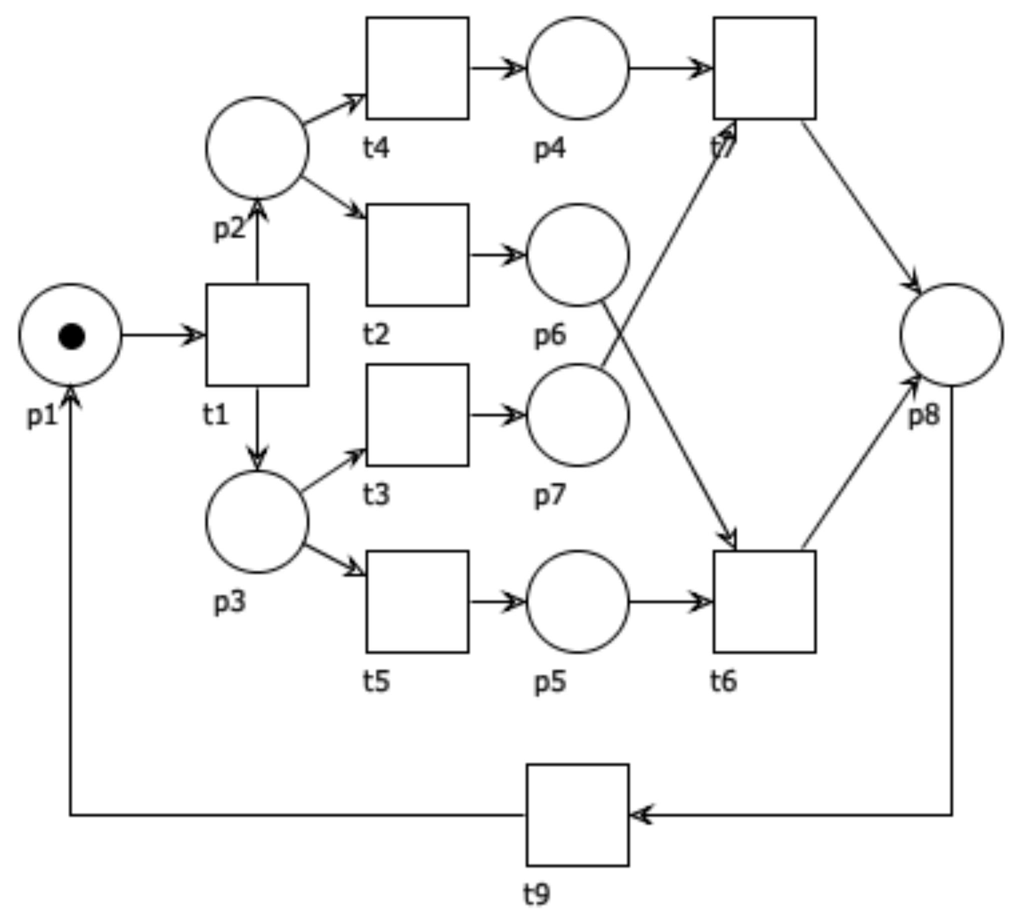

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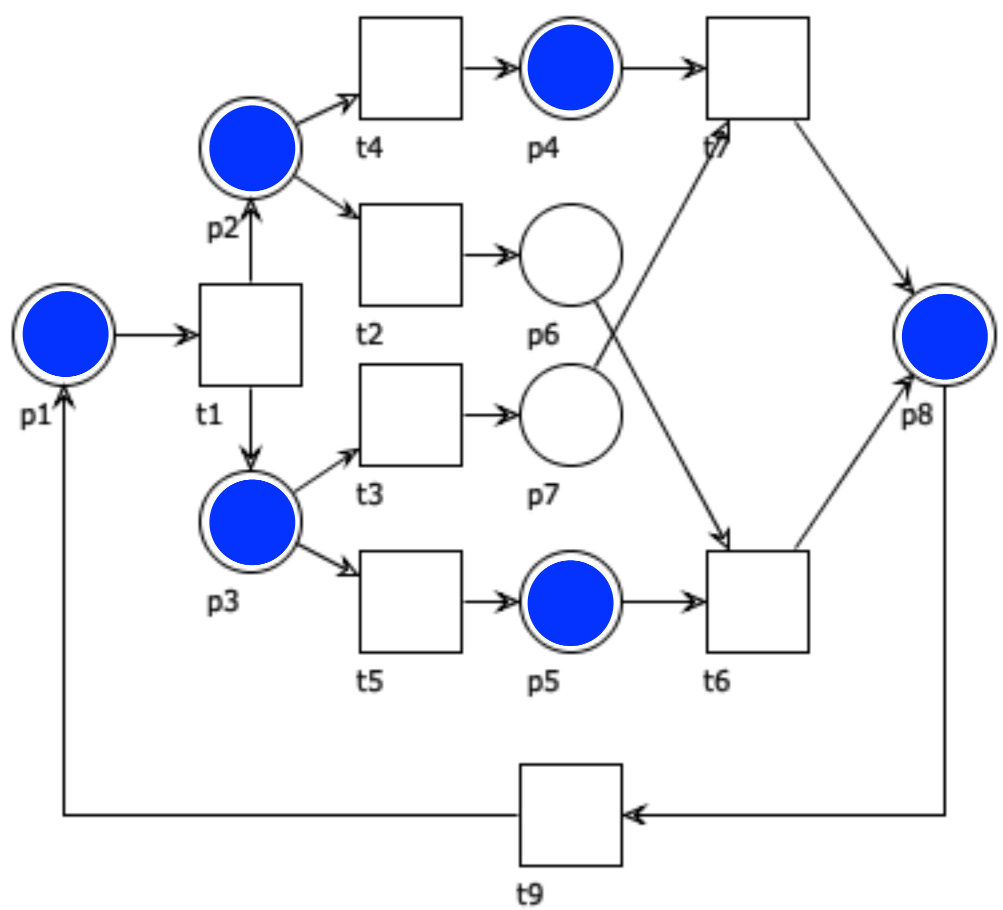

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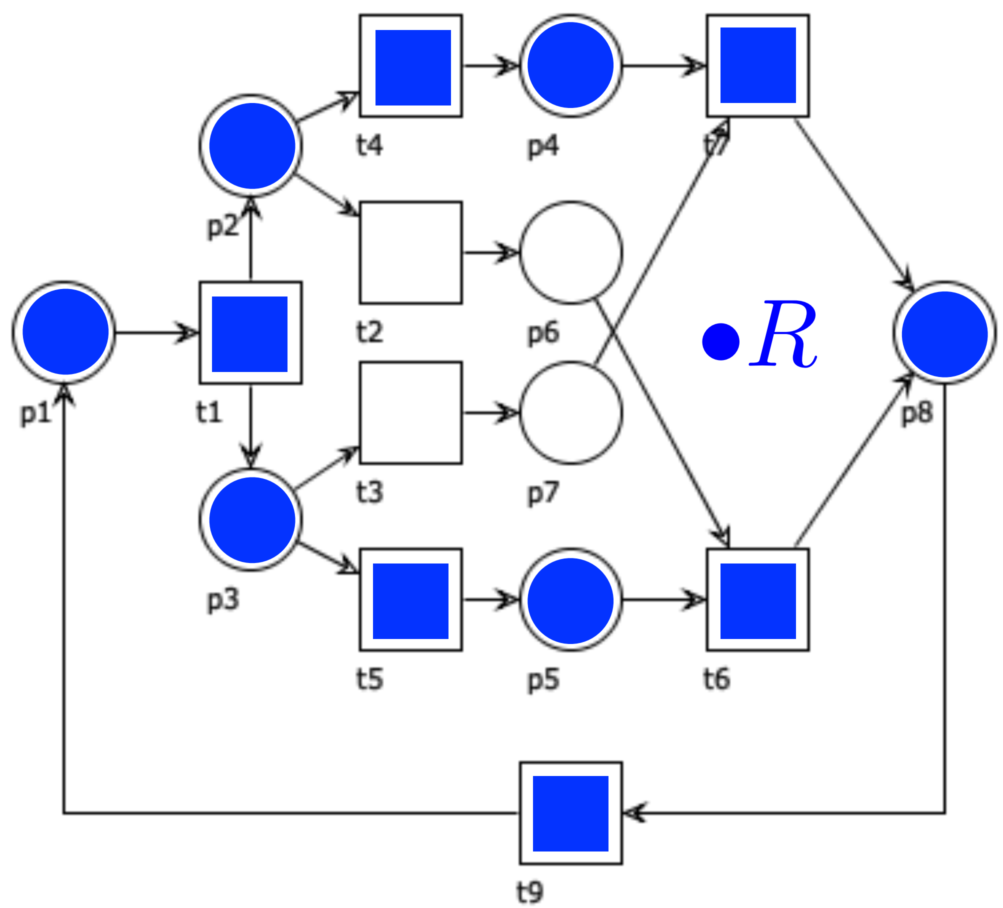

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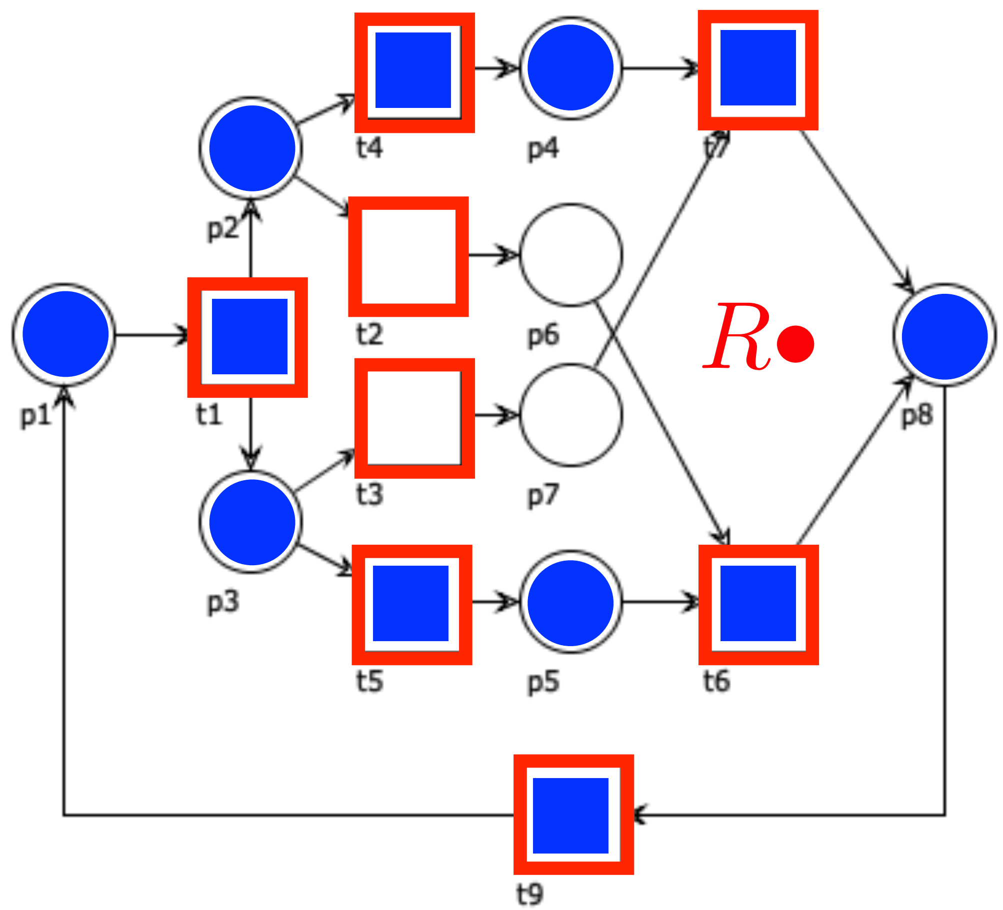

$\bullet R \subseteq R \bullet$

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trap

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The system below is free-choice and non-live:
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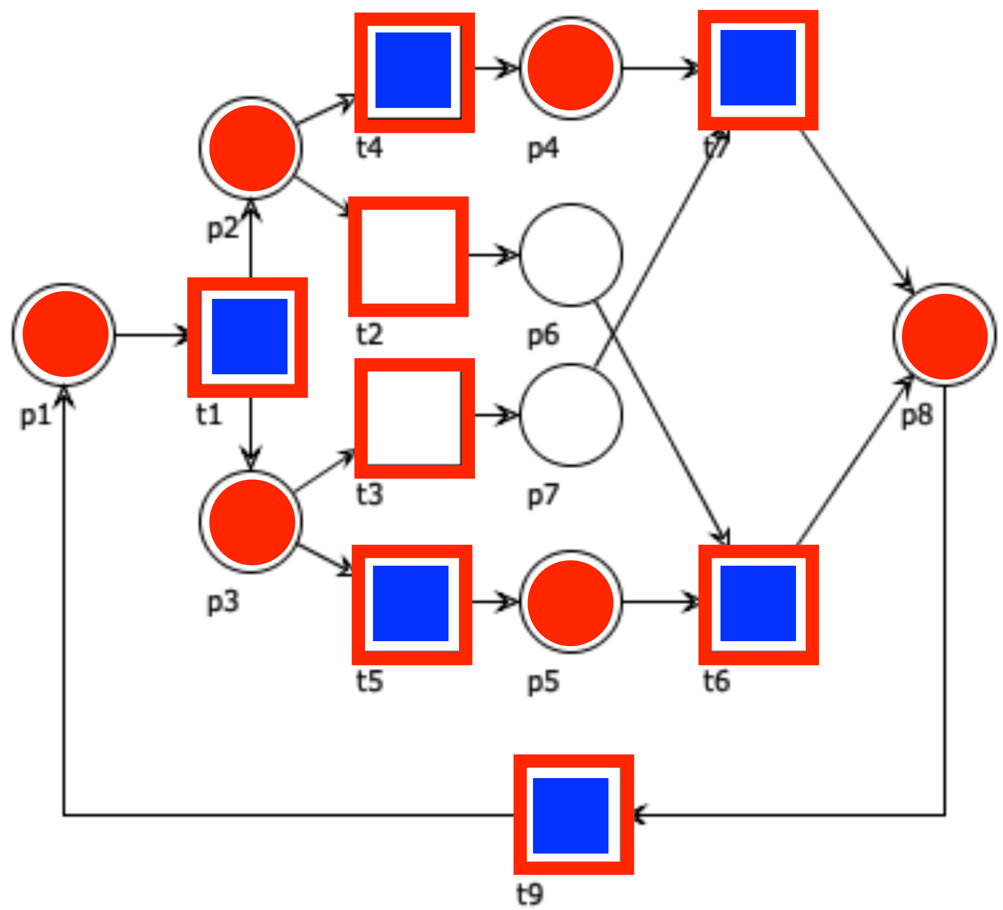

$Q=\{p_1,p_2,p_3,p_4,p_5,p_8\}$
not a trap

$$\bullet Q \not\supseteq Q \bullet$$

because of t_2,t_3
need to remove p_2,p_3

Exercise

$\bullet Q \supseteq Q \bullet$

trap

The system below is free-choice and non-live:
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Hint: take
$R=\{p_1,p_2,p_3,p_4,p_5,p_8\}$
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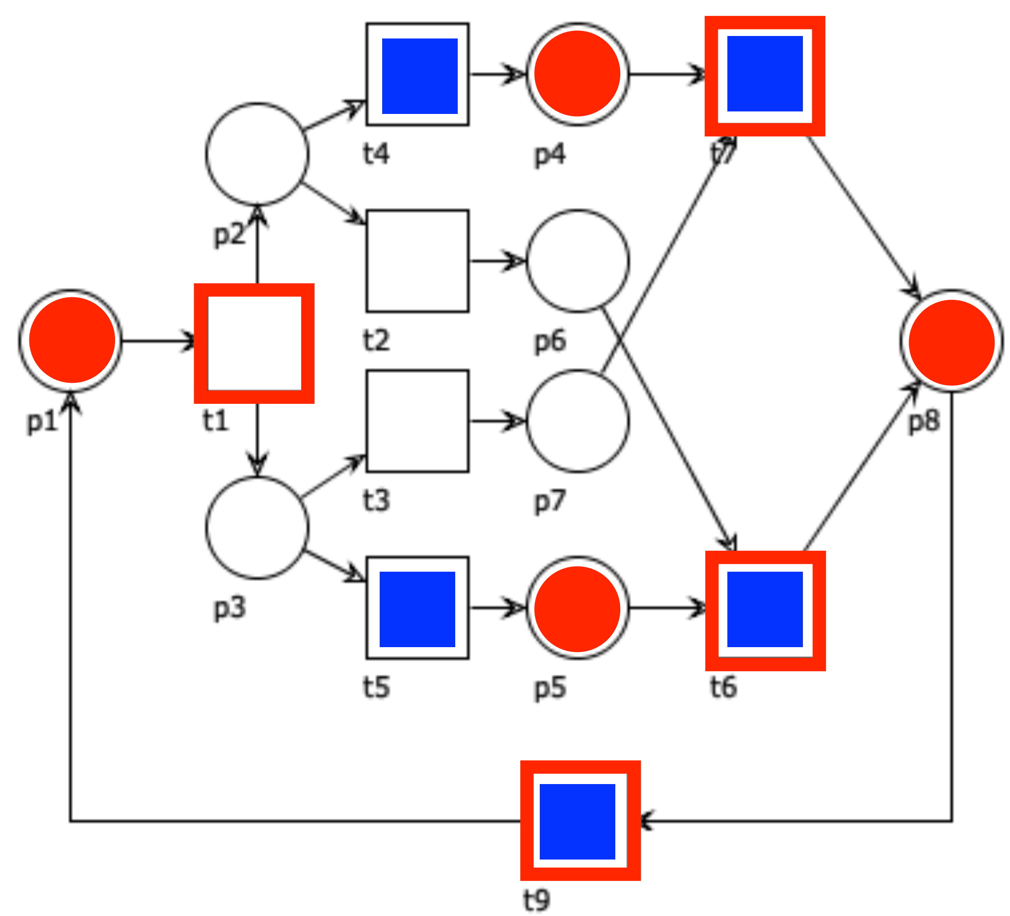

$Q=\{p_1,p_4,p_5,p_8\}$
not a trap

$\bullet Q \not\supseteq Q \bullet$

because of t_1
need to remove p_1

$\bullet Q \supseteq Q \bullet$
trap

Exercise

The system below is free-choice and non-live:
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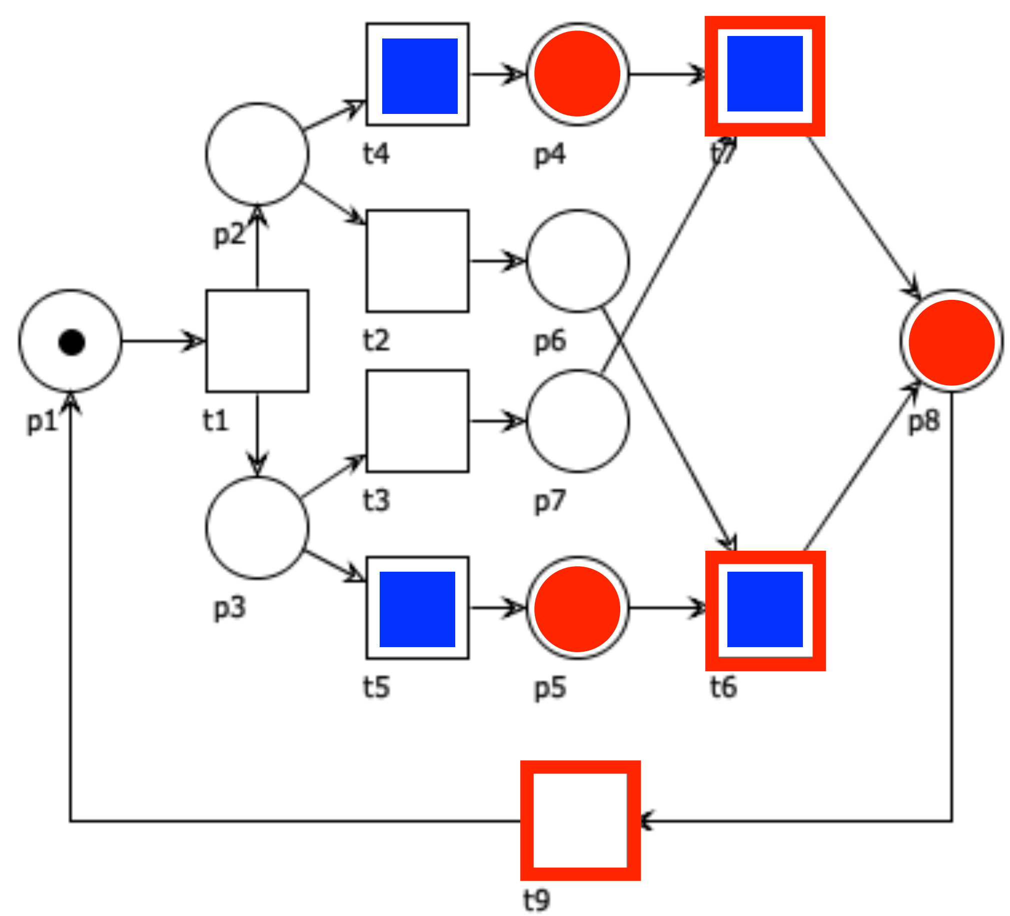
and show that:
it contains no trap



$Q=\{p_4,p_5,p_8\}$
not a trap

$\bullet Q \not\supseteq Q \bullet$

because of t_9
need to remove p_8

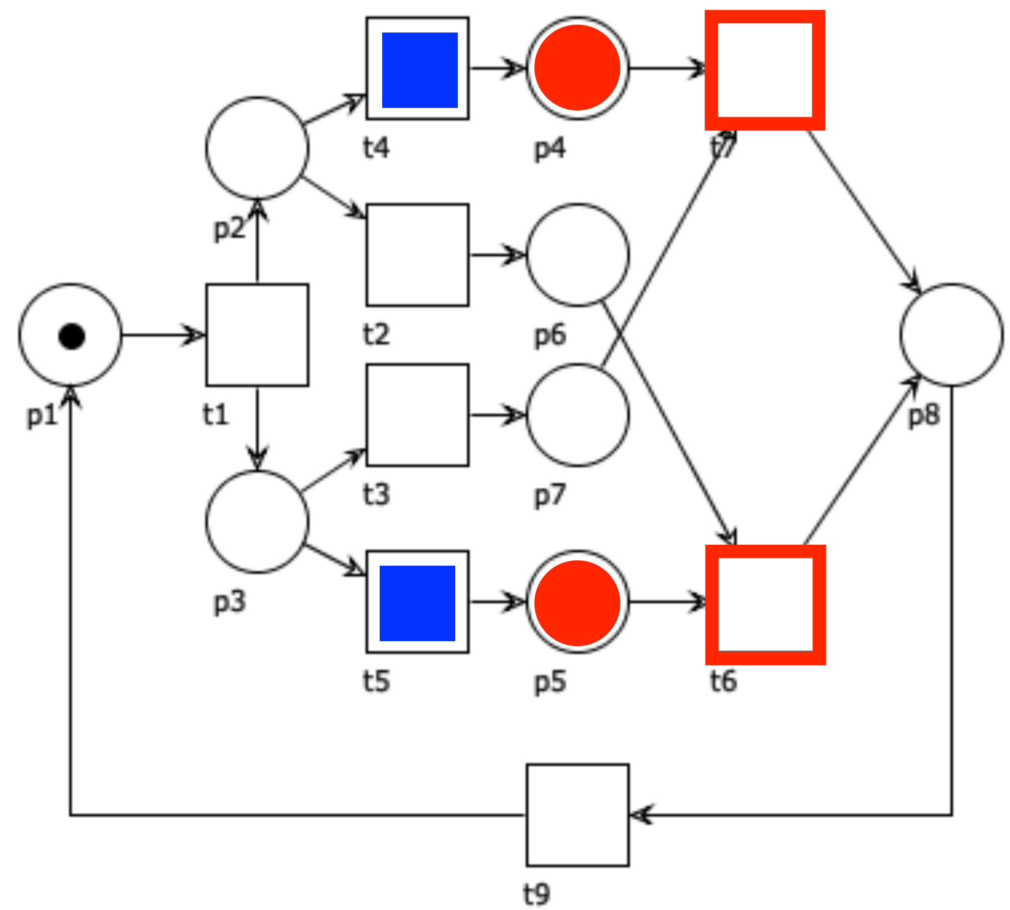
$$\bullet Q \supseteq Q \bullet$$

trap

Exercise

The system below is free-choice and non-live:
find a proper siphon that does not include a marked trap

Hint: take
$R=\{p_1,p_2,p_3,p_4,p_5,p_8\}$
and show that:
it contains no trap



$Q=\{p_4,p_5\}$
not a trap

$$\bullet Q \not\supseteq Q \bullet$$

because of t_6,t_7
need to remove p_4,p_5

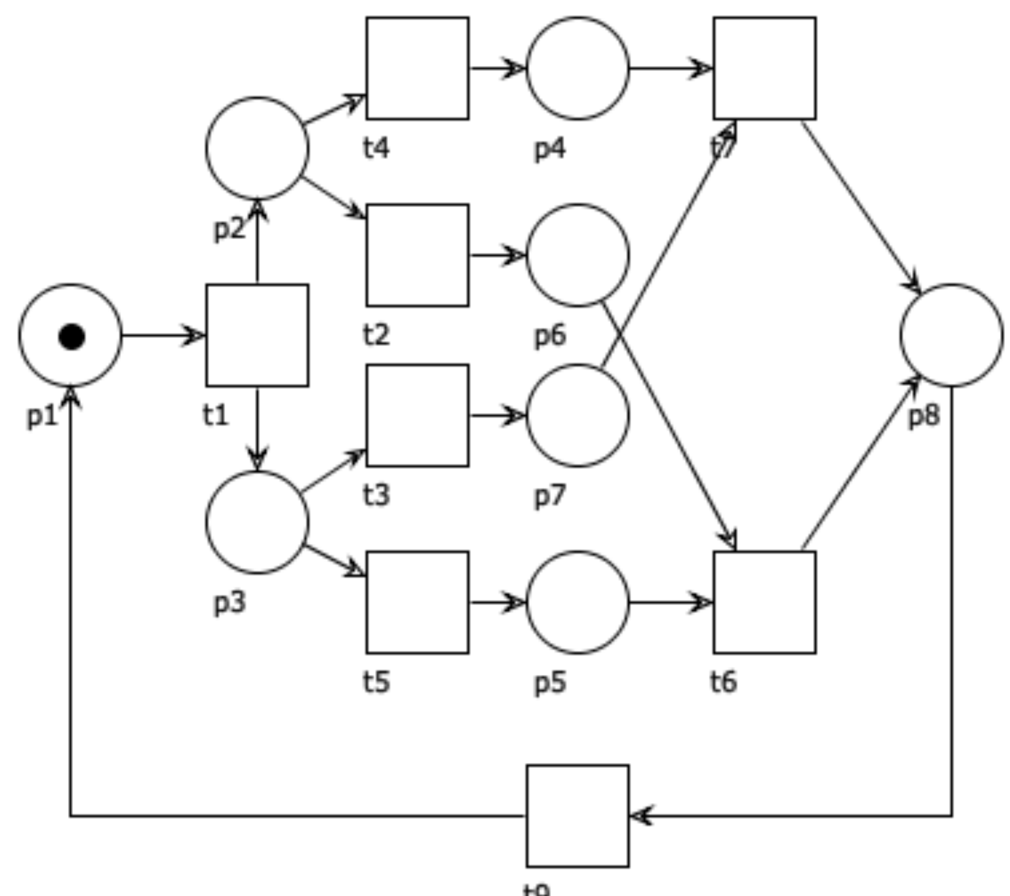
Exercise

$\bullet Q \supseteq Q \bullet$

trap

The system below is free-choice and non-live:
find a proper siphon that does not include a marked trap

Hint: take
$R=\{p_1,p_2,p_3,p_4,p_5,p_8\}$
and show that:
it contains no trap



Q={}
not a proper trap

The only trap contained in the siphon is the empty one!

$\bullet R \subseteq R\bullet$

siphon

empty siphons remain empty

Exercise

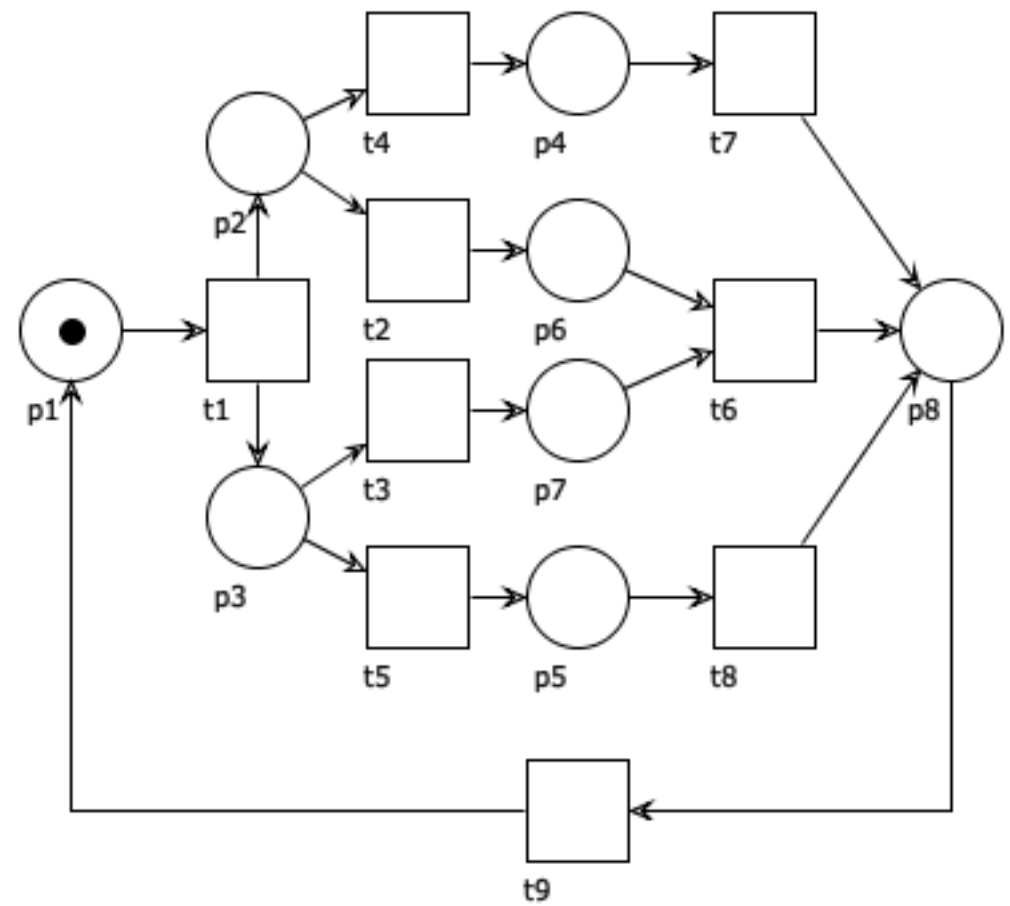
$\bullet Q \supseteq Q\bullet$

trap

marked traps remain marked

The system below is free-choice and live: show that every proper siphon includes a marked trap

Hint: the only proper siphons are $R_1=\{p_1,p_2,p_3,p_4,p_5,p_7,p_8\}$ and $R_2=\{p_1,p_2,p_3,p_4,p_5,p_6,p_8\}$



$\bullet R \subseteq R\bullet$

siphon

empty siphons remain empty

Exercise

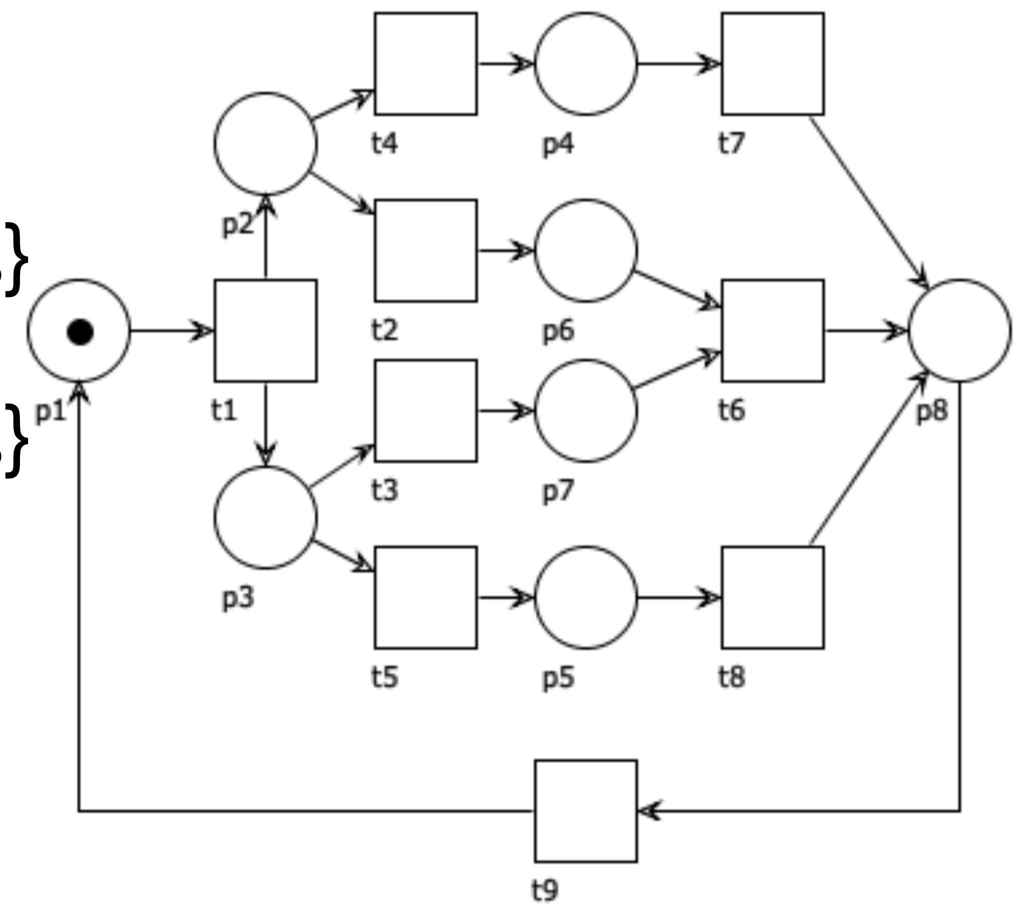
$\bullet Q \supseteq Q\bullet$

trap

marked traps remain marked

The system below is free-choice and live:
show that every proper siphon includes a marked trap

Hint: the only proper siphons are $R_1=\{p_1,p_2,p_3,p_4,p_5,p_7,p_8\}$ and $R_2=\{p_1,p_2,p_3,p_4,p_5,p_6,p_8\}$



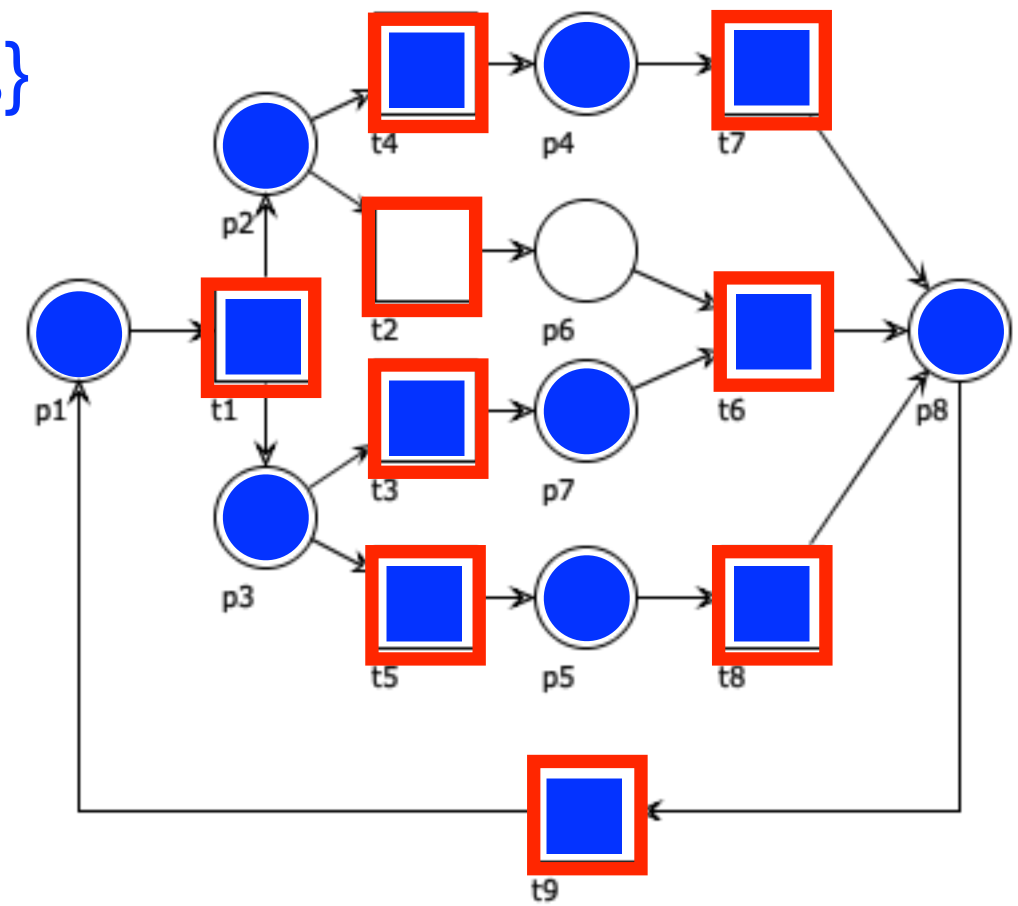
$\bullet R \subseteq R\bullet$

siphon

Exercise

The system below is free-choice and live:
show that every proper siphon includes a marked trap

$R_1=\{p_1,p_2,p_3,p_4,p_5,p_7,p_8\}$



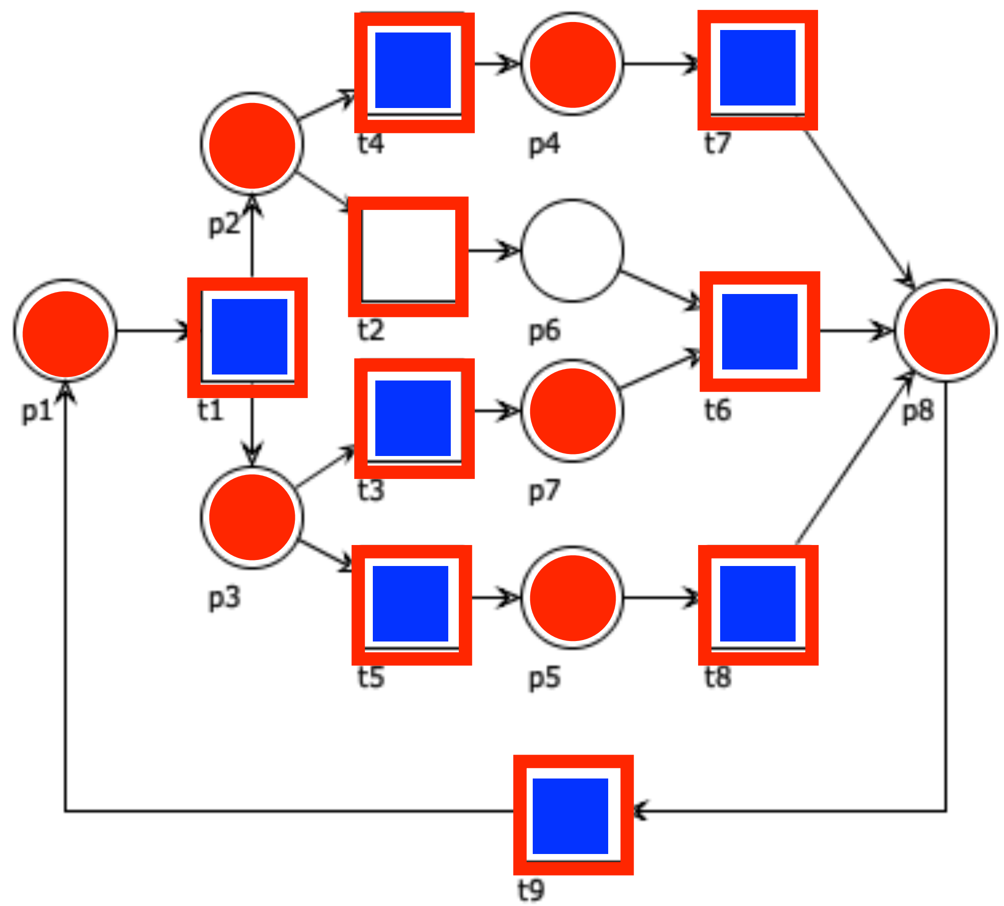
$\bullet R \subseteq R\bullet$
is a proper siphon

$$\bullet Q \supseteq Q \bullet$$

trap

Exercise

The system below is free-choice and live:
show that every proper siphon includes a marked trap



Q={p_1,p_2,p_3,p_4,p_5,p_7,p_8}
not a trap

$$\bullet Q \not\supseteq Q \bullet$$

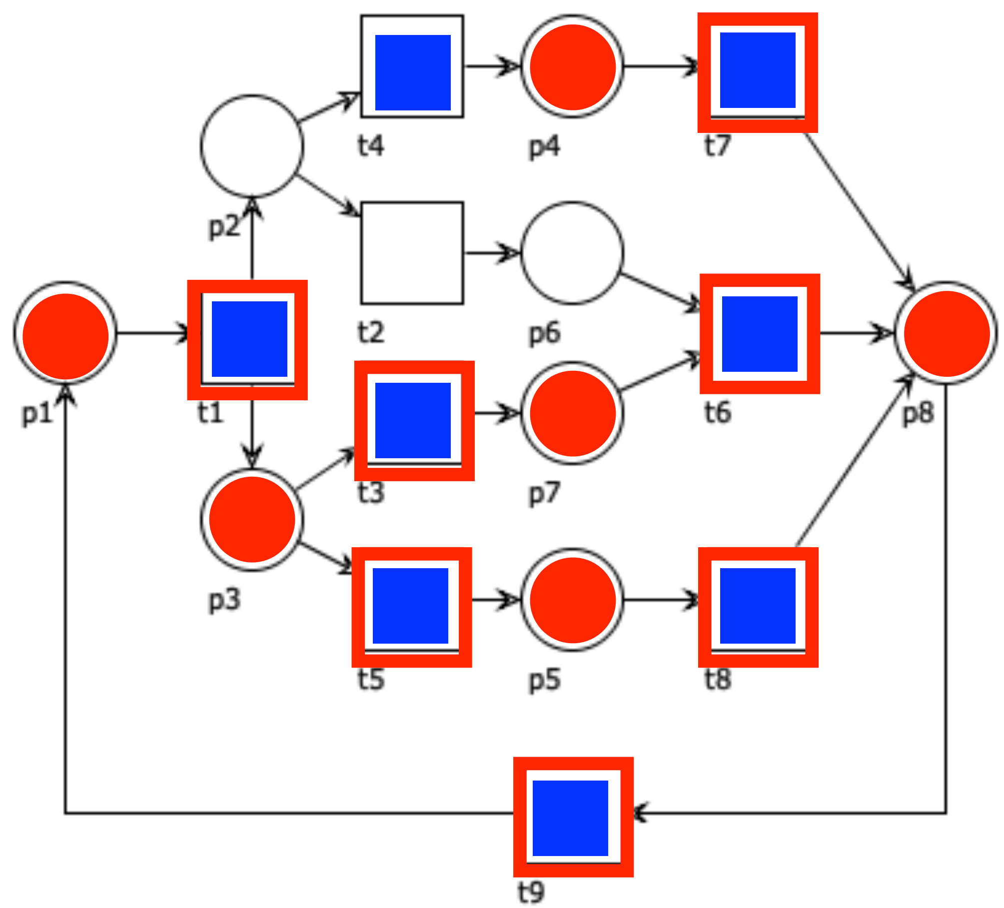
because of t_2
need to remove p_2

$$\bullet Q \supseteq Q\bullet$$

trap

Exercise

The system below is free-choice and live:
show that every proper siphon includes a marked trap



$Q=\{p_1,p_3,p_4,p_5,p_7,p_8\}$
is a (maximal) trap

$$\bullet Q \supseteq Q\bullet$$

(similarly for R_2)