

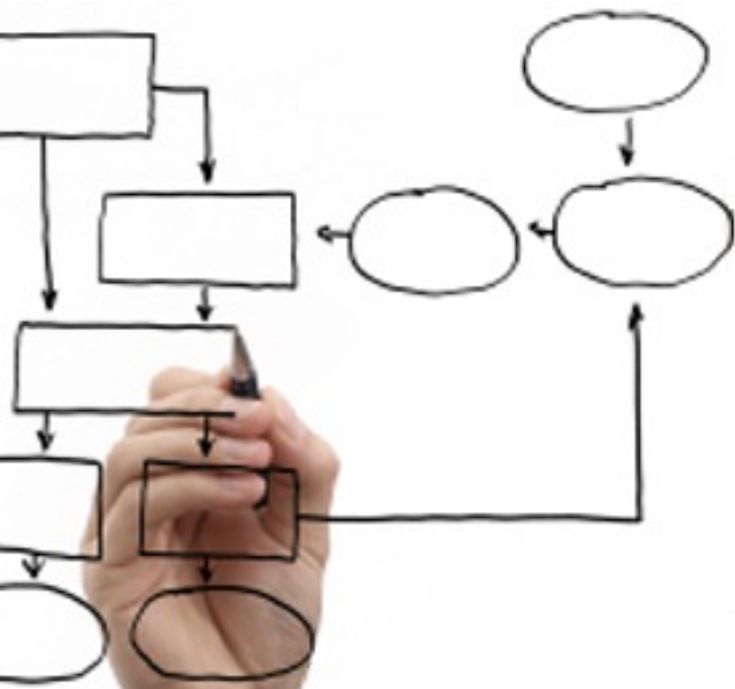
Business Processes Modelling

MPB (6 cfu, 295AA)

Roberto Bruni

<http://www.di.unipi.it/~bruni>

13 - Sound by construction



Object



We show a technique to build sound
Workflow nets

Soundness proof by construction

Idea

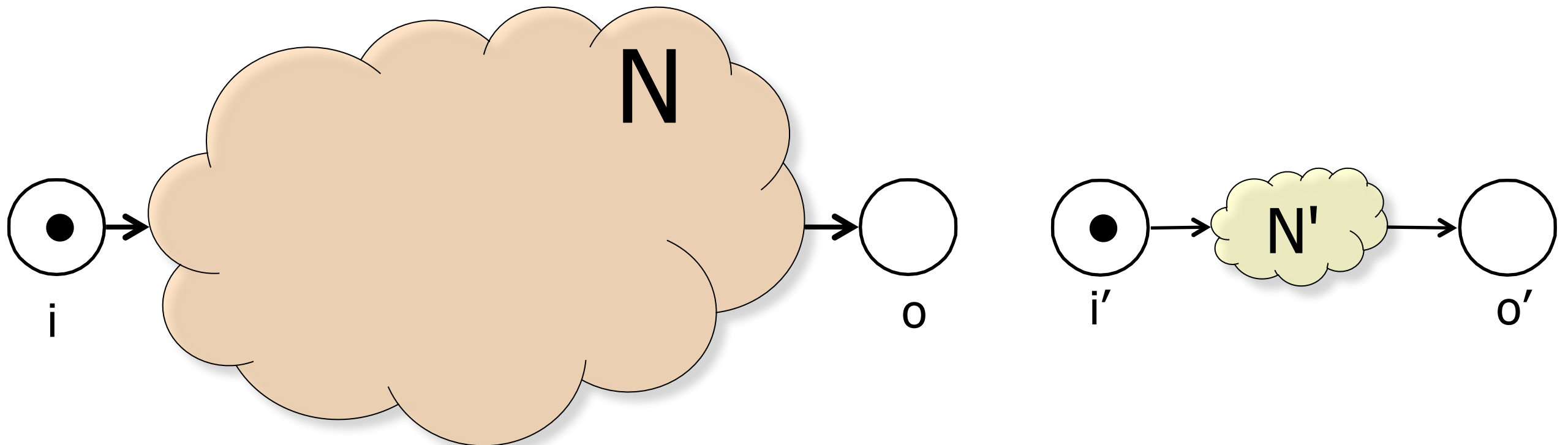
1. Find a suitable set of "building blocks"

they are (small) workflow nets
that can be (easily) proved
to be **sound** and
to be **safe** (1-bounded)

2. Define composition patterns so that
by composing **safe and sound** WF nets
we get **safe and sound** WF nets

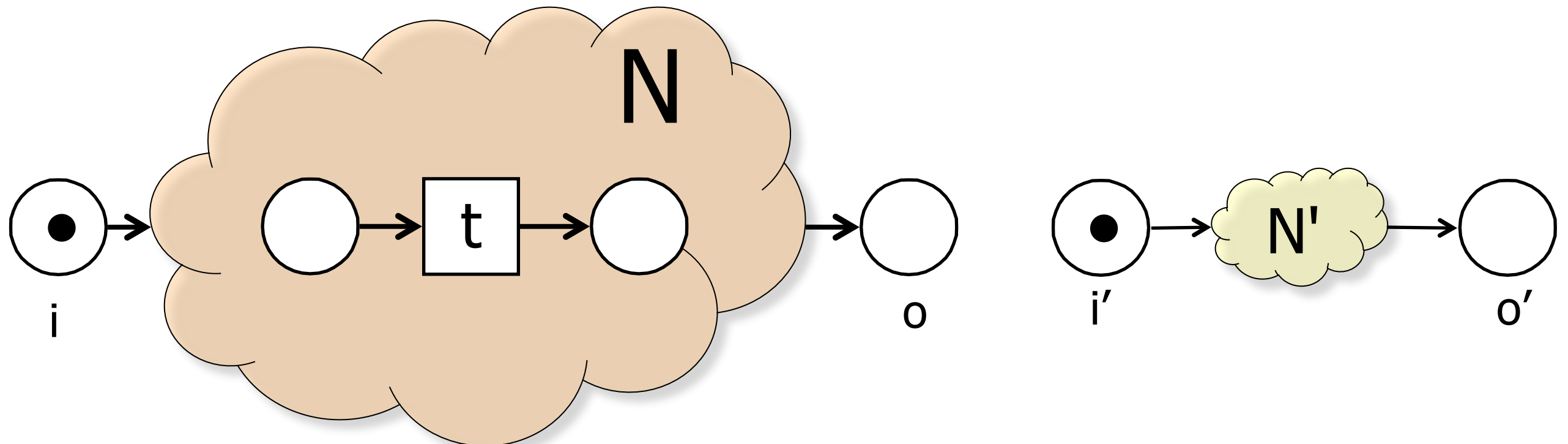
Sound and safe by composition

Let N , N' be two safe and sound workflow nets



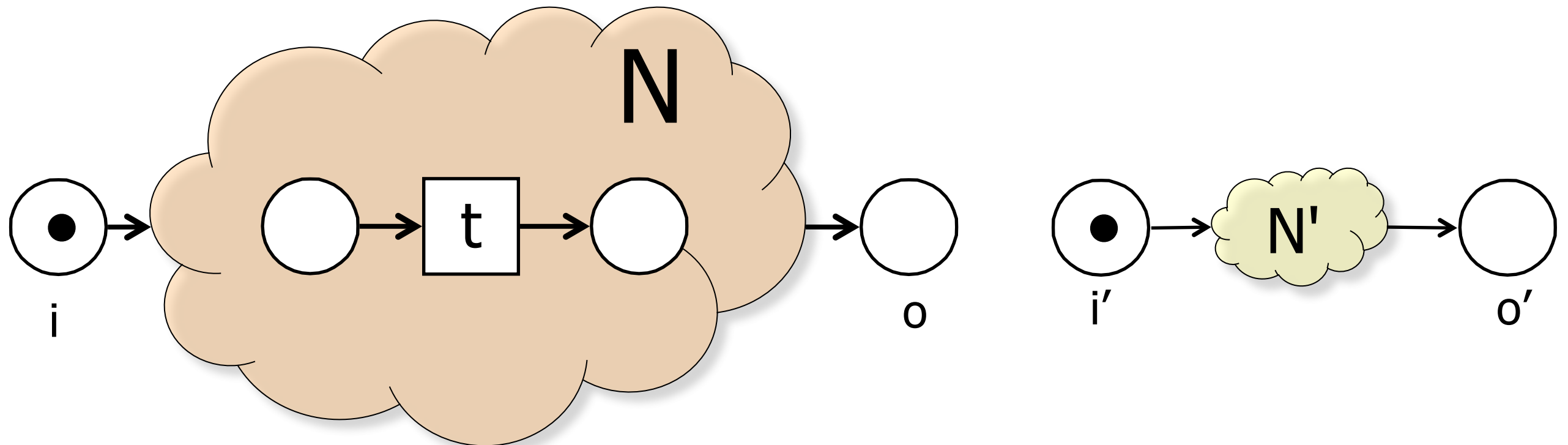
Sound and safe by composition

Let t be a task of N with exactly one input and one output place



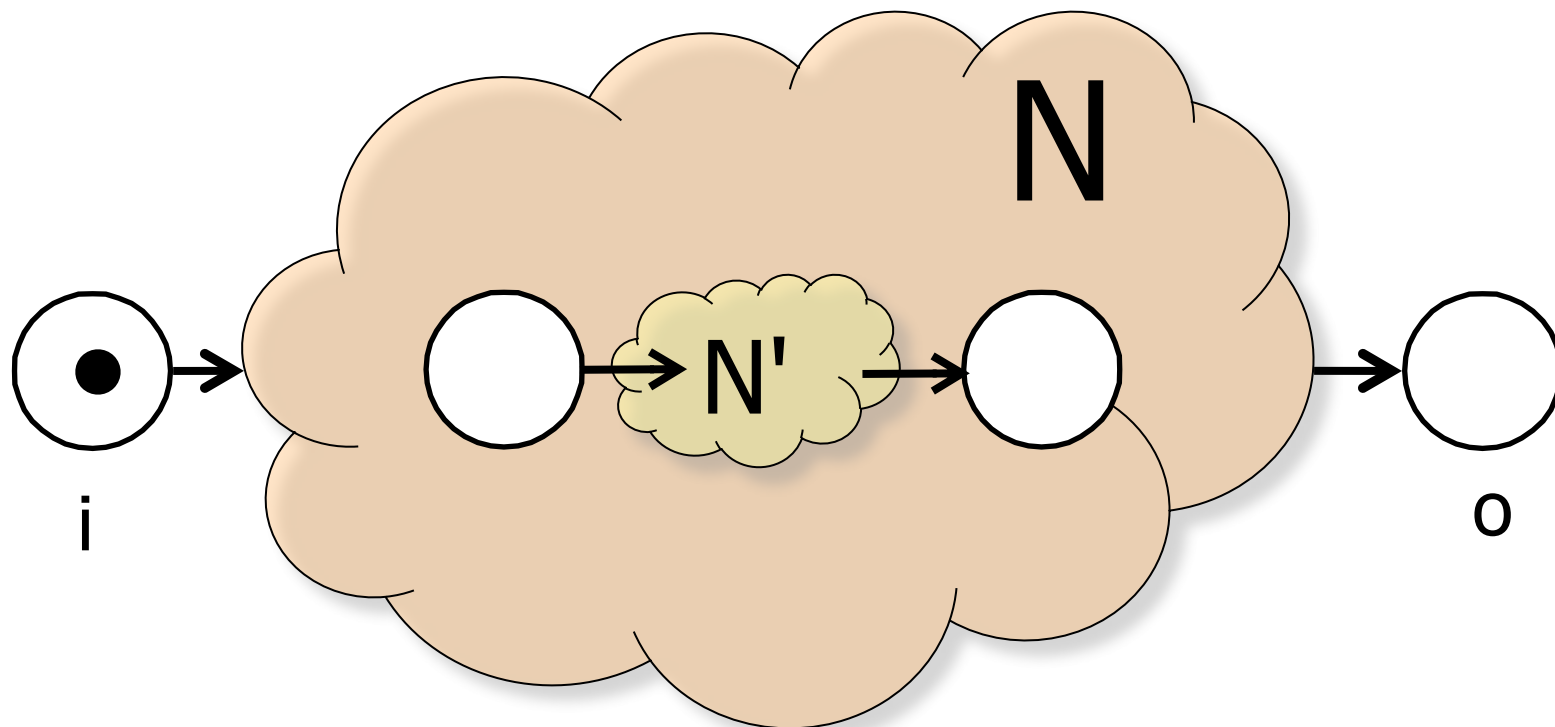
Sound and safe by composition

Let $N[N'/t]$ denote the net obtained by replacing the task t in N by N'

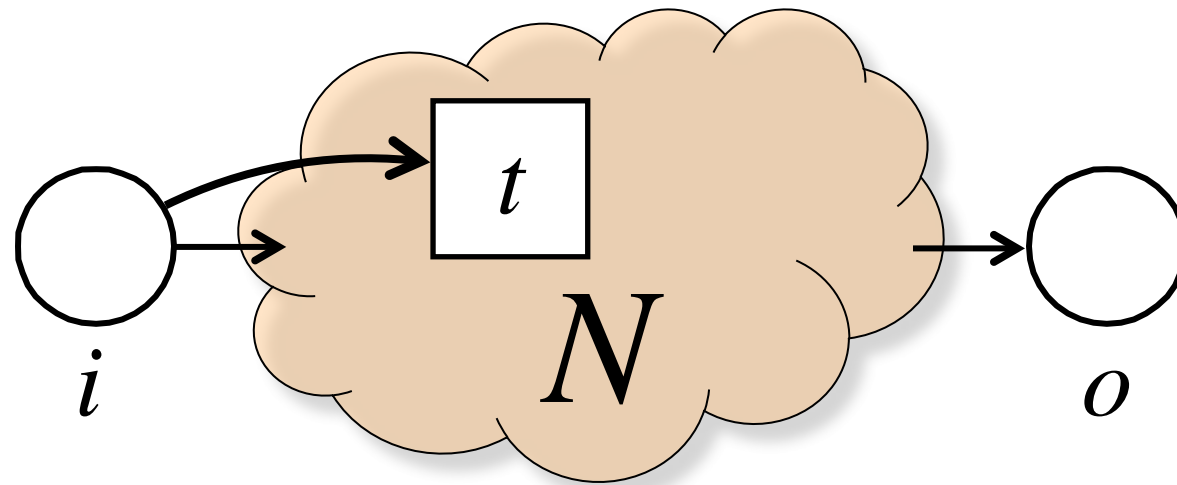


Sound and safe by composition

The net $N[N'/t]$ is
a **sound and safe workflow net**
(proof omitted)

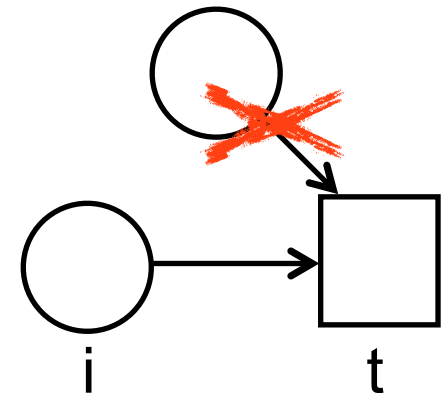


Initial transitions



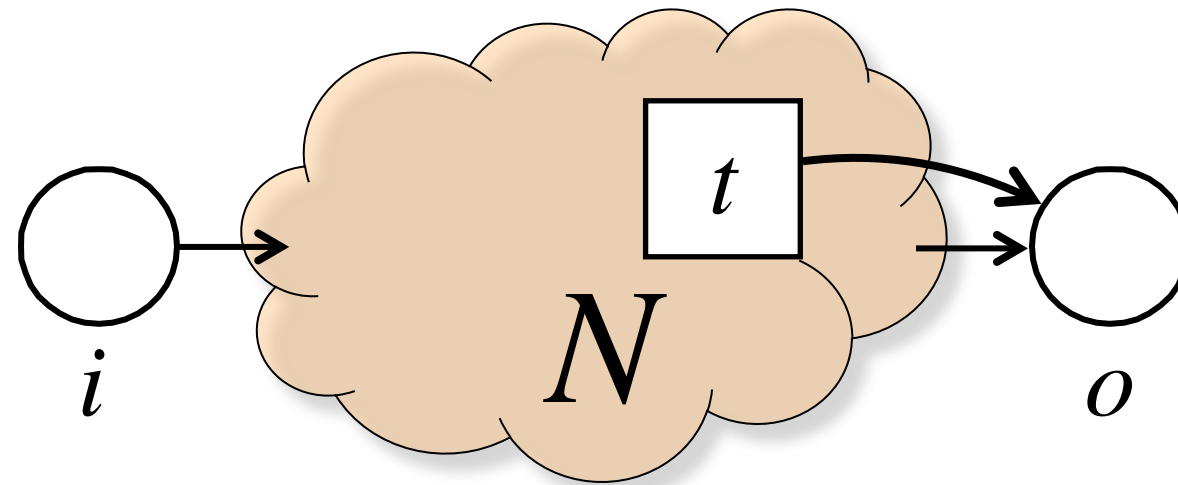
A transition t of a workflow net N is called **initial** if there is an arc from the initial place i to the transition t

Lemma: Let N be a sound WF net.
If the transition t is initial then $\bullet t = \{i\}$



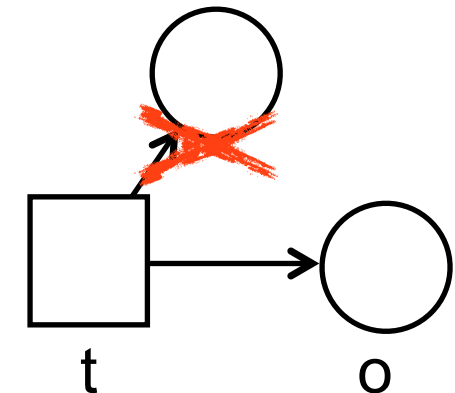
(otherwise t would be a dead transition)

Final transitions



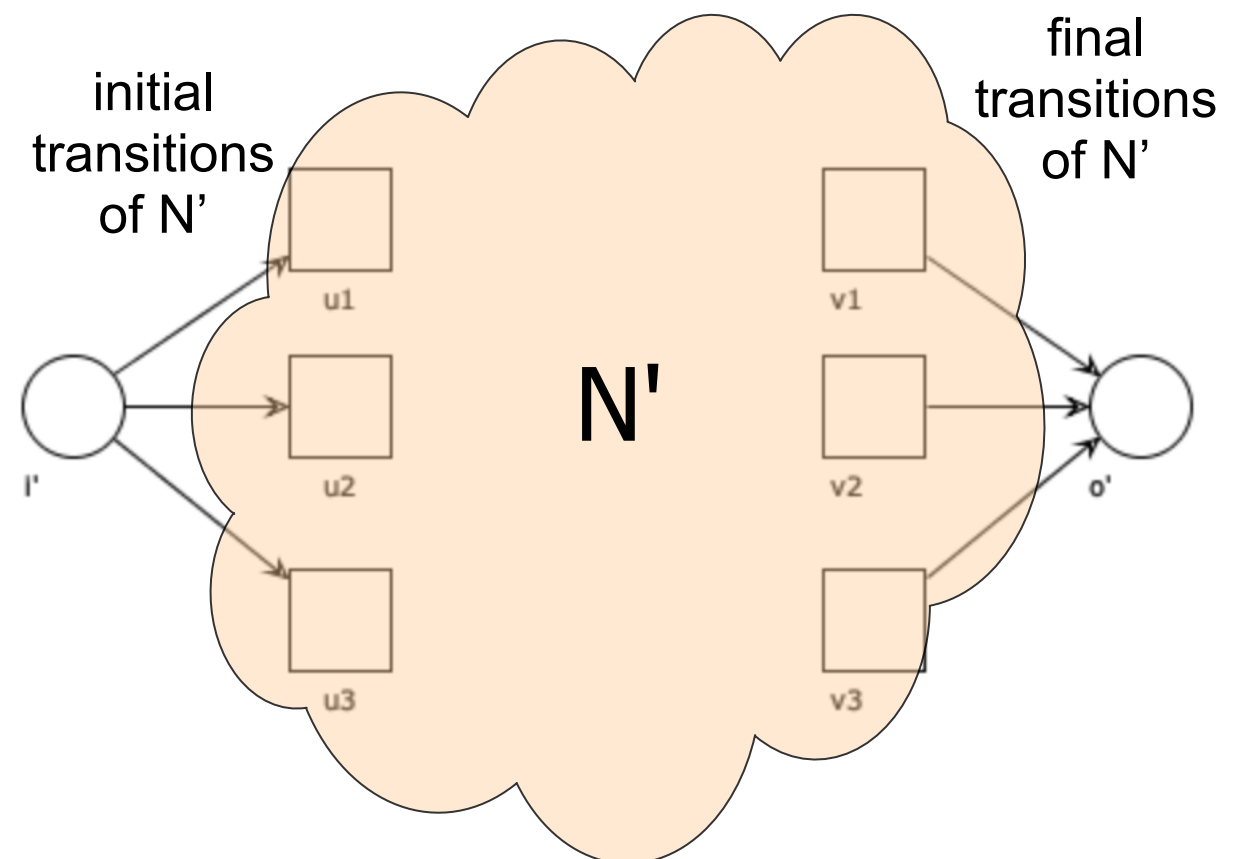
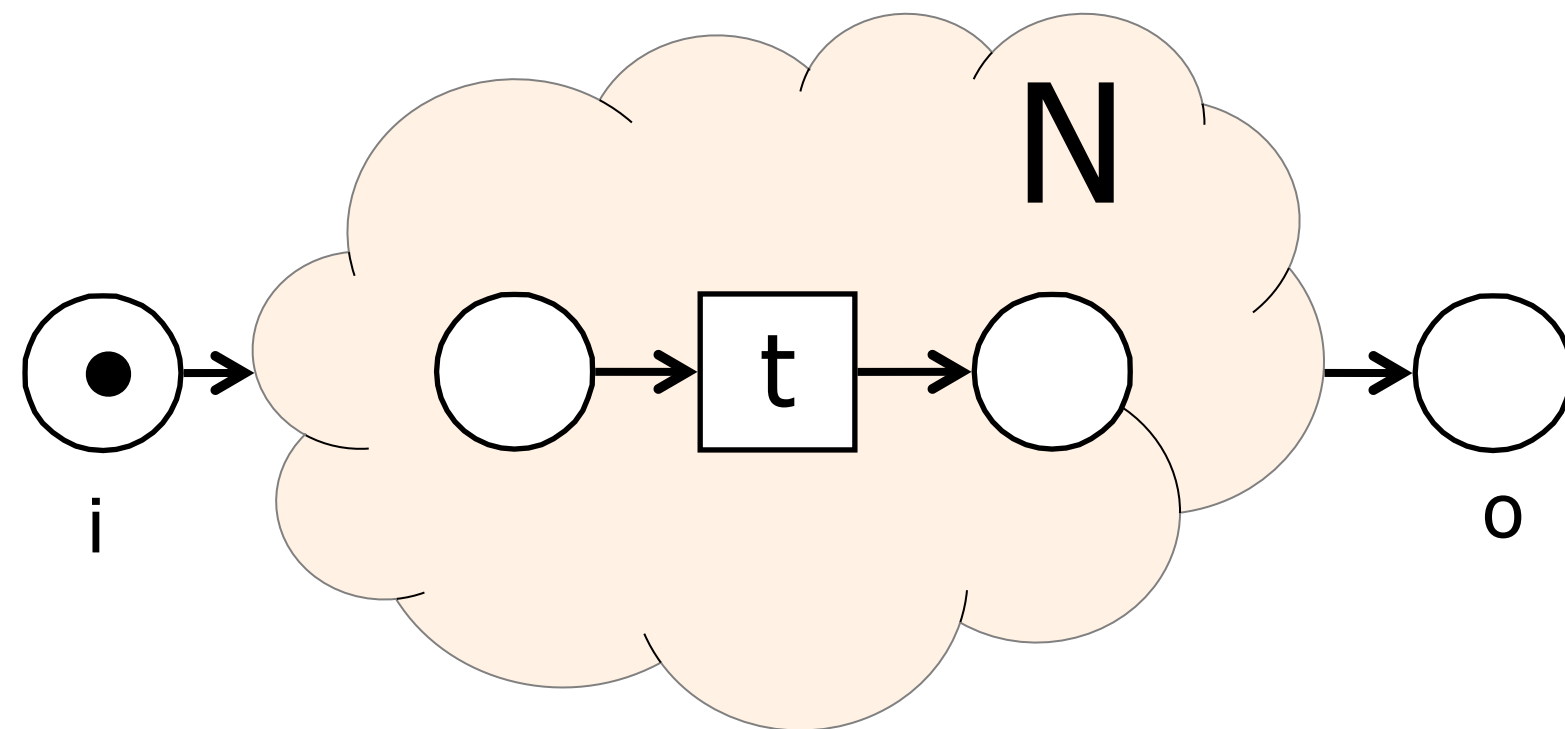
A transition t of a workflow net N is called **final** if there is an arc from the transition t to the final place o

Lemma: Let N be a sound WF net.
If the transition t is final then $t \bullet = \{o\}$

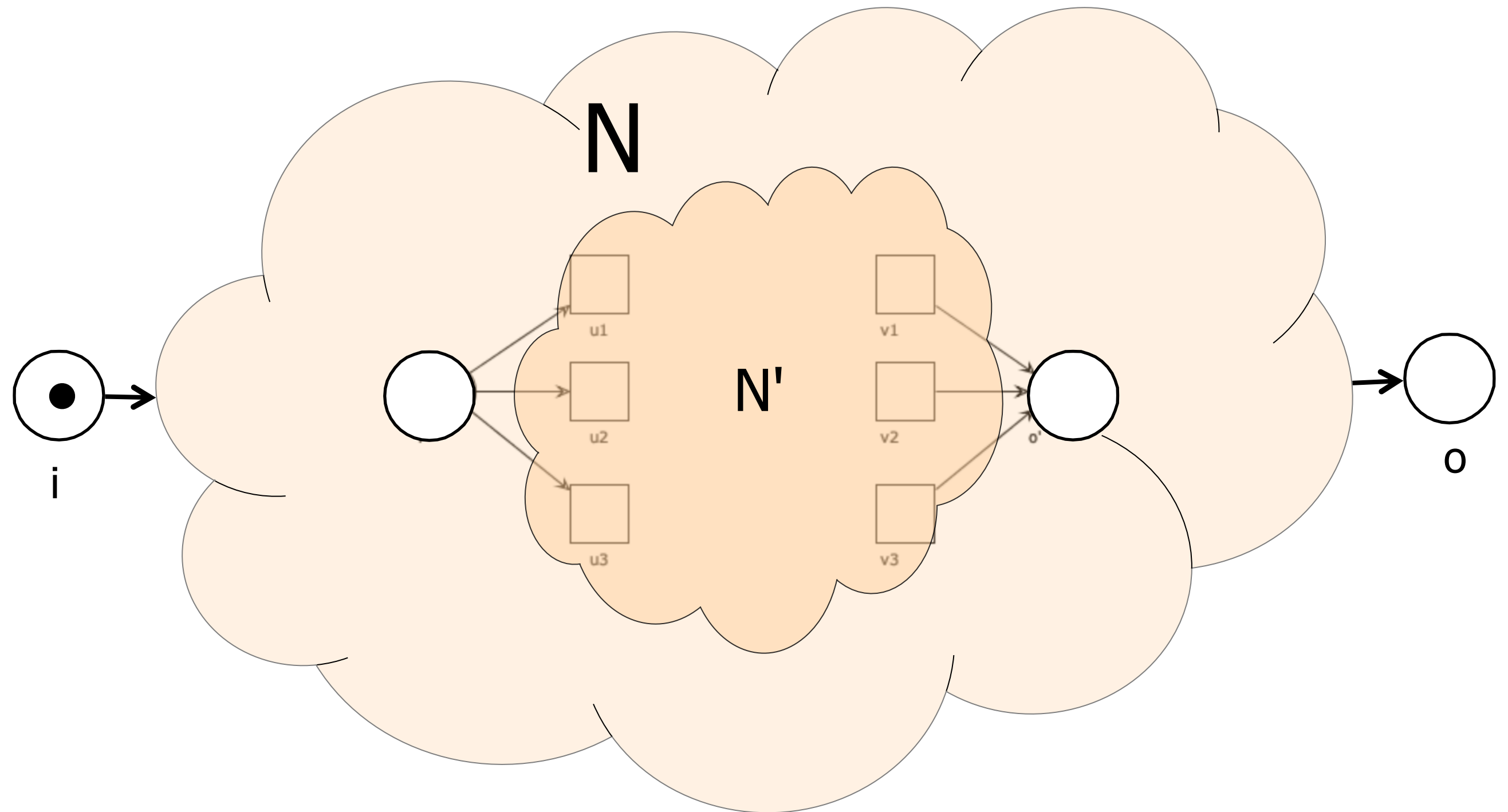


(otherwise t would be dead or proper completion would not hold)

Sketching the idea



Sketching the idea



Proof sketch

Intuitively

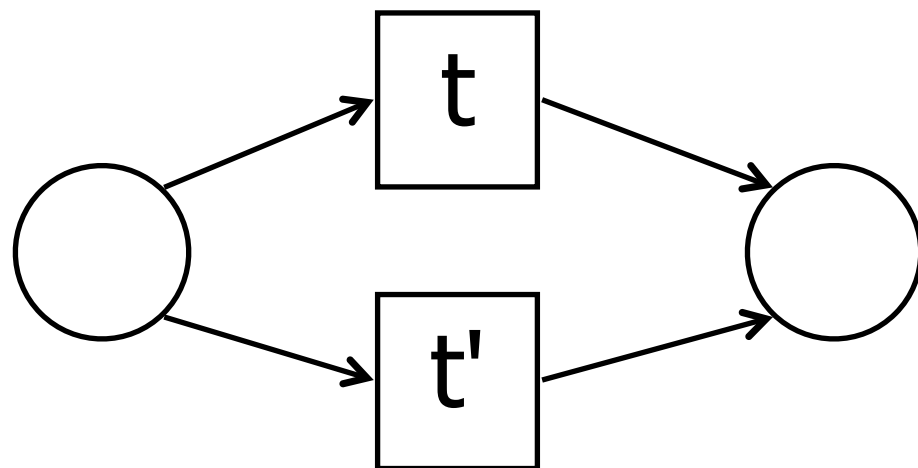
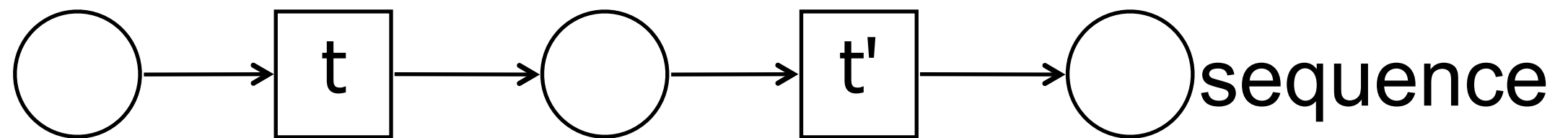
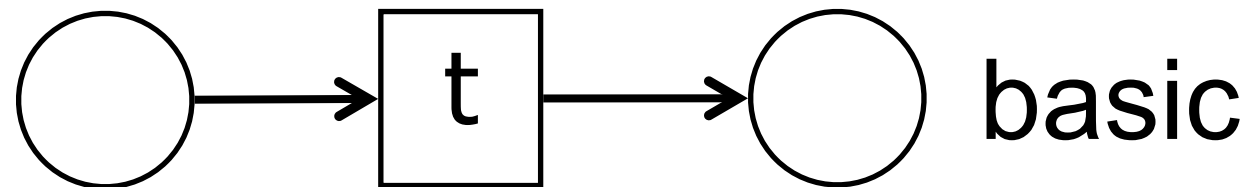
a sound workflow net behaves as a transition:
it takes one token from its input place and
it produces one token to its output place
(but not atomically)

Formally

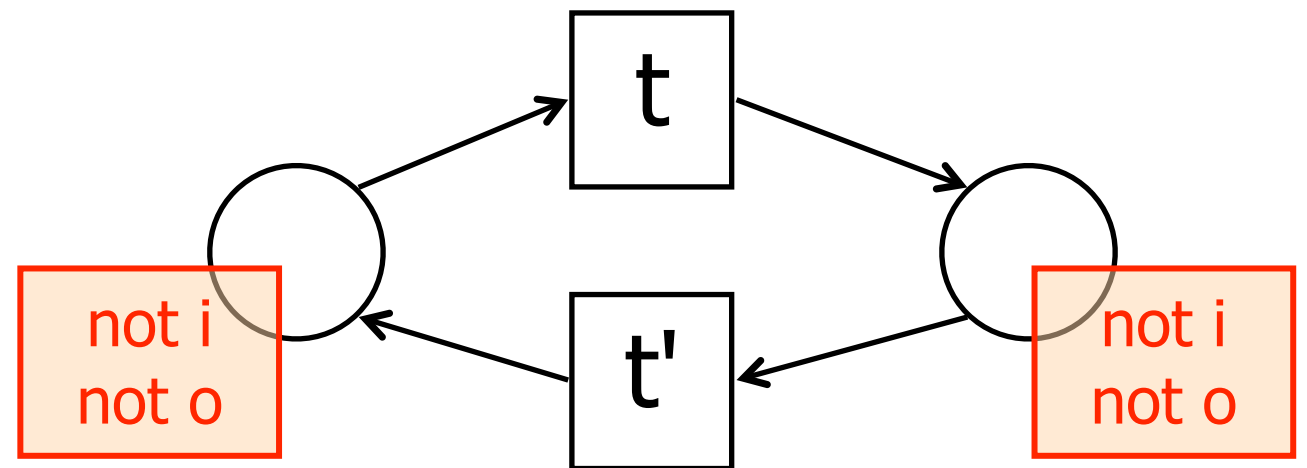
the crux of the proof is showing a bijective correspondence
between

markings of the composed net $N[N'/t]$
and the pairs of markings in N and N'

Some Building Blocks 1

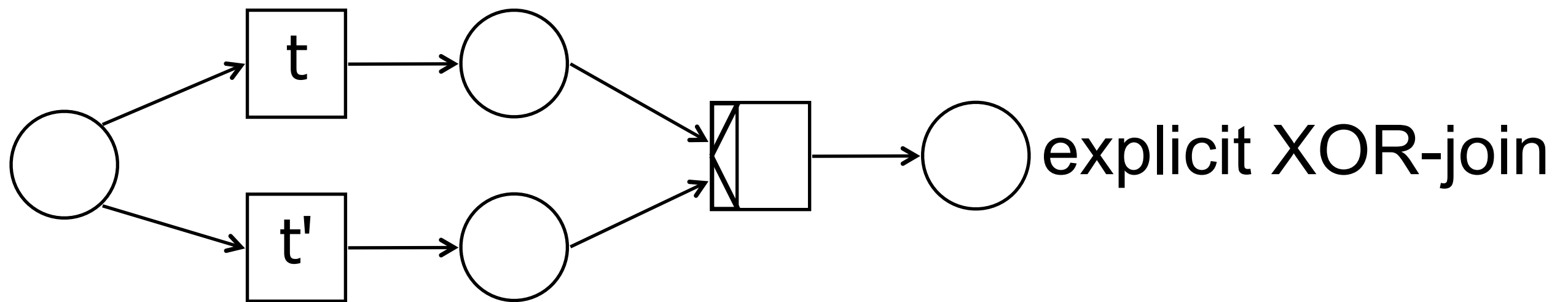
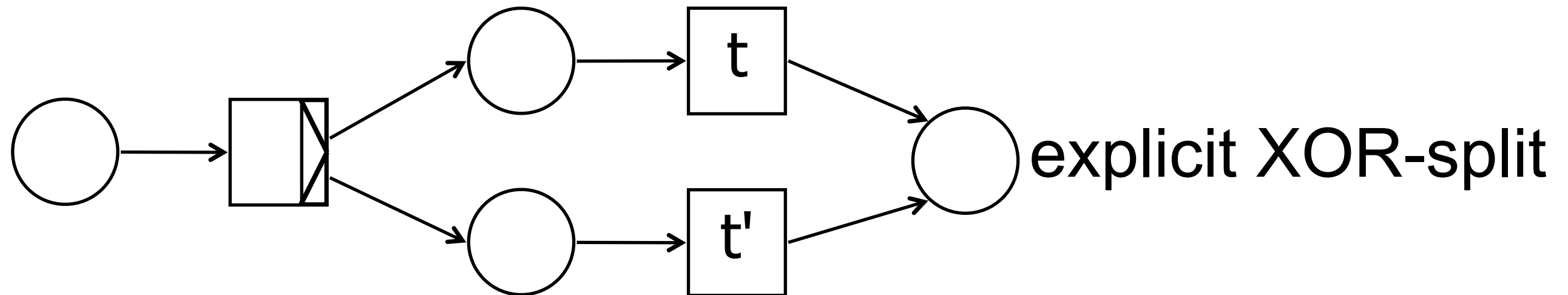


implicit XOR

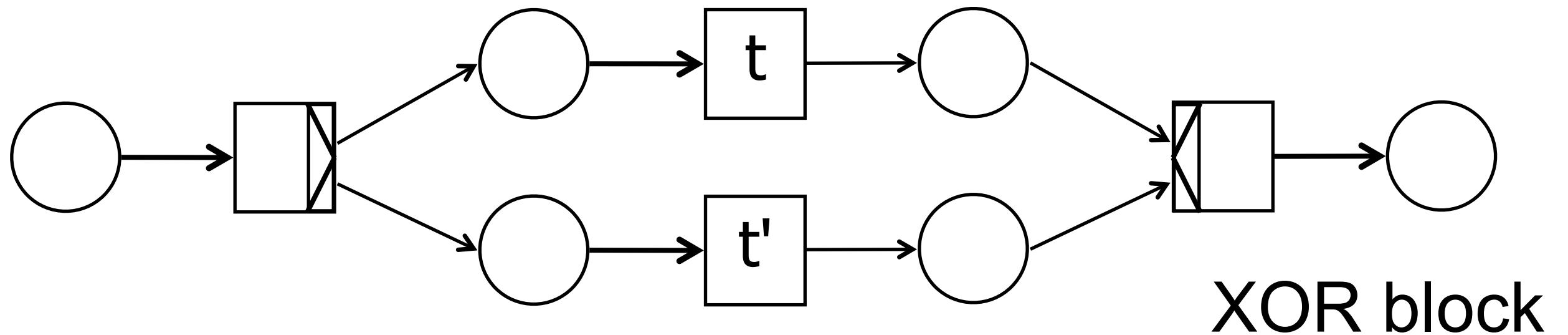


iteration

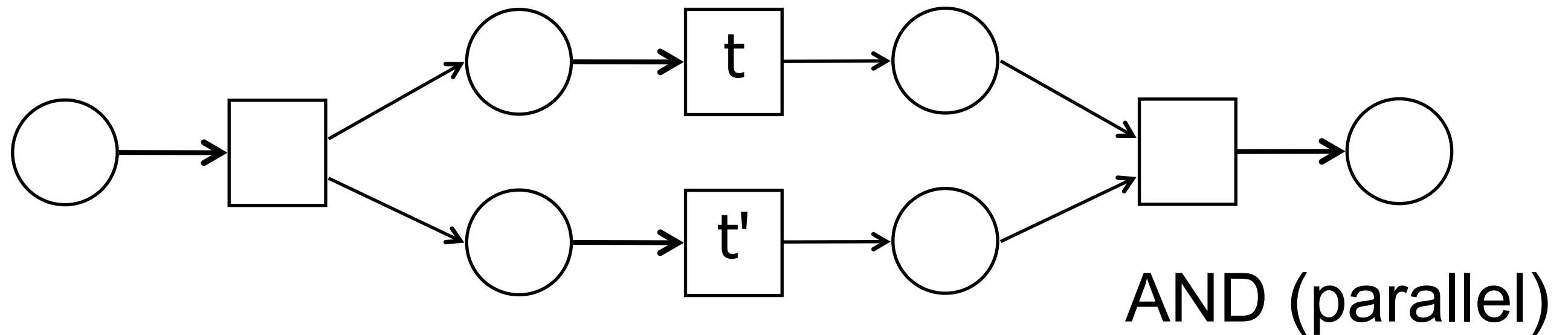
Some Building Blocks 2



Some Building Blocks 3

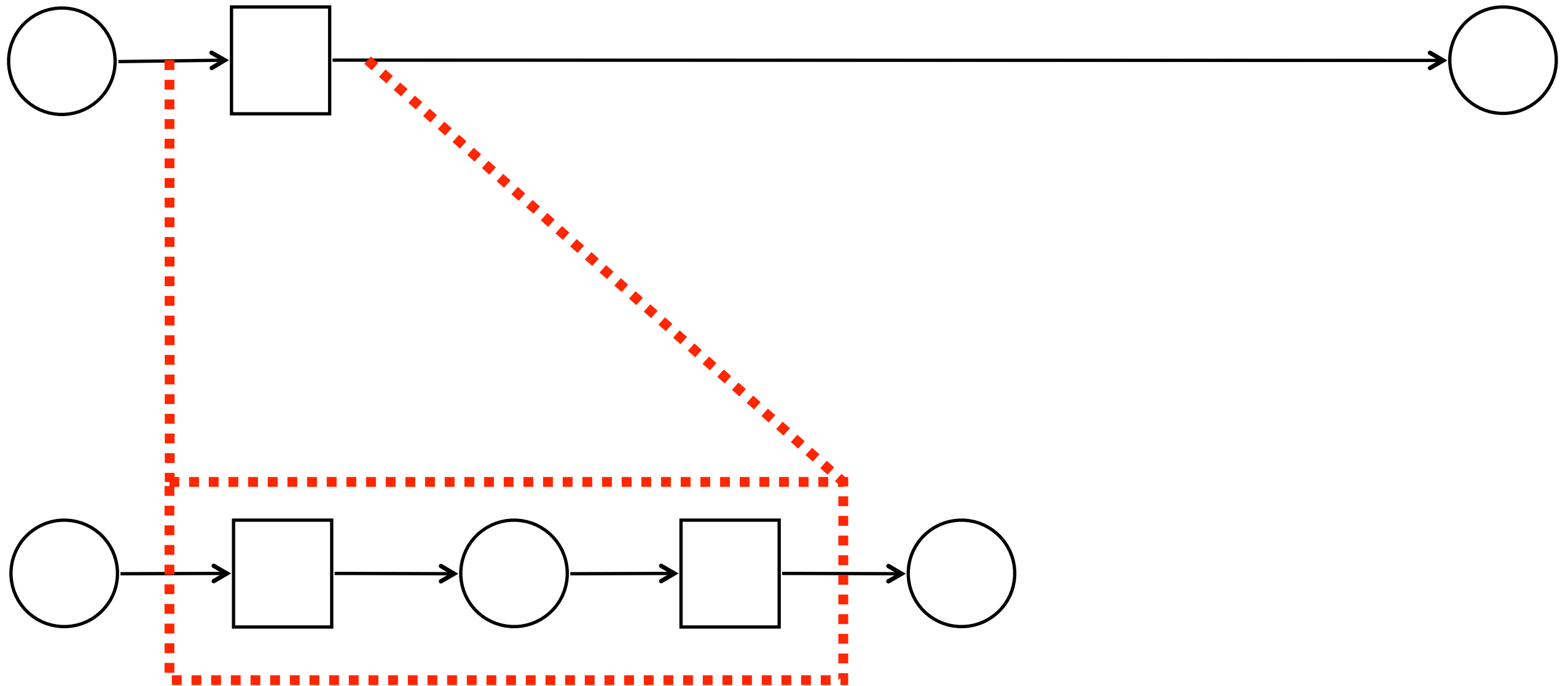


Some Building Blocks 4

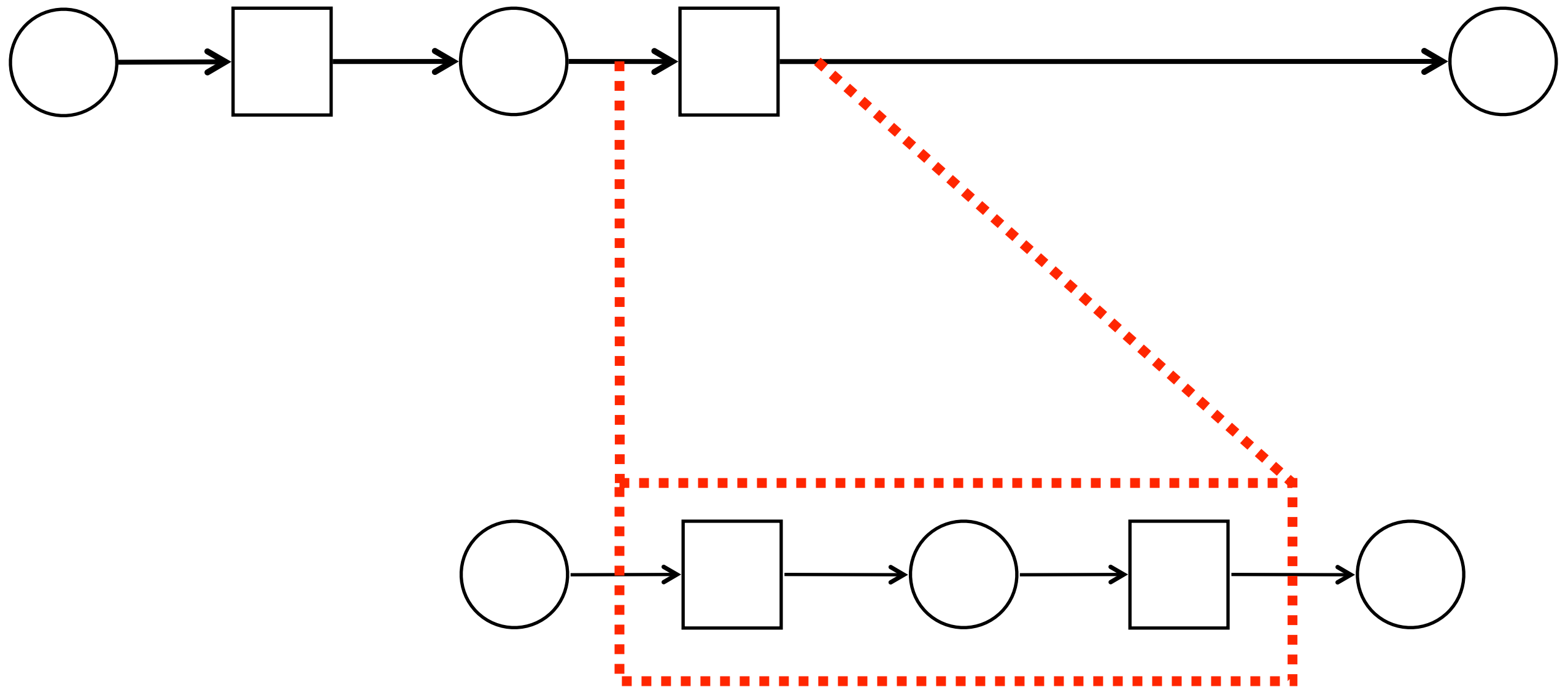


But you can define more blocks on your own

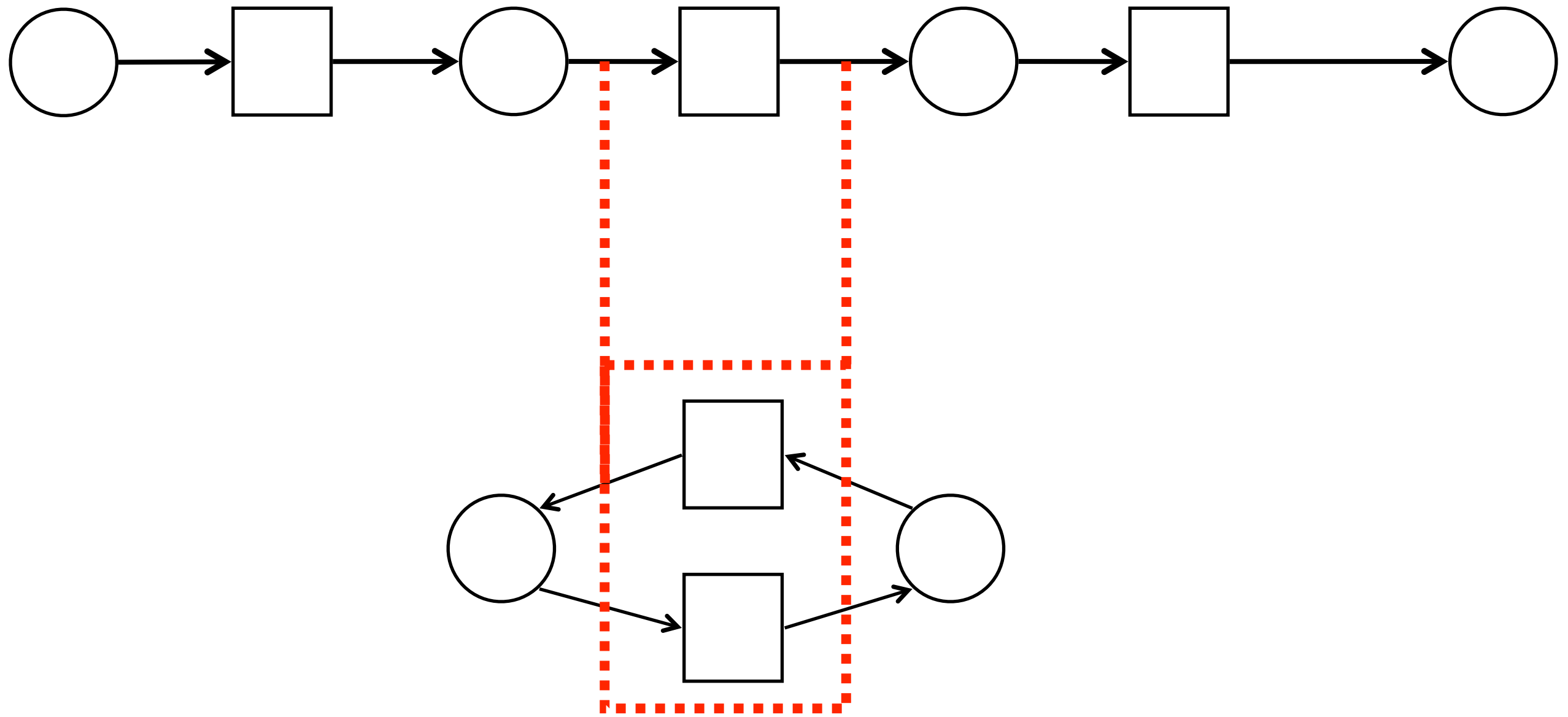
Example: refinement



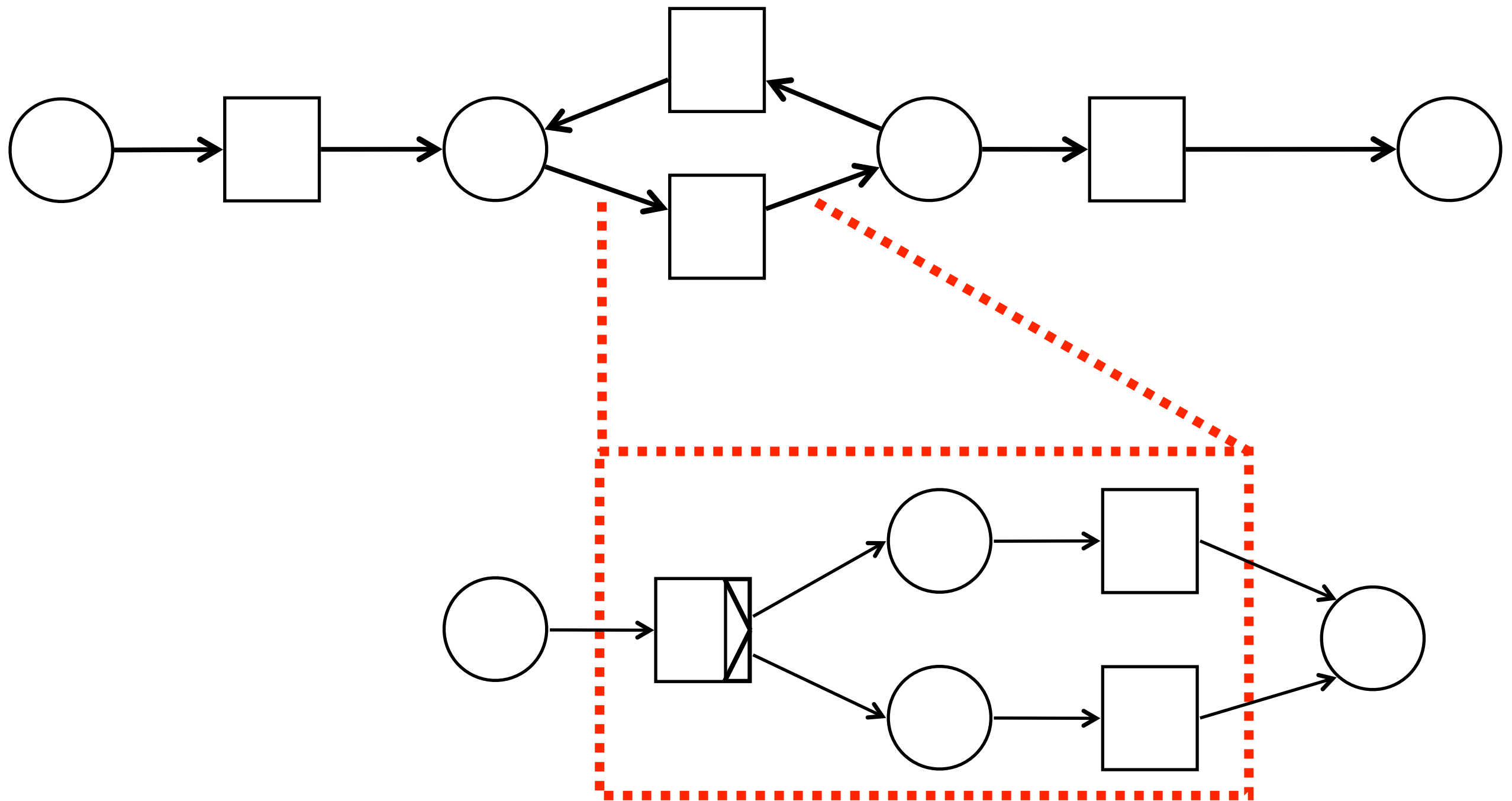
Example: refinement



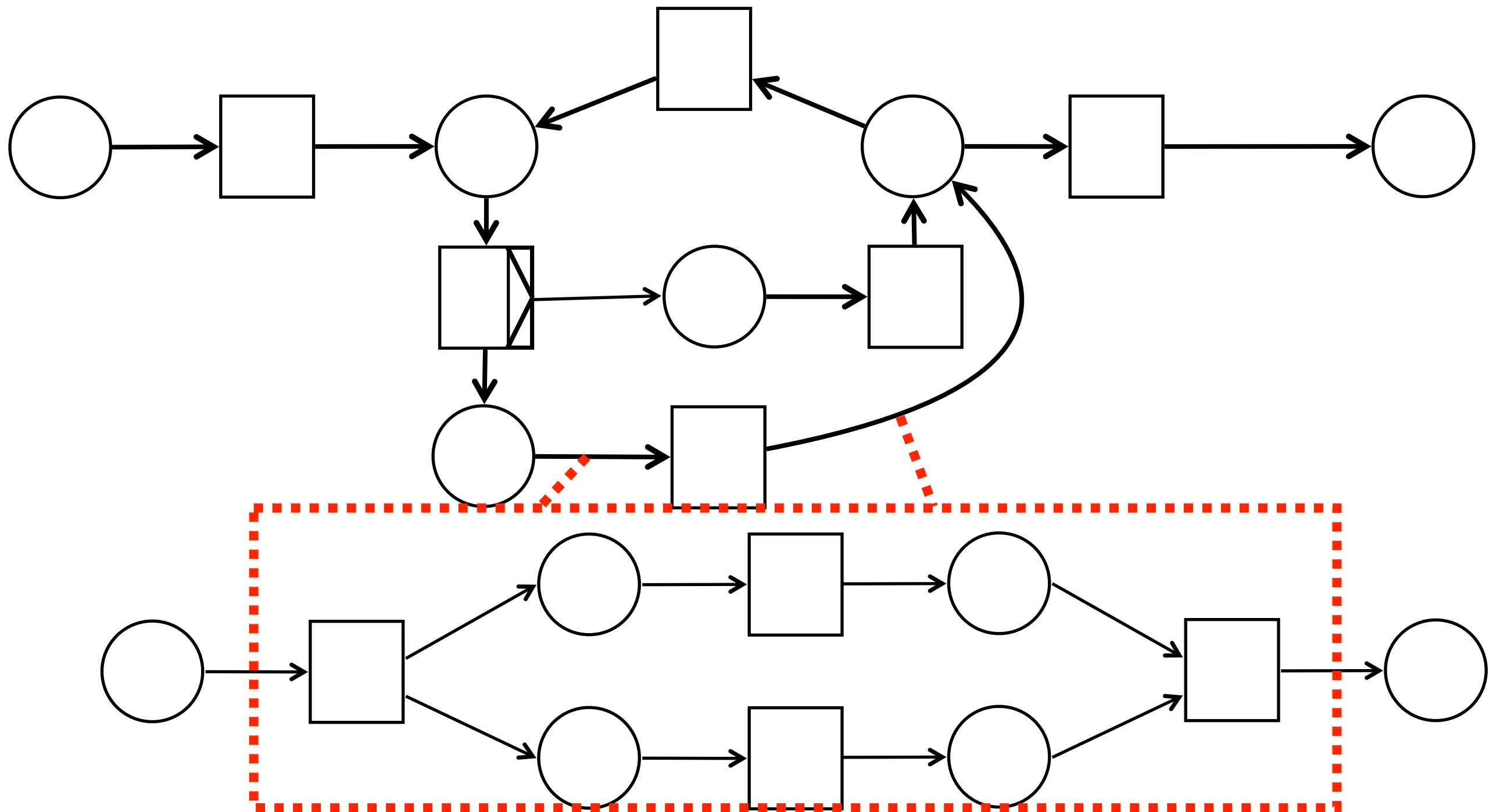
Example: refinement



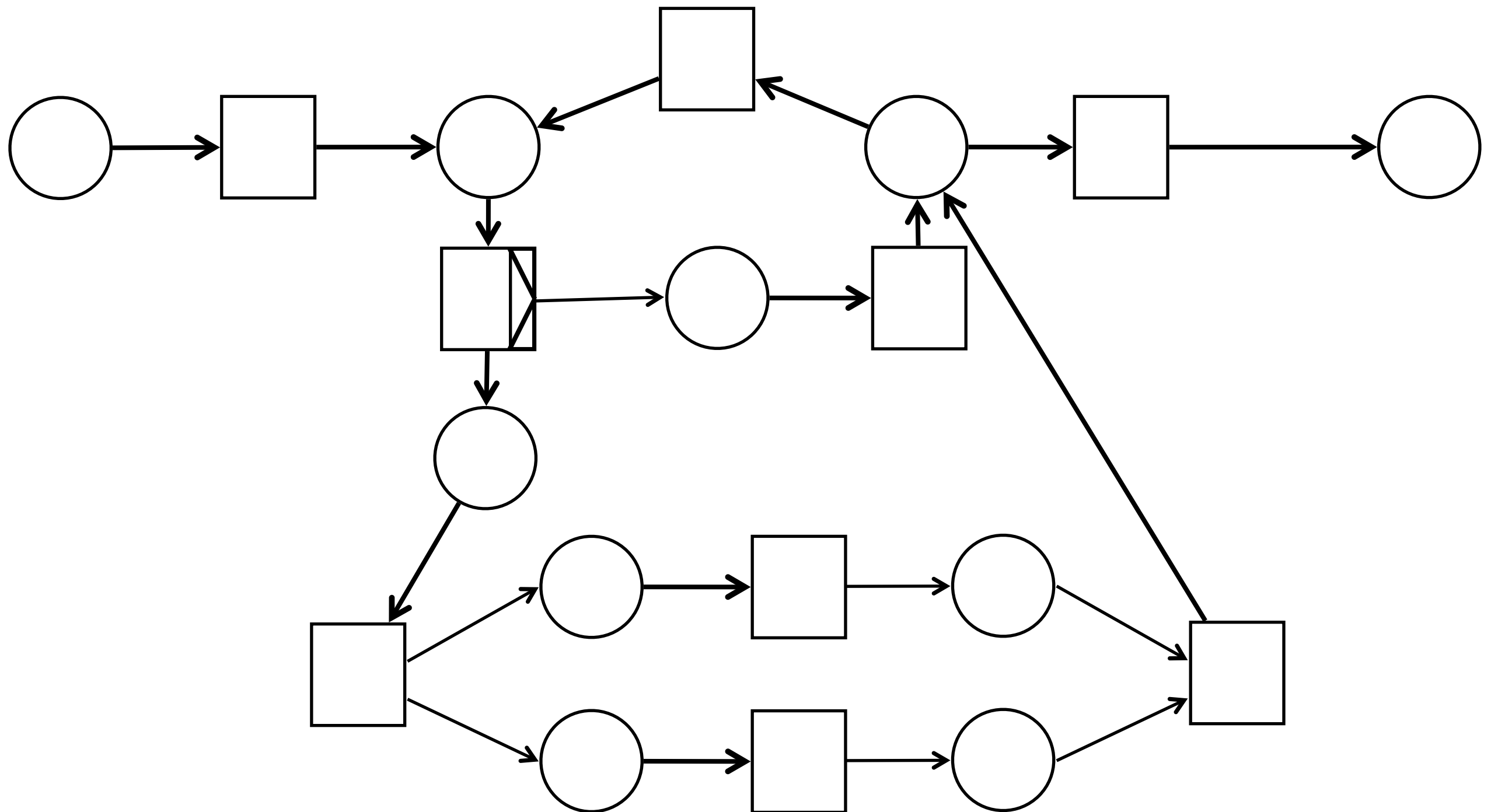
Example: refinement



Example: refinement

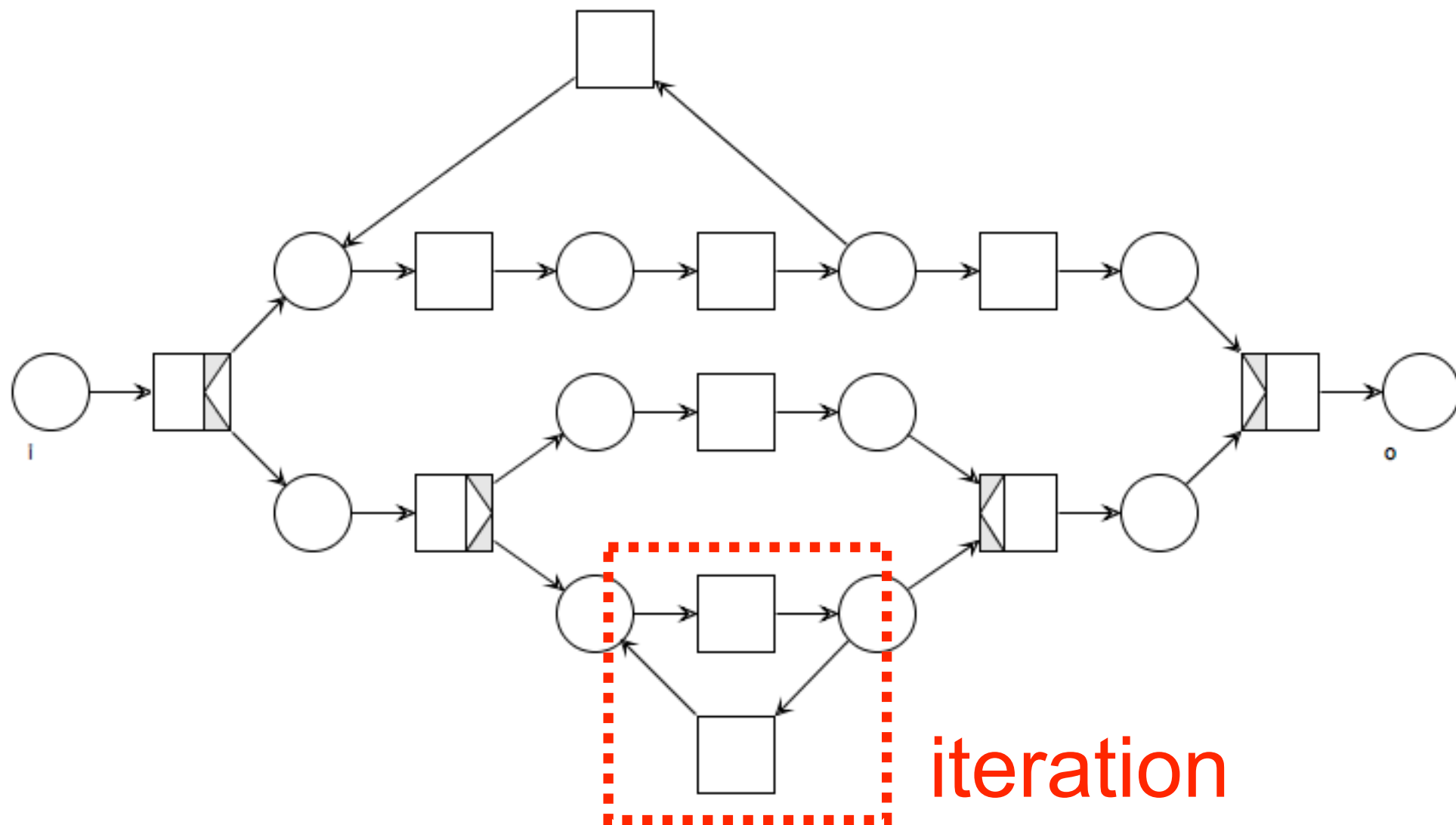


Example: refinement



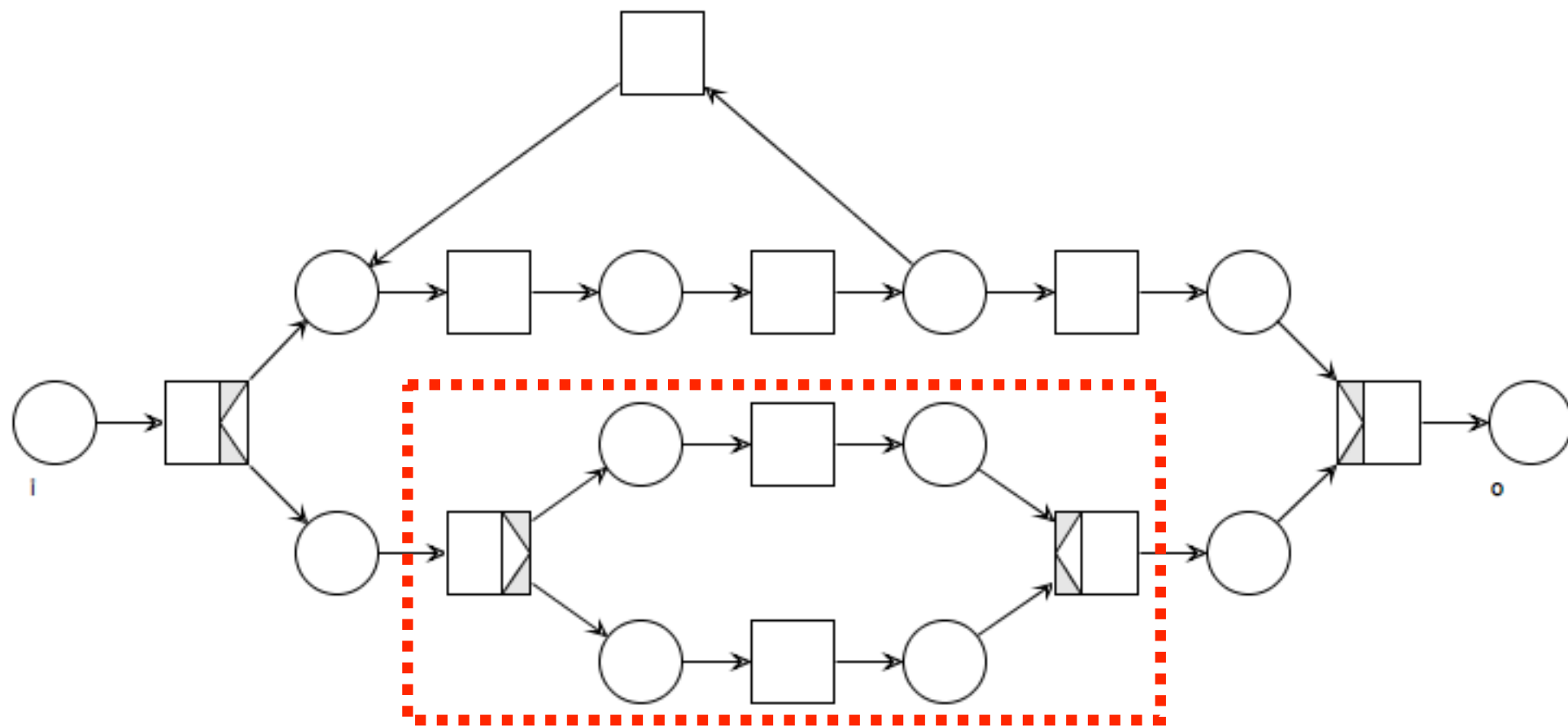
Example: abstraction

Prove that the net below is
a safe and sound workflow net



Example: abstraction

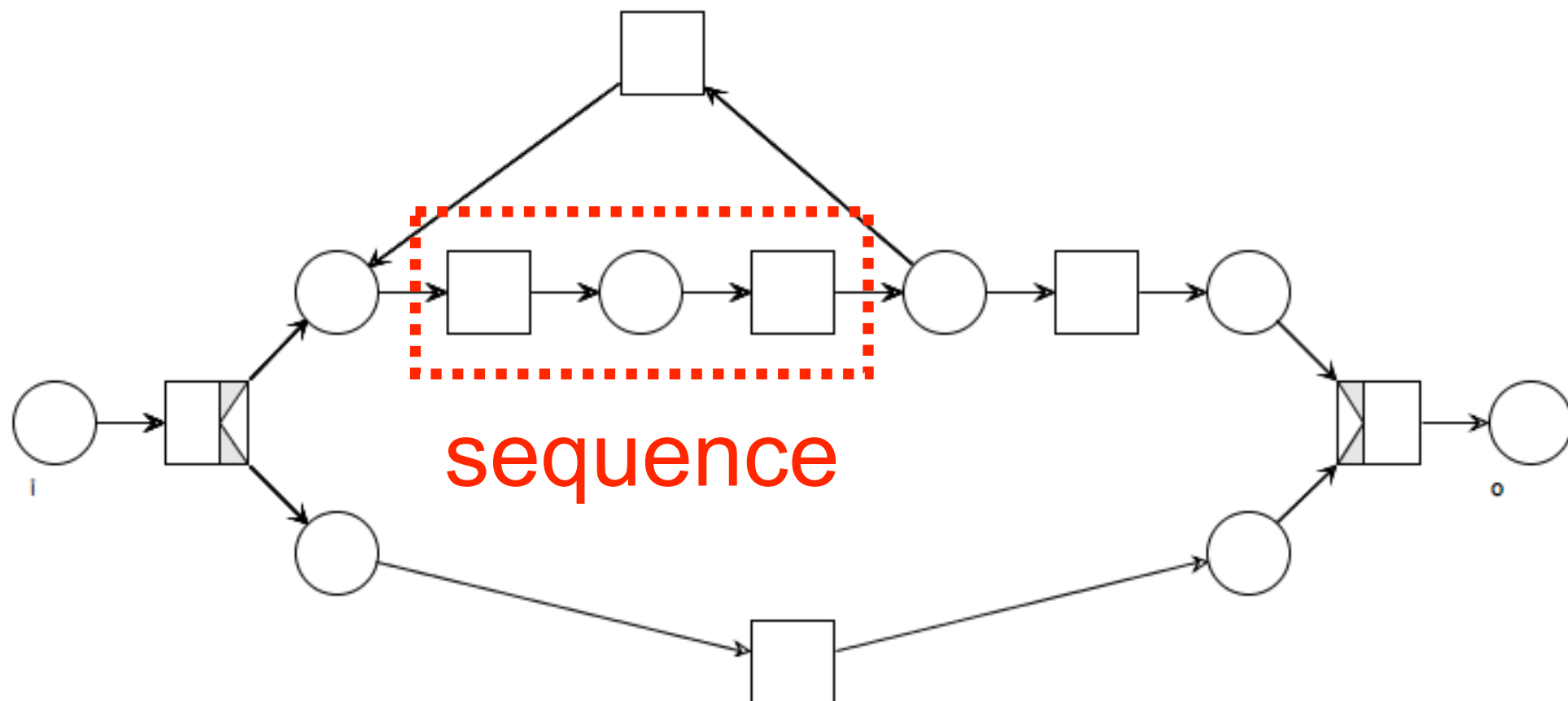
Prove that the net below is
a safe and sound workflow net



explicit XOR block

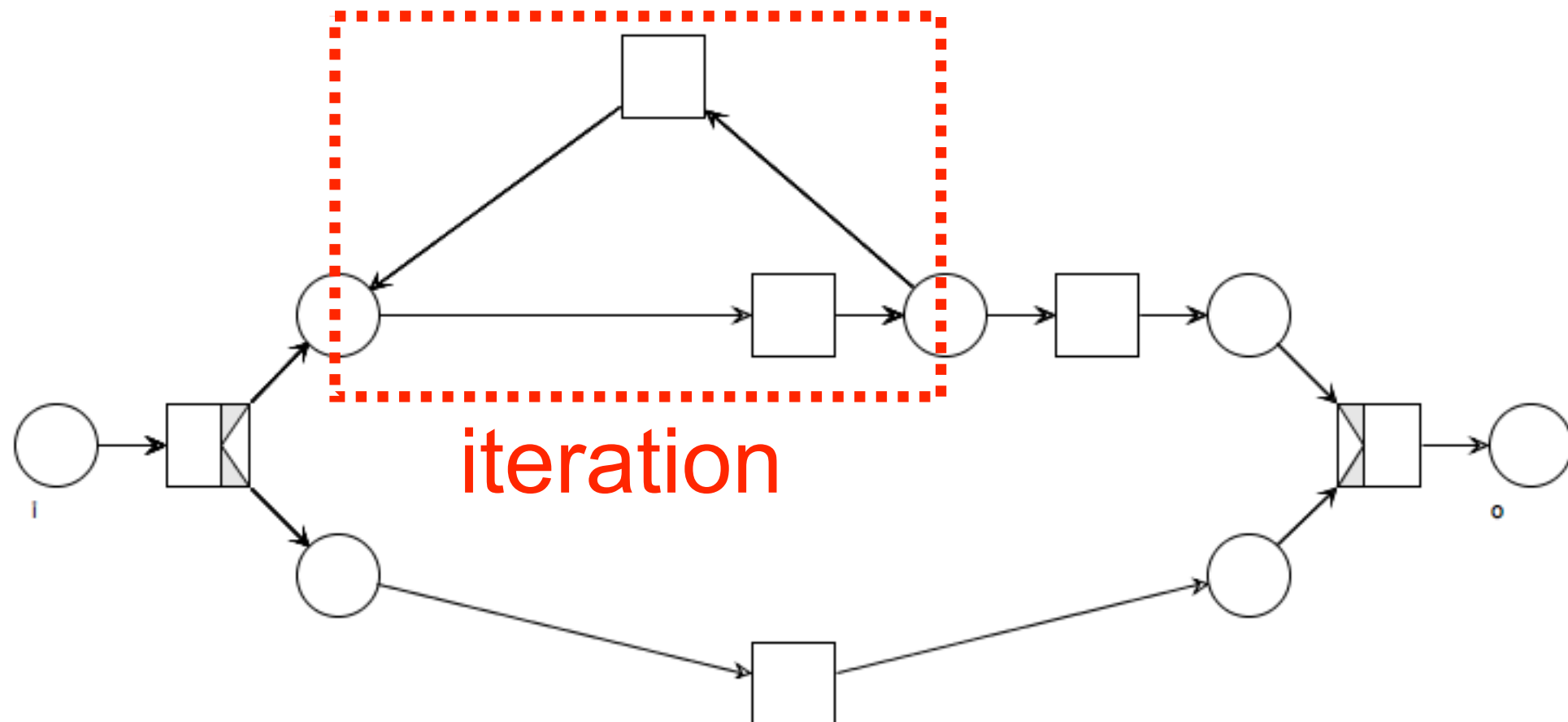
Example: abstraction

Prove that the net below is
a safe and sound workflow net



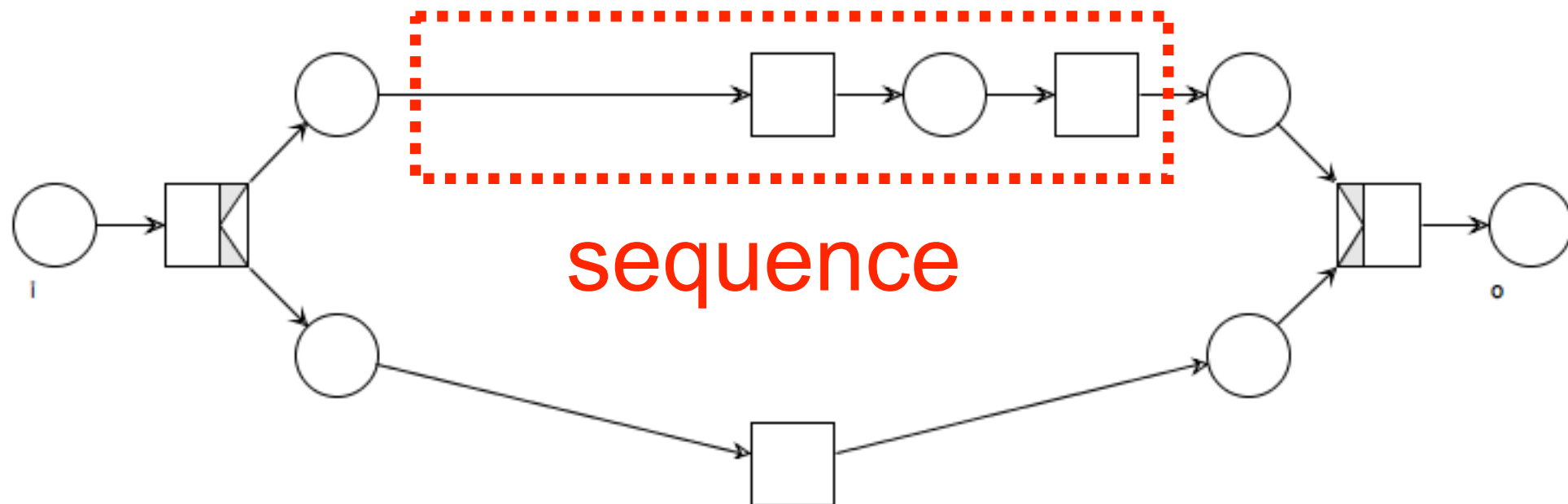
Example: abstraction

Prove that the net below is
a safe and sound workflow net



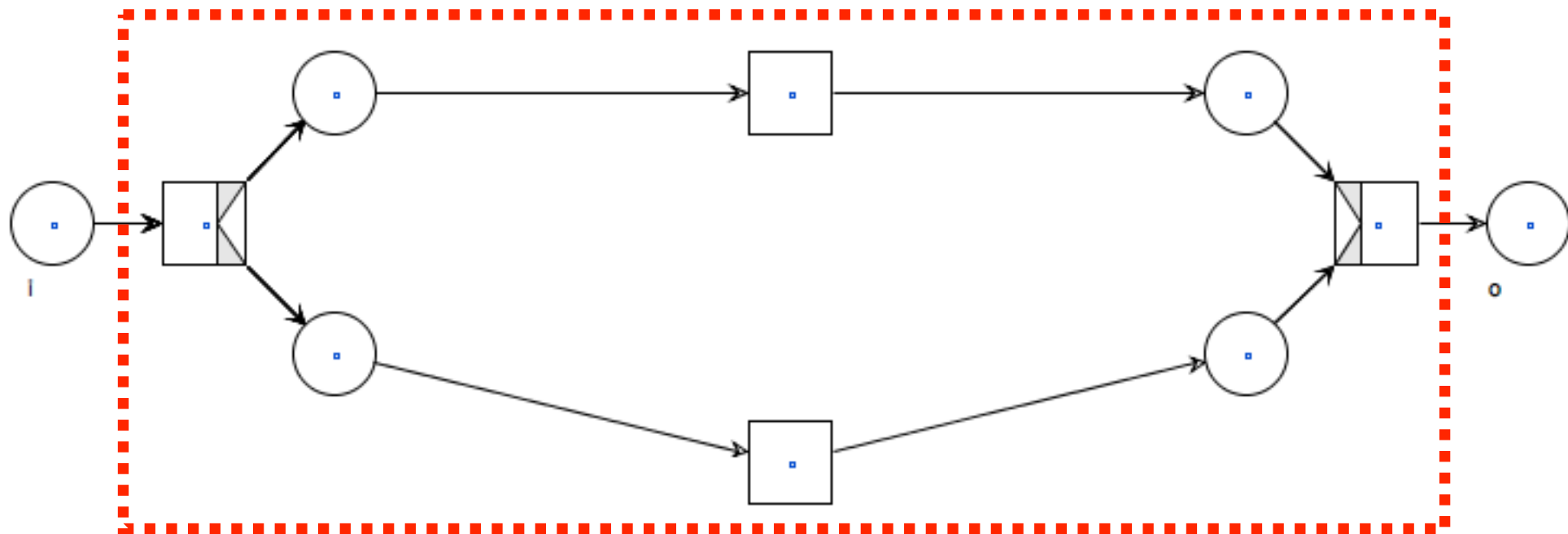
Example: abstraction

Prove that the net below is
a safe and sound workflow net



Example: abstraction

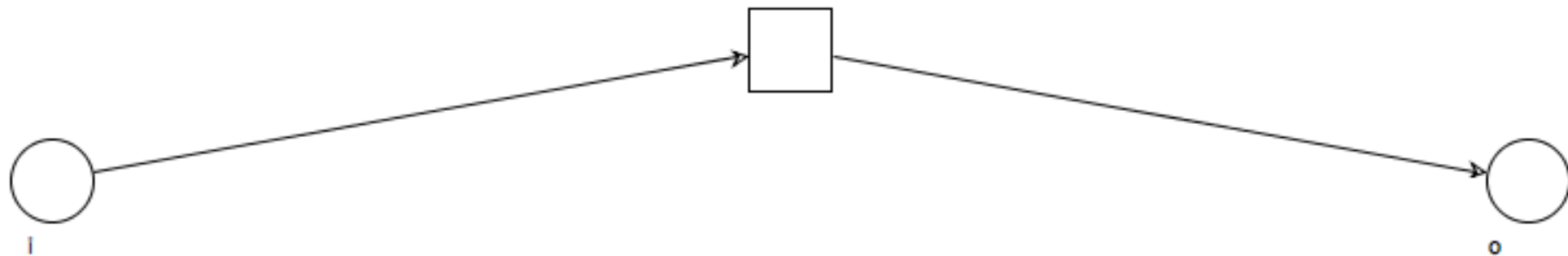
Prove that the net below is
a safe and sound workflow net



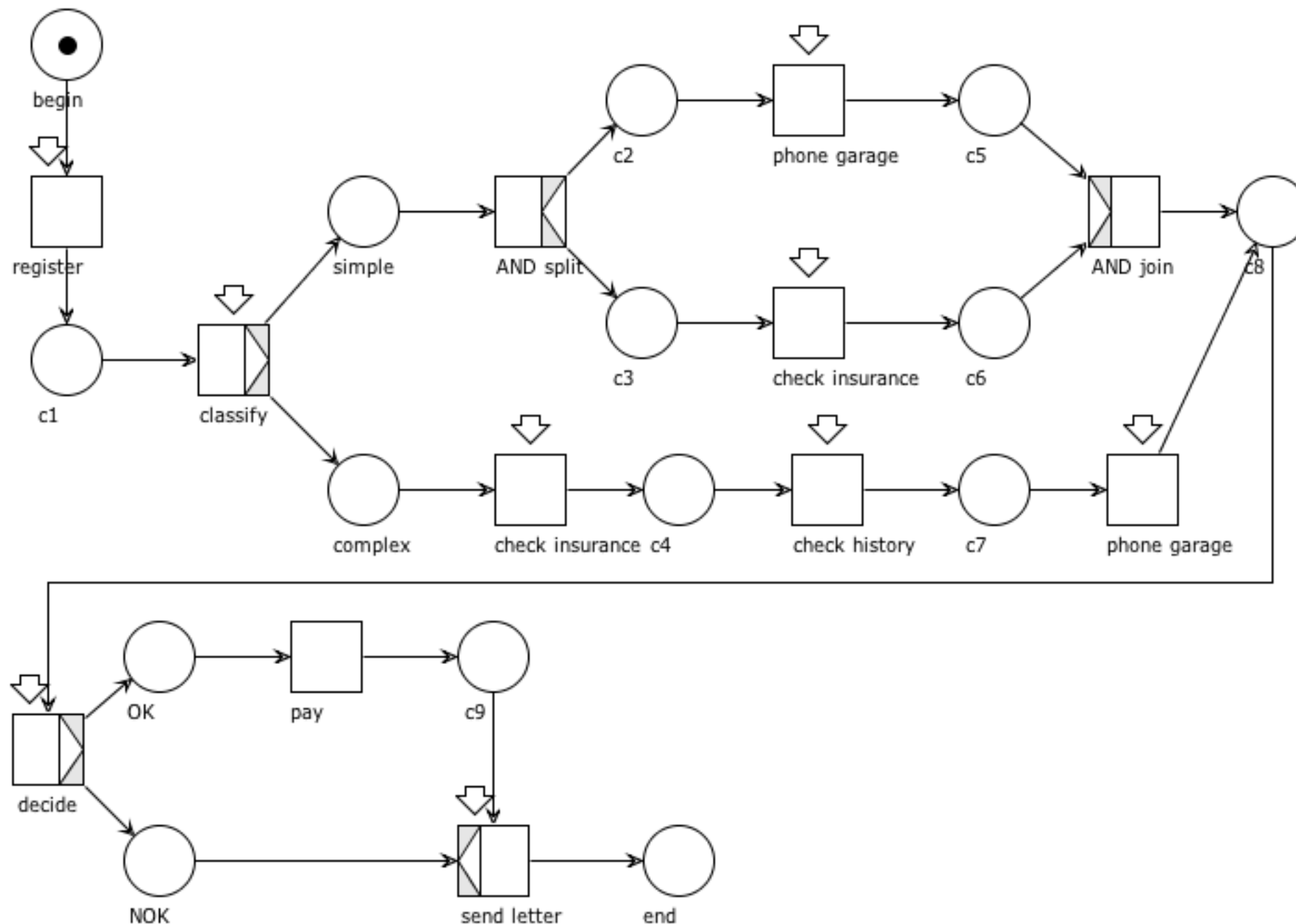
parallel (AND) block

Example: abstraction

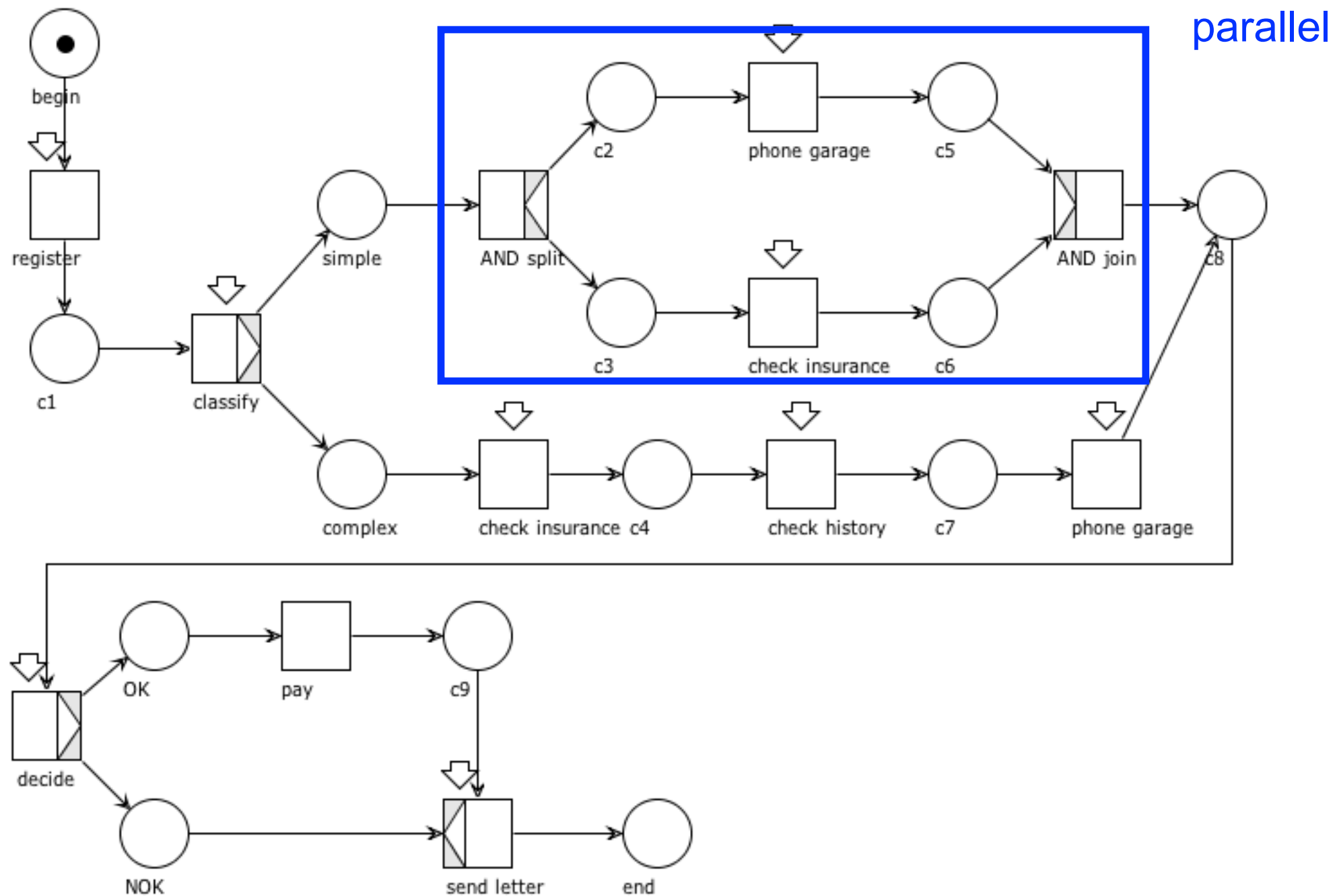
Prove that the net below is
a safe and sound workflow net



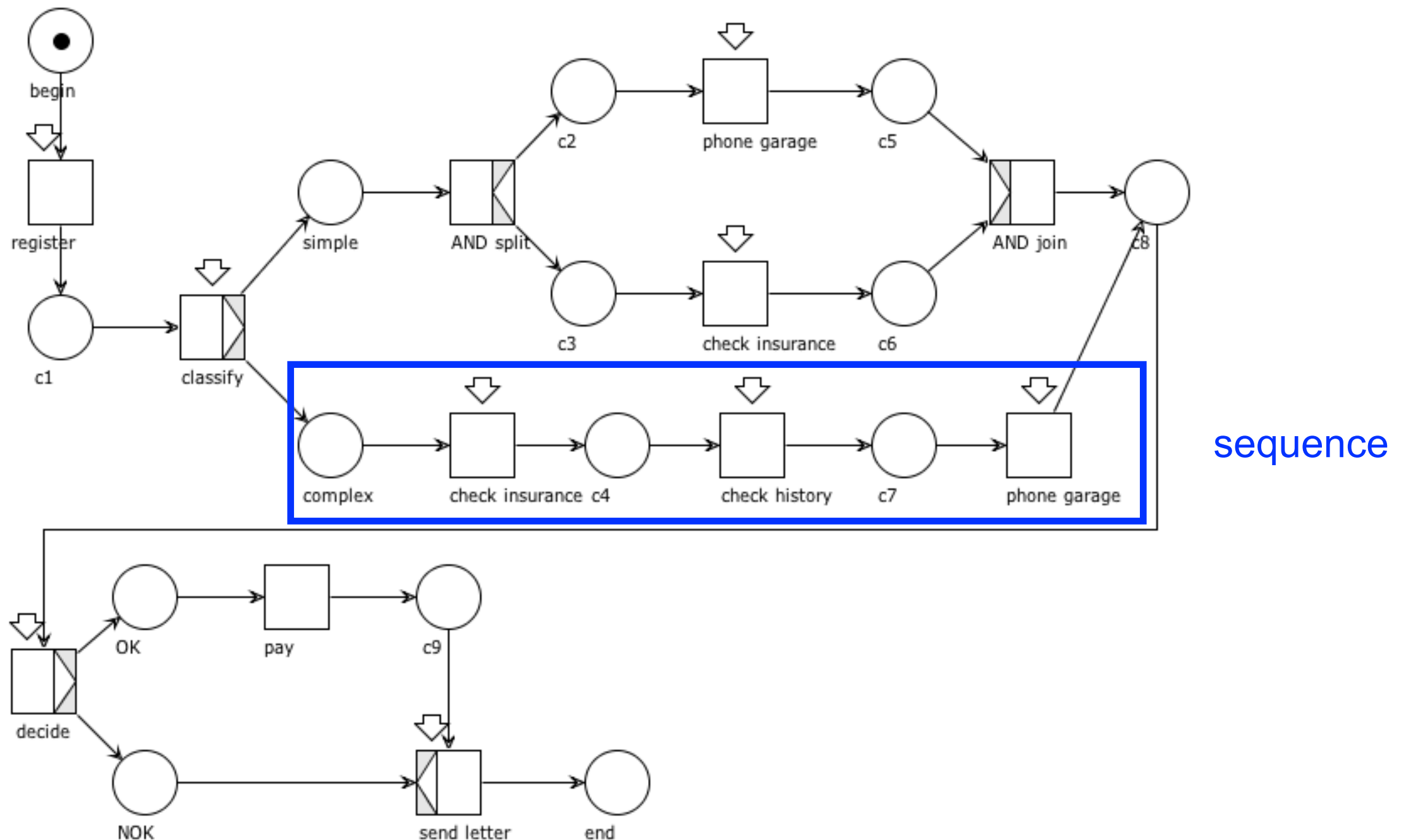
Design Example: Car Damage



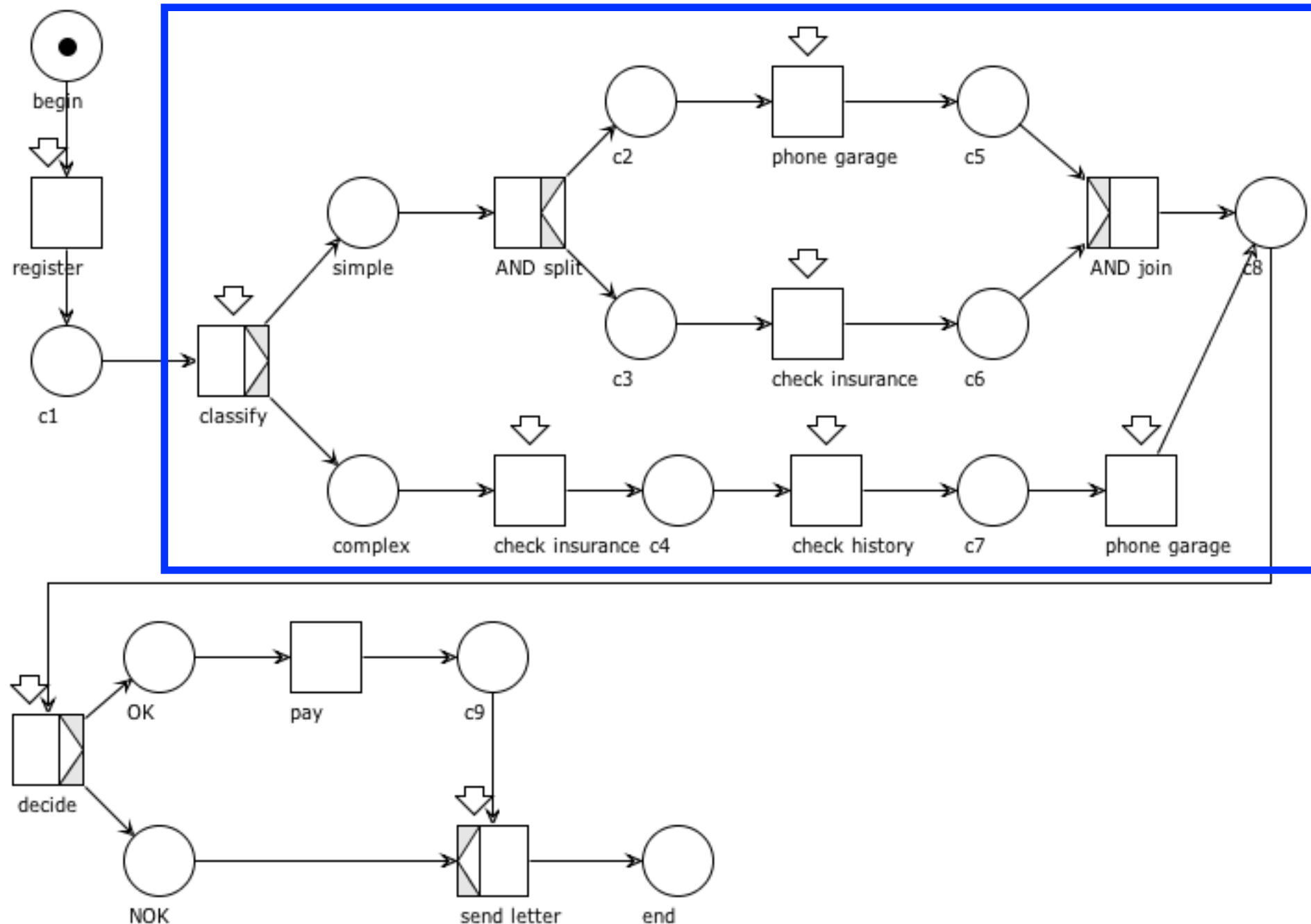
Design Example: Car Damage



Design Example: Car Damage

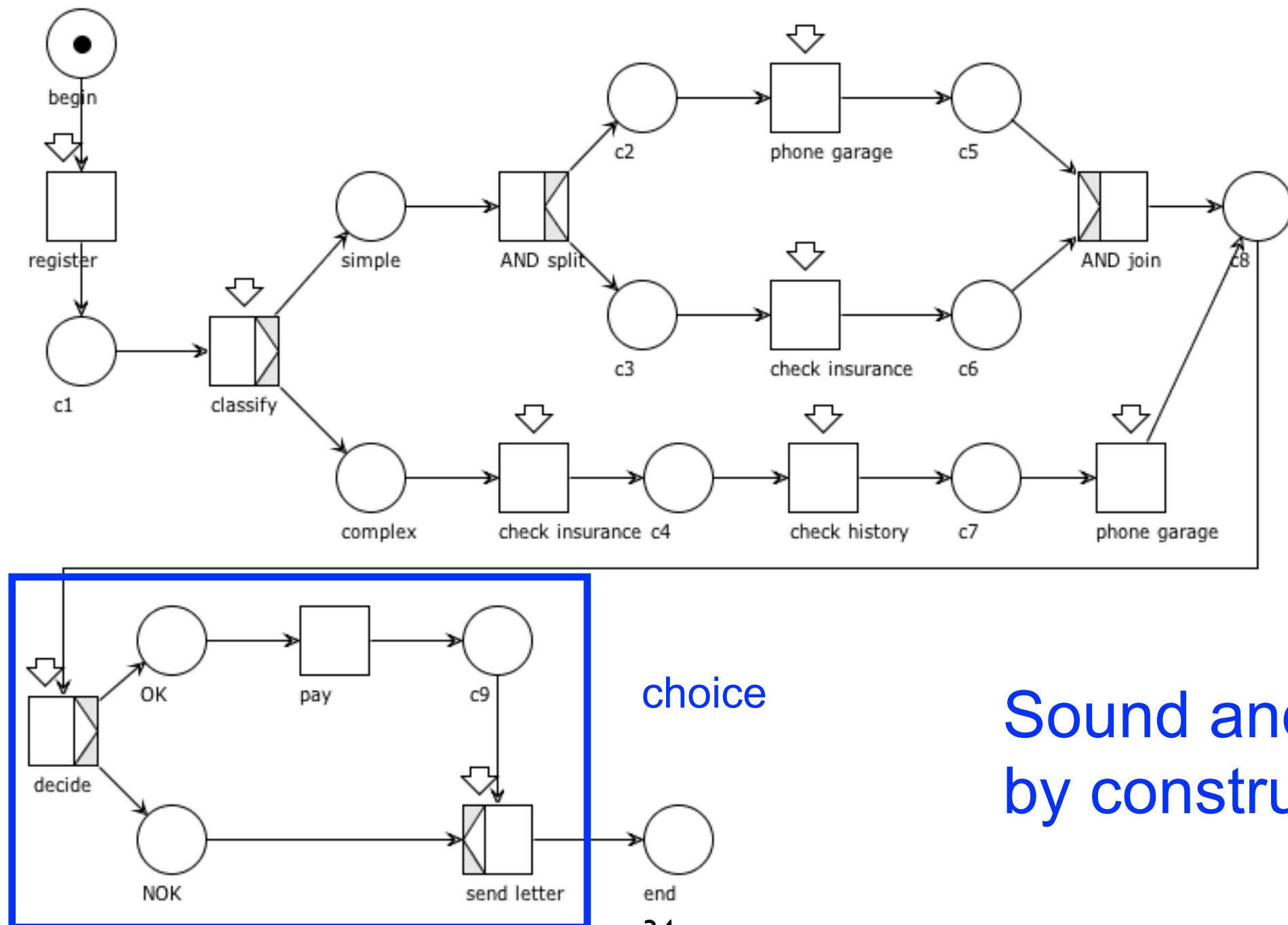


Design Example: Car Damage



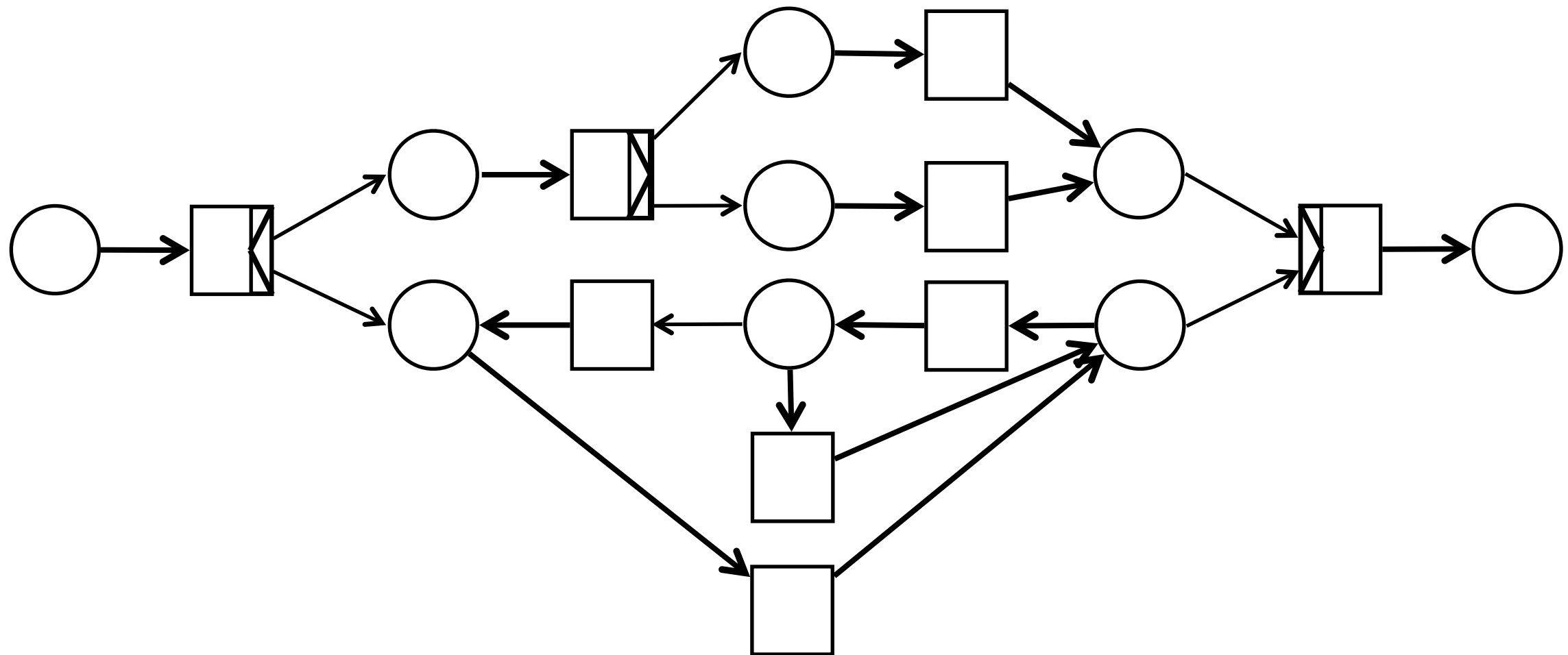
choice

Design Example: Car Damage



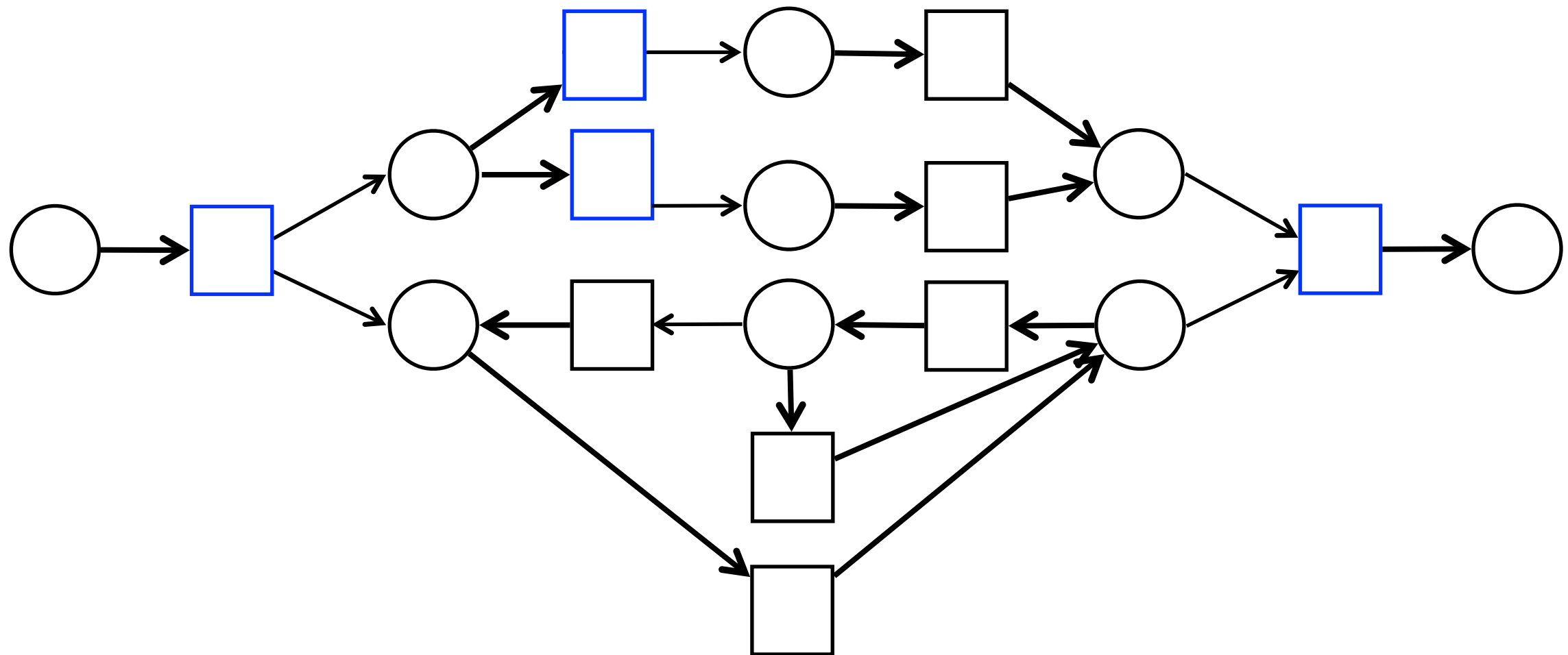
Exercise

Prove that the net below is
a safe and sound workflow net



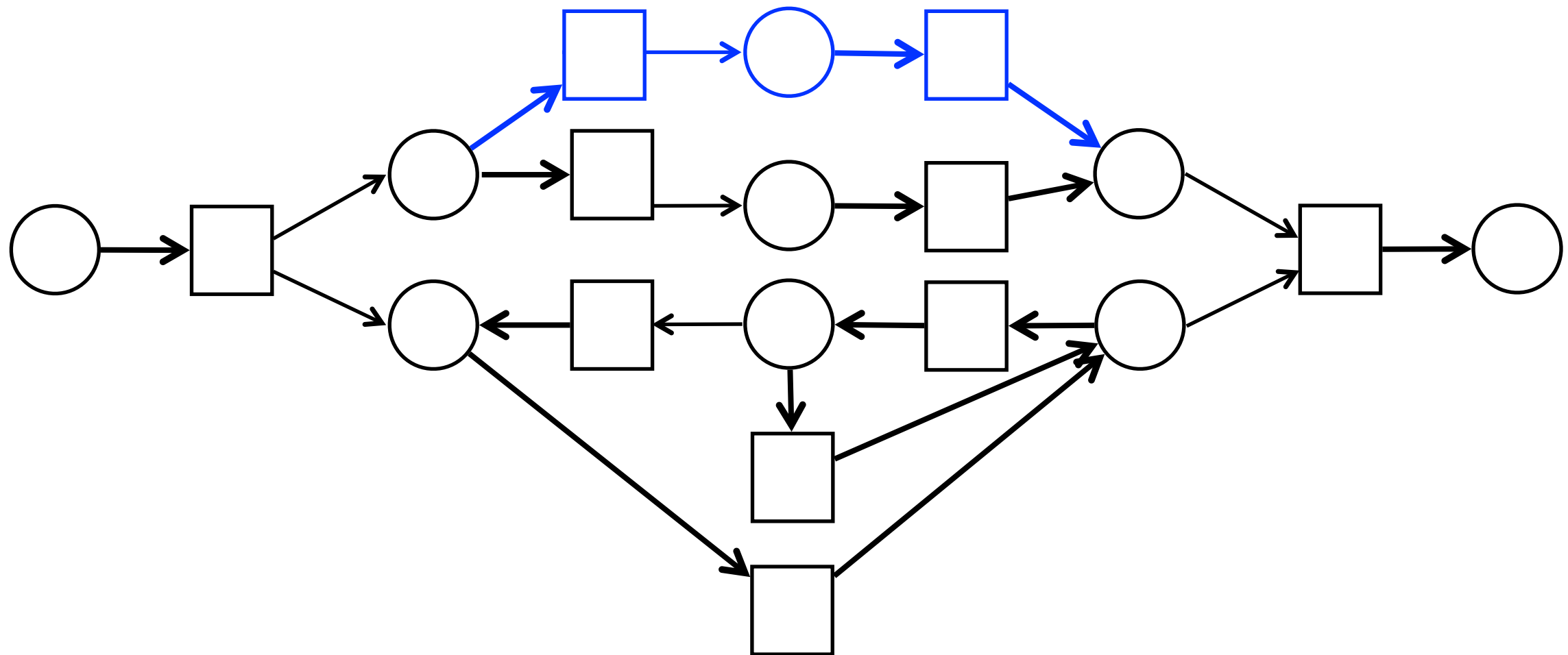
Exercise

Prove that the desugarized net below is a safe and sound workflow net



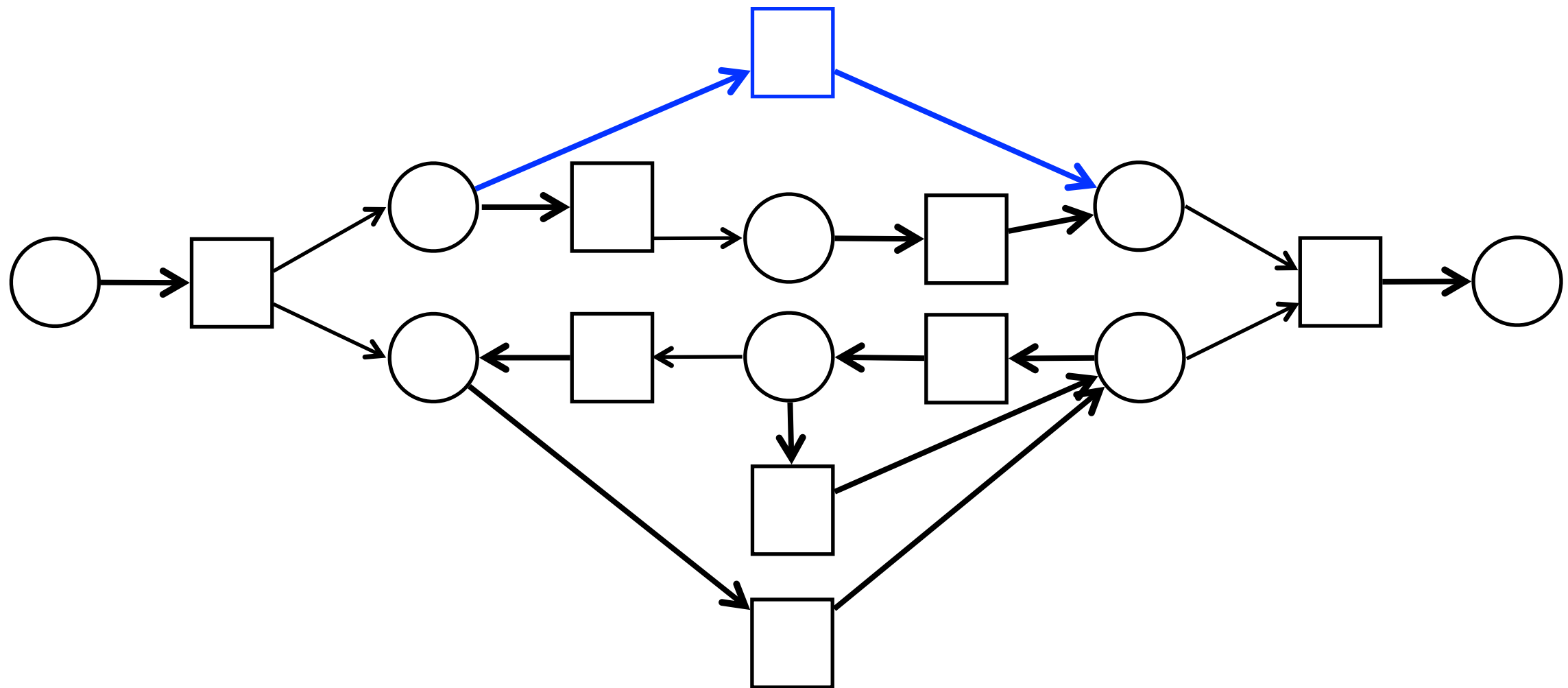
Exercise

Prove that the net below is
a safe and sound workflow net



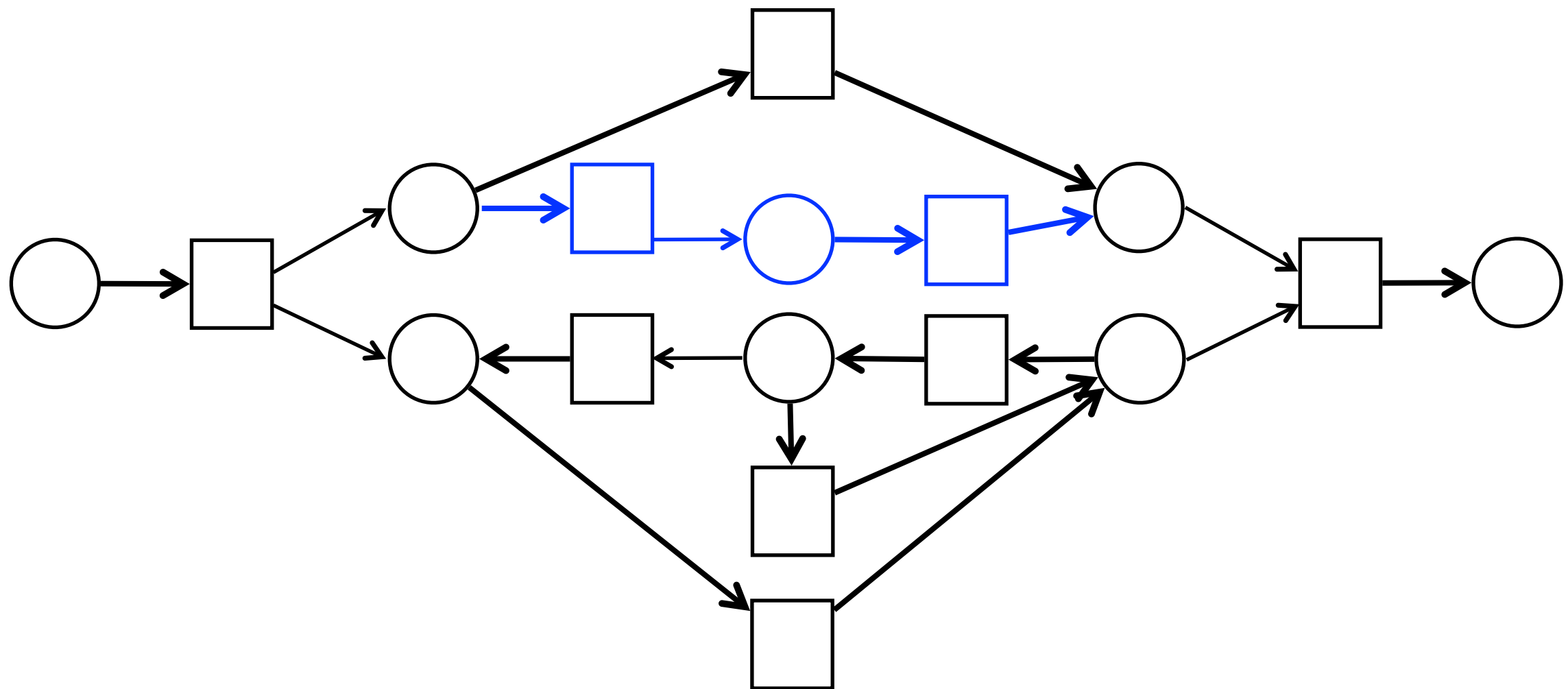
Exercise

Prove that the net below is
a safe and sound workflow net



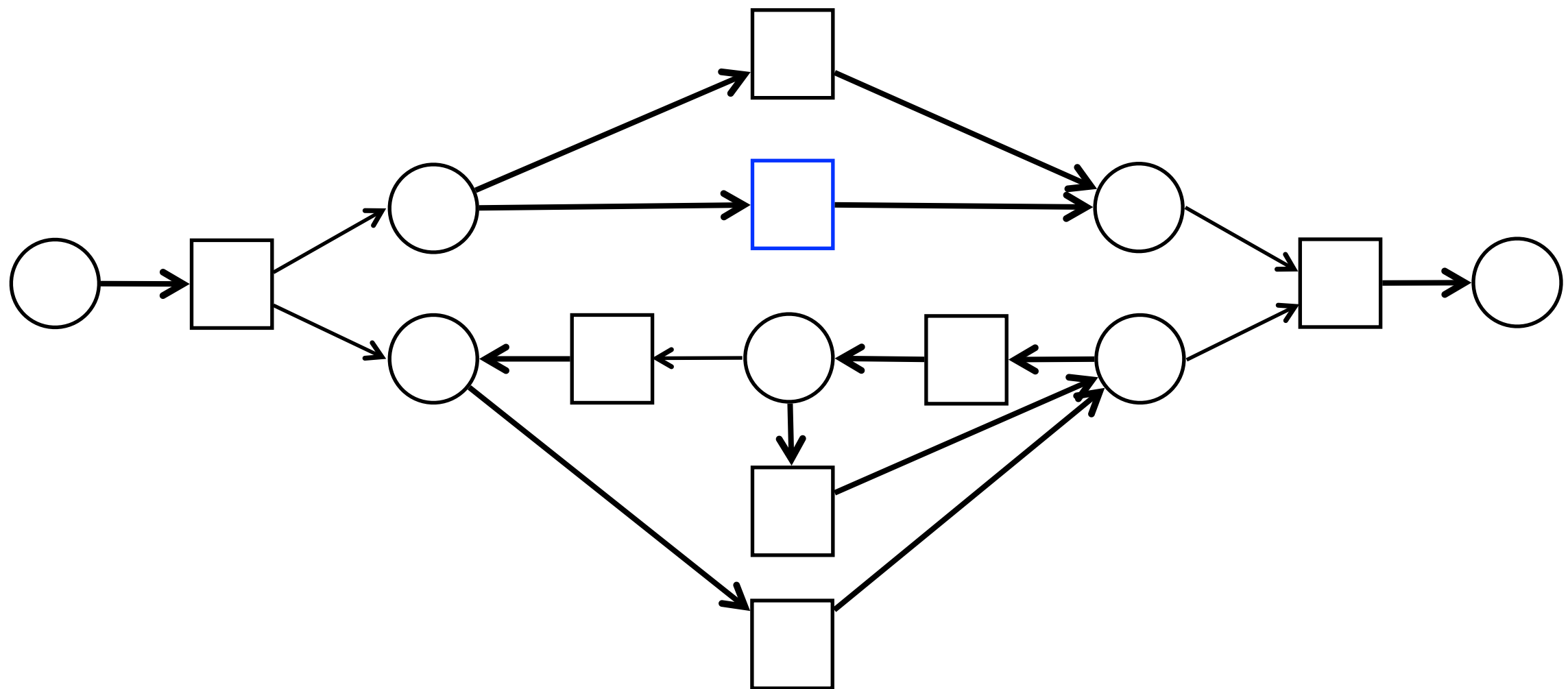
Exercise

Prove that the net below is
a safe and sound workflow net



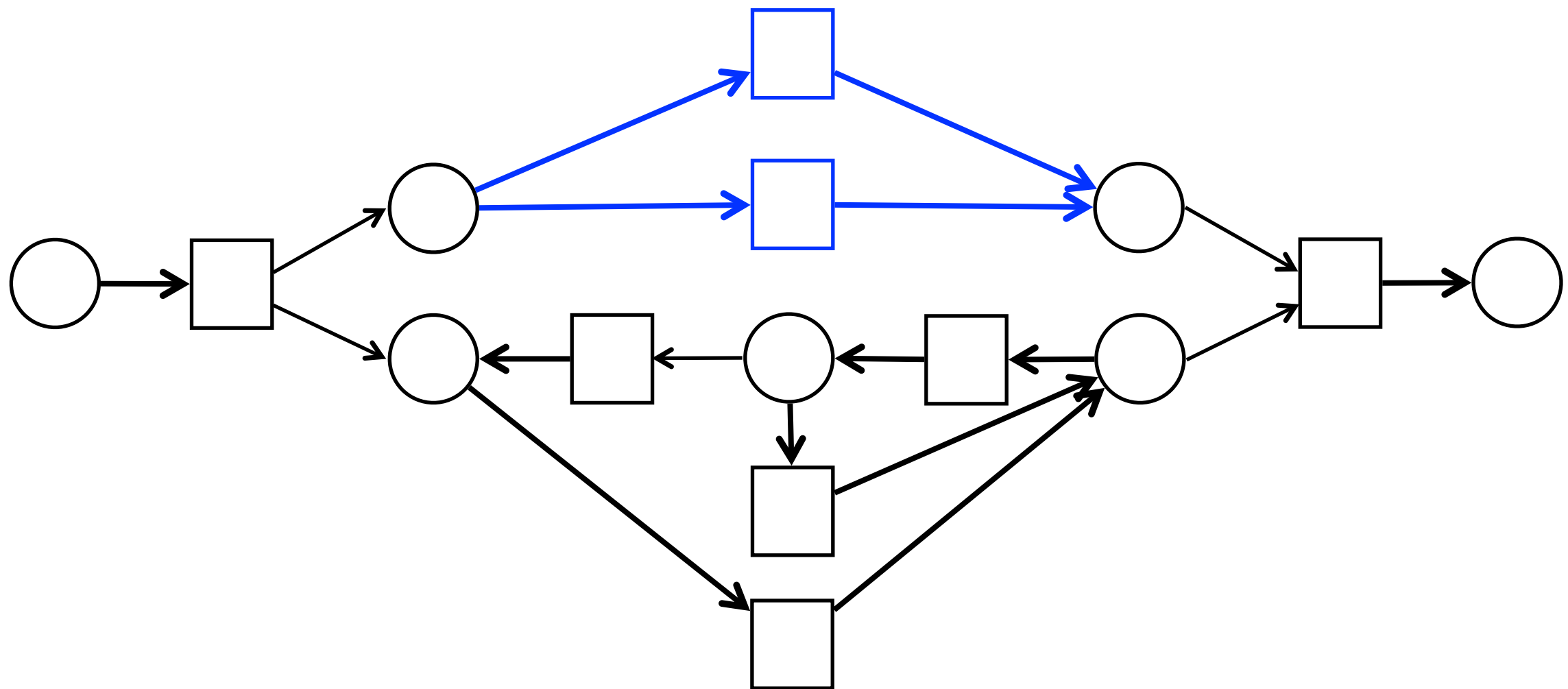
Exercise

Prove that the net below is a safe and sound workflow net



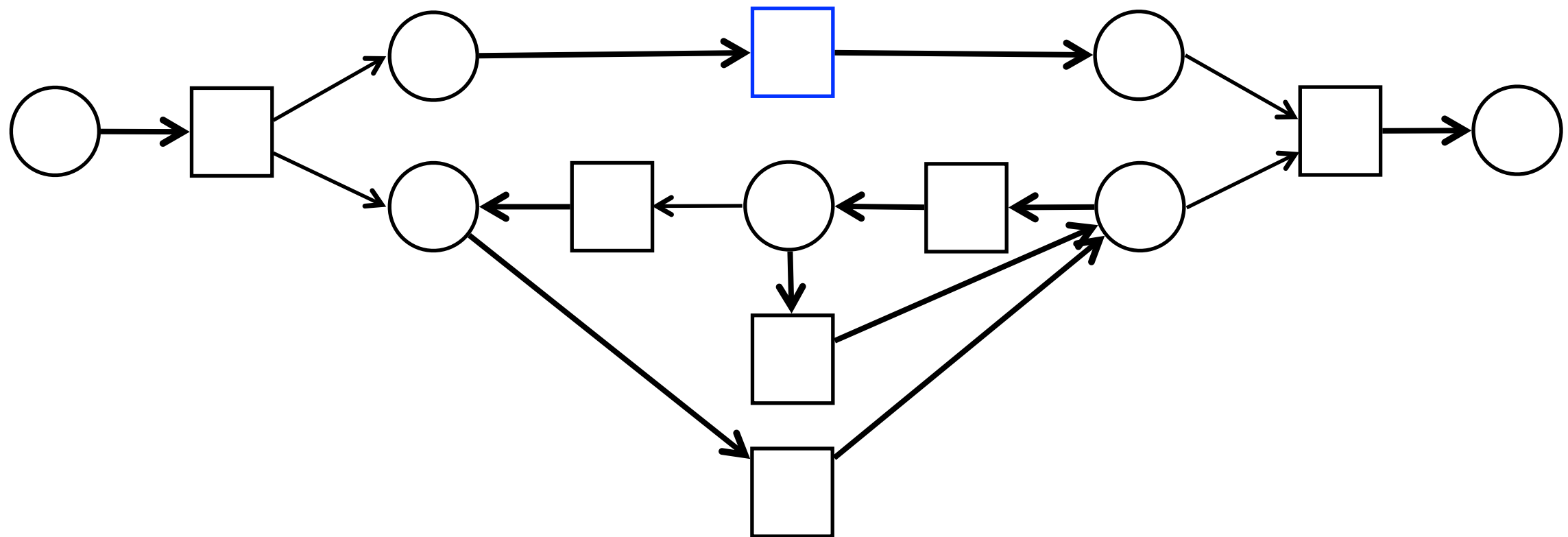
Exercise

Prove that the net below is
a safe and sound workflow net



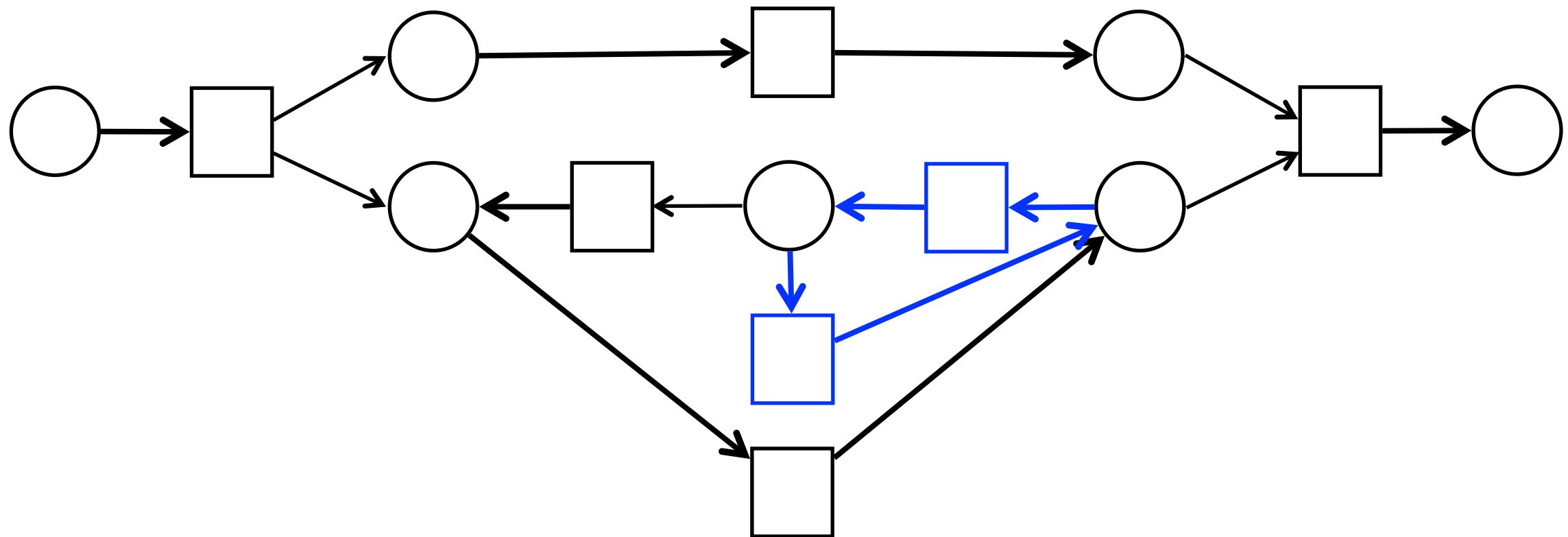
Exercise

Prove that the net below is
a safe and sound workflow net



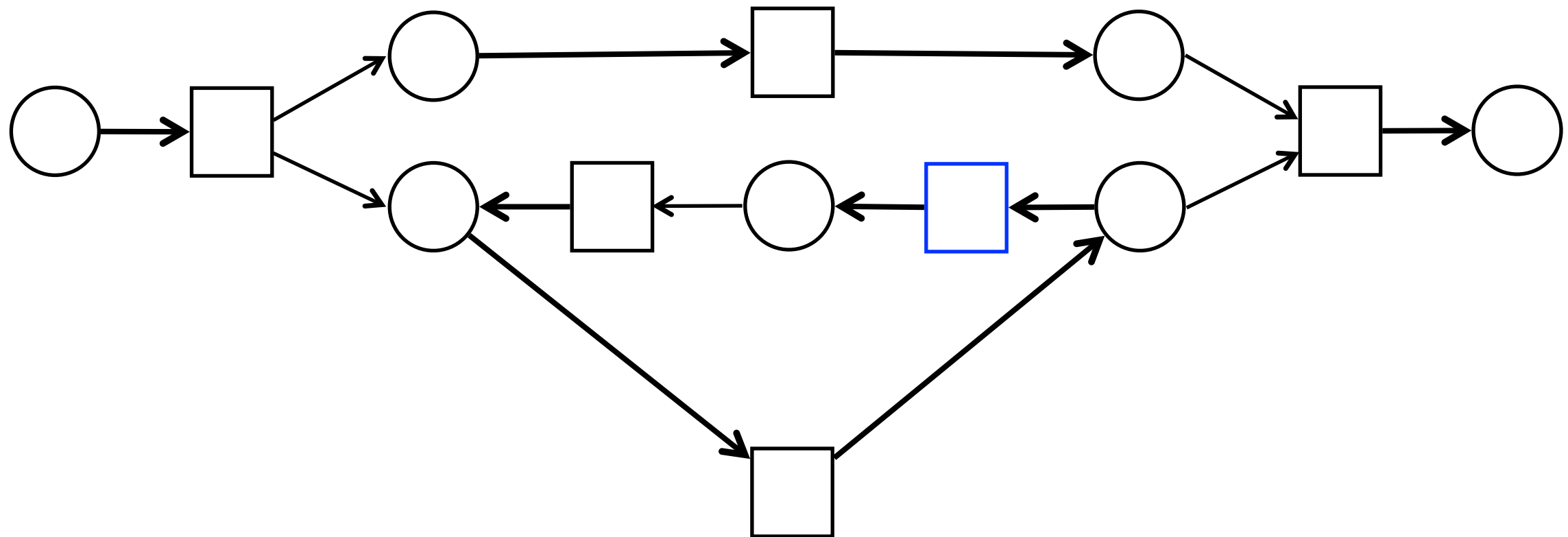
Exercise

Prove that the net below is
a safe and sound workflow net



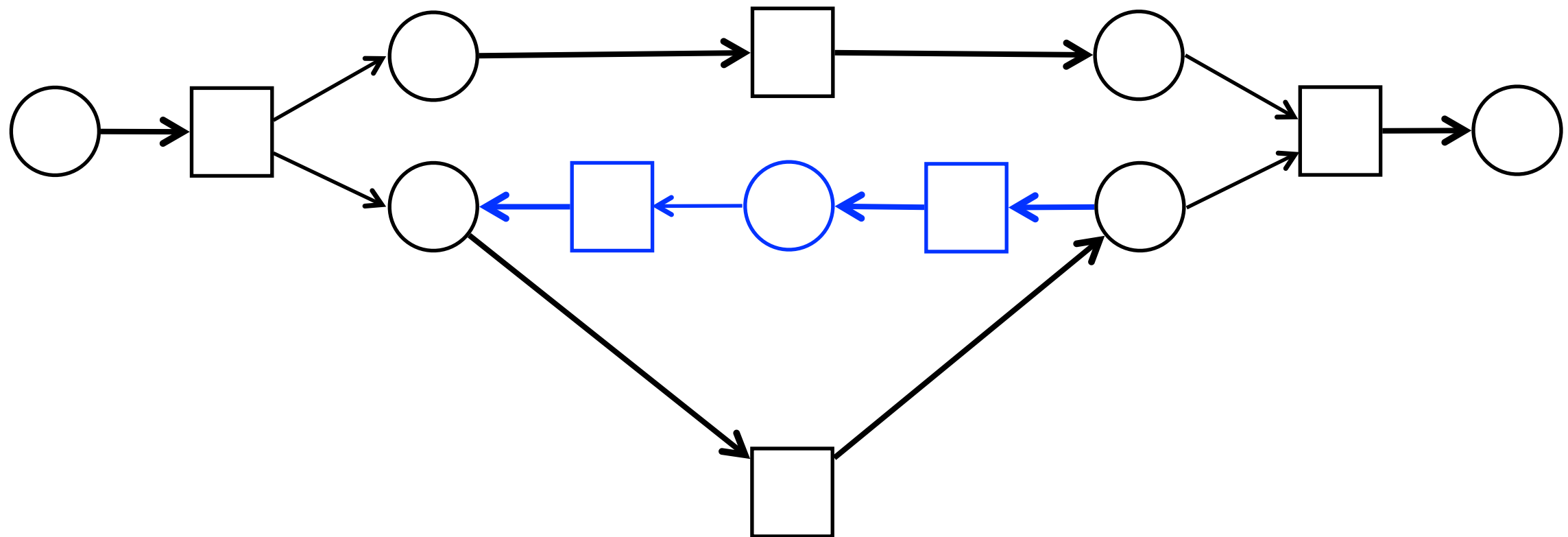
Exercise

Prove that the net below is
a safe and sound workflow net



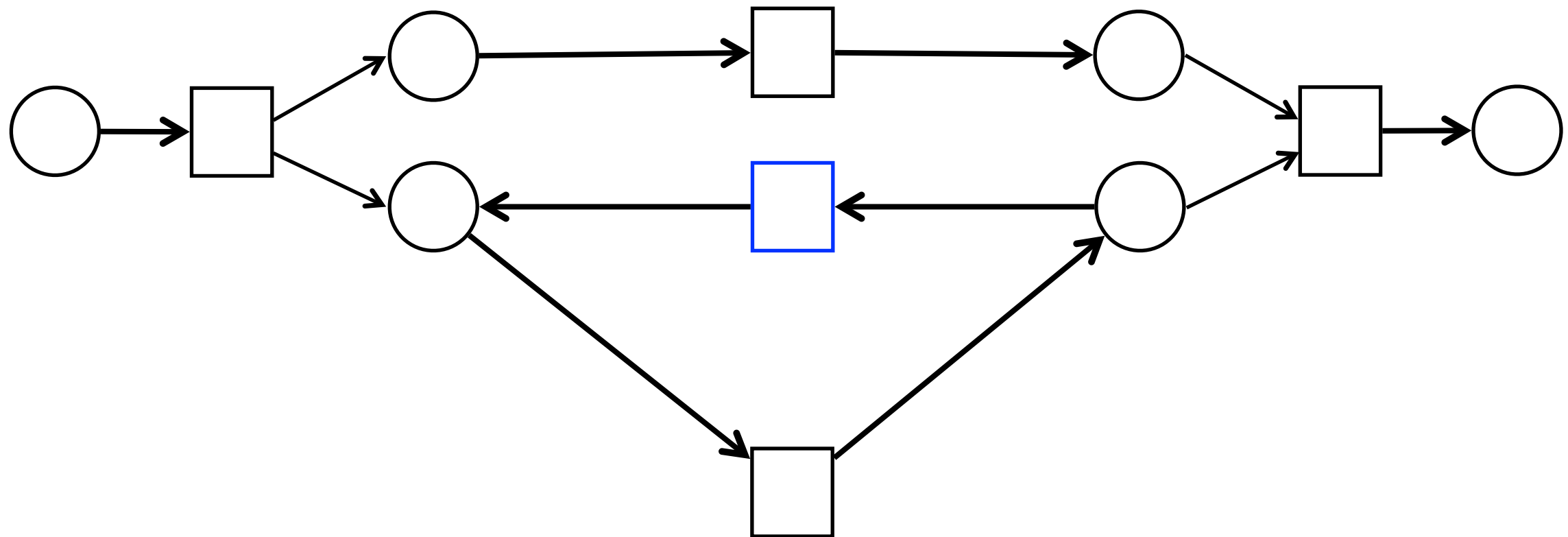
Exercise

Prove that the net below is
a safe and sound workflow net



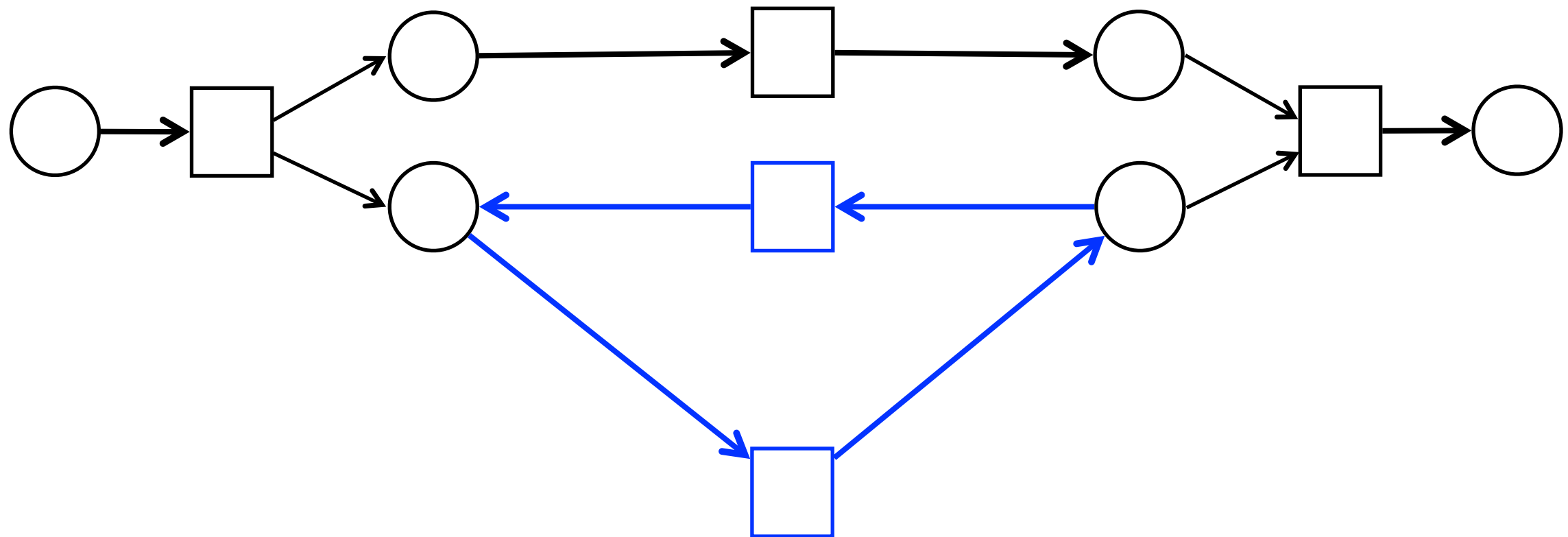
Exercise

Prove that the net below is
a safe and sound workflow net



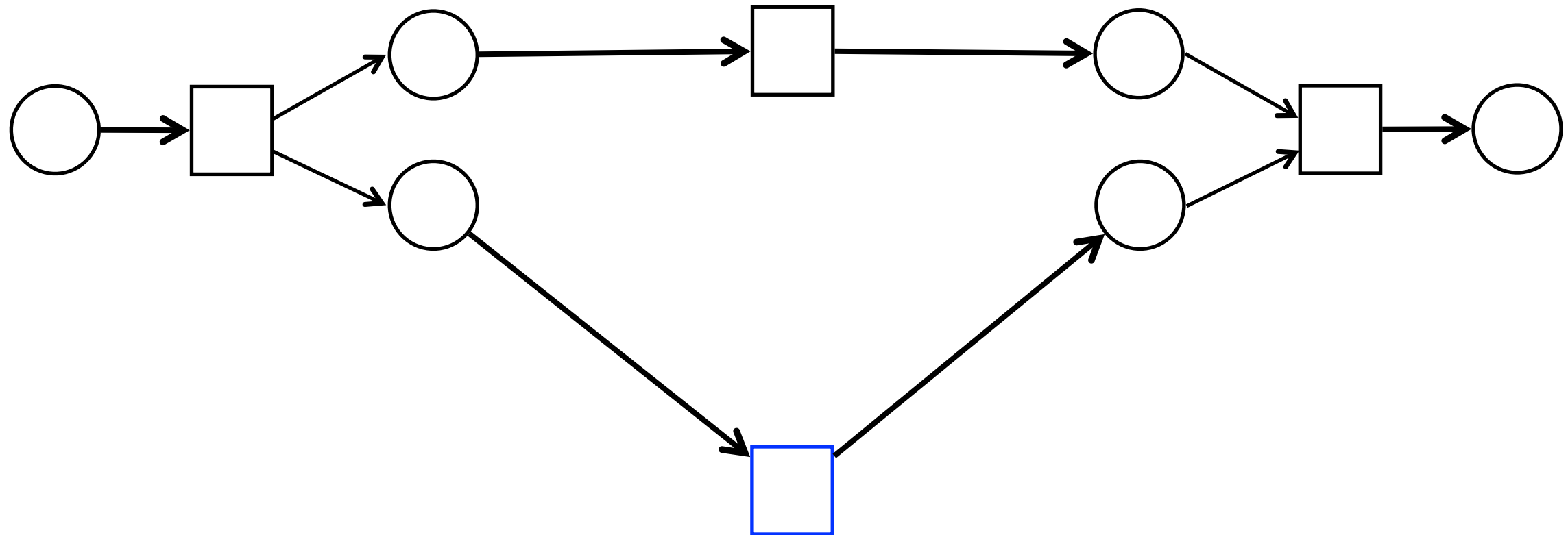
Exercise

Prove that the net below is
a safe and sound workflow net



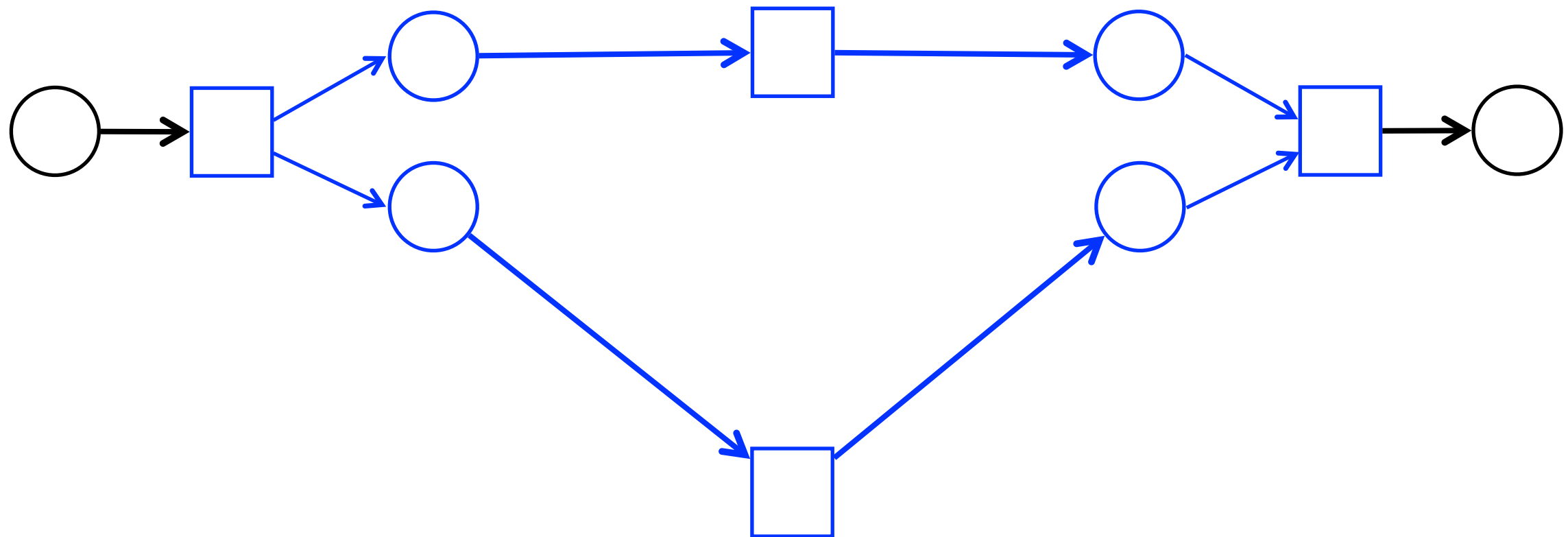
Exercise

Prove that the net below is
a safe and sound workflow net



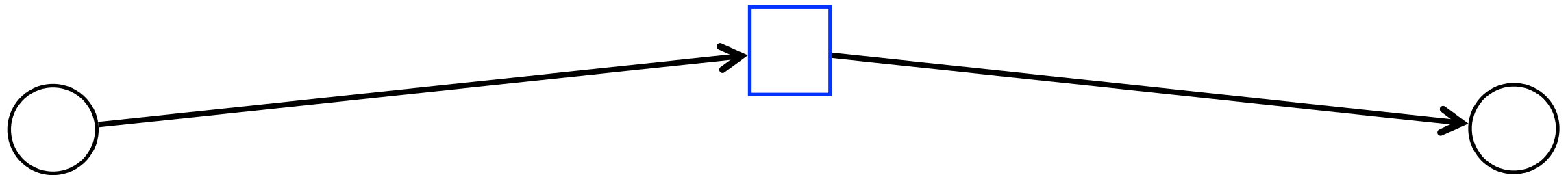
Exercise

Prove that the net below is
a safe and sound workflow net



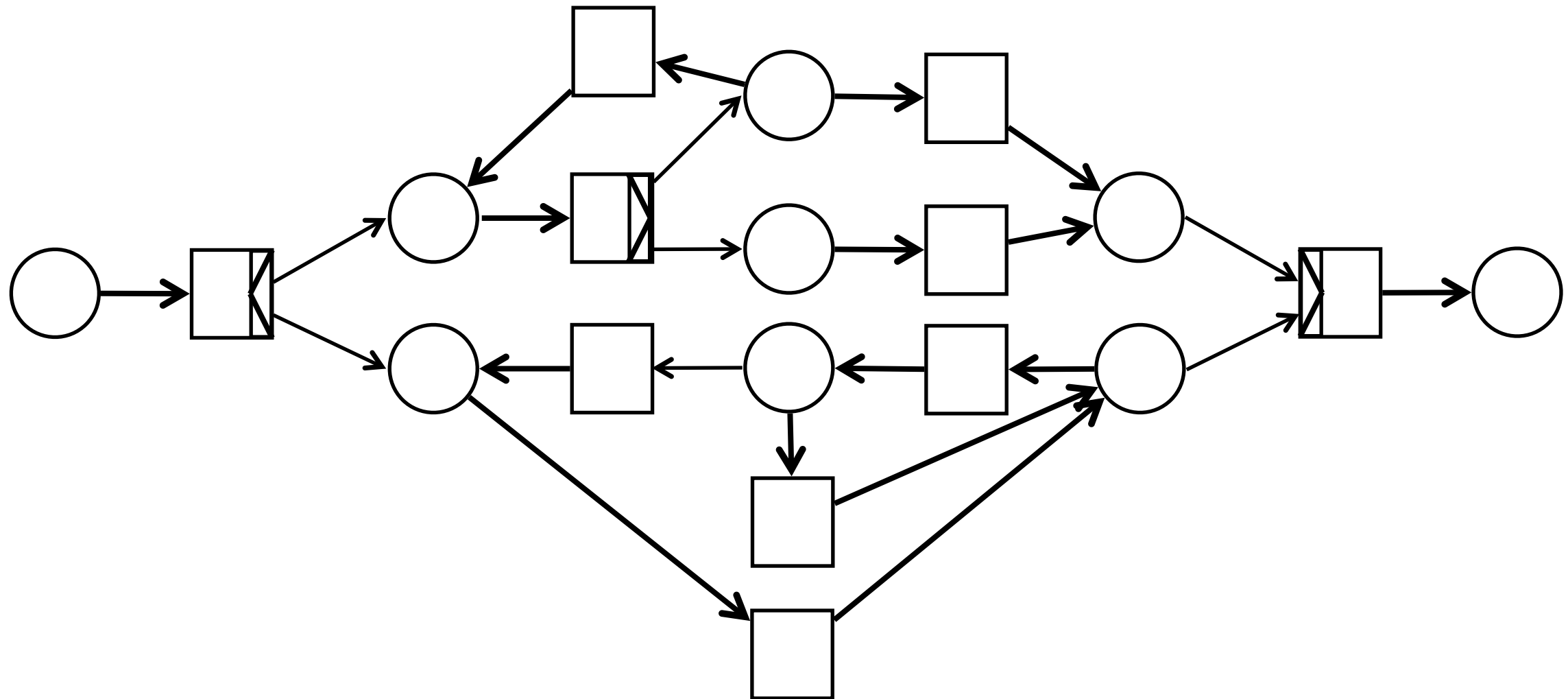
Exercise

Prove that the net below is
a safe and sound workflow net



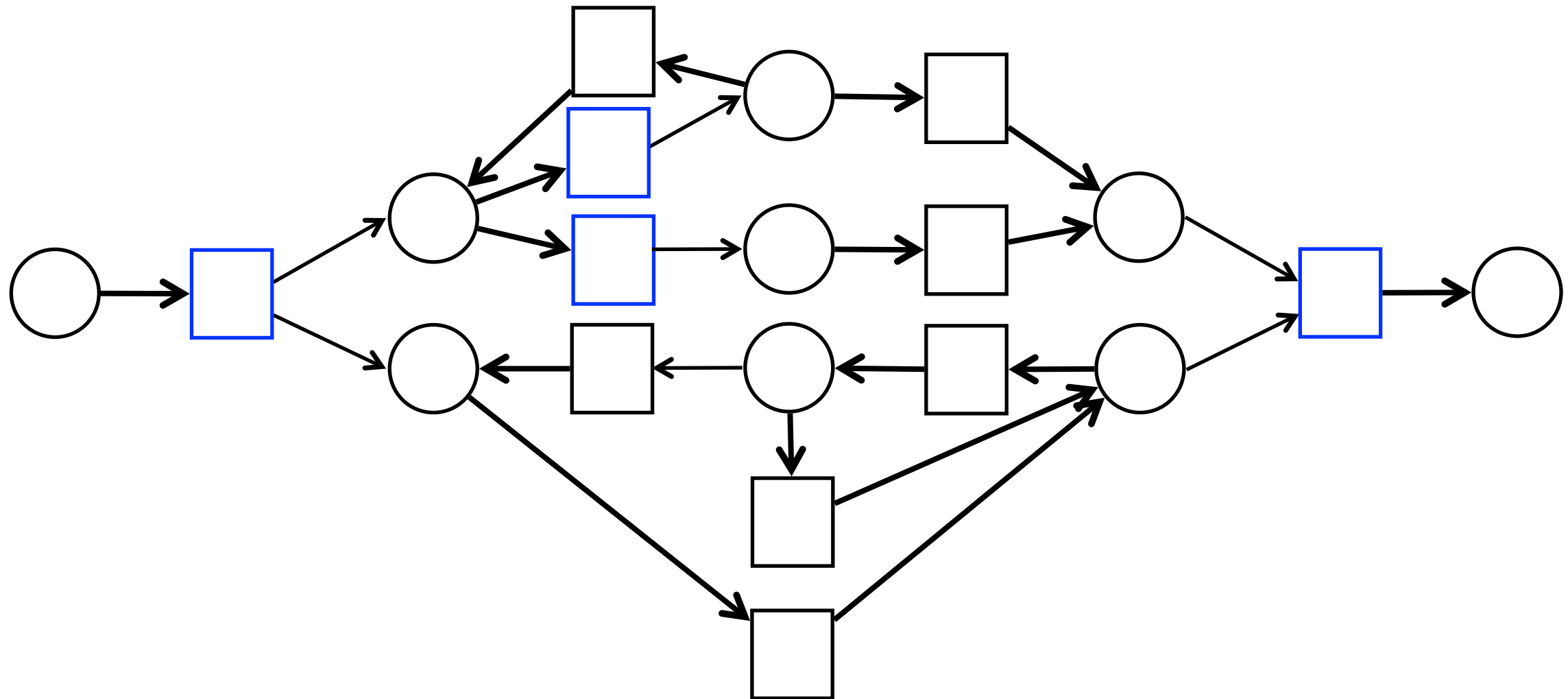
Exercise

Prove that the net below is a safe and sound workflow net (hint: "desugar" it)



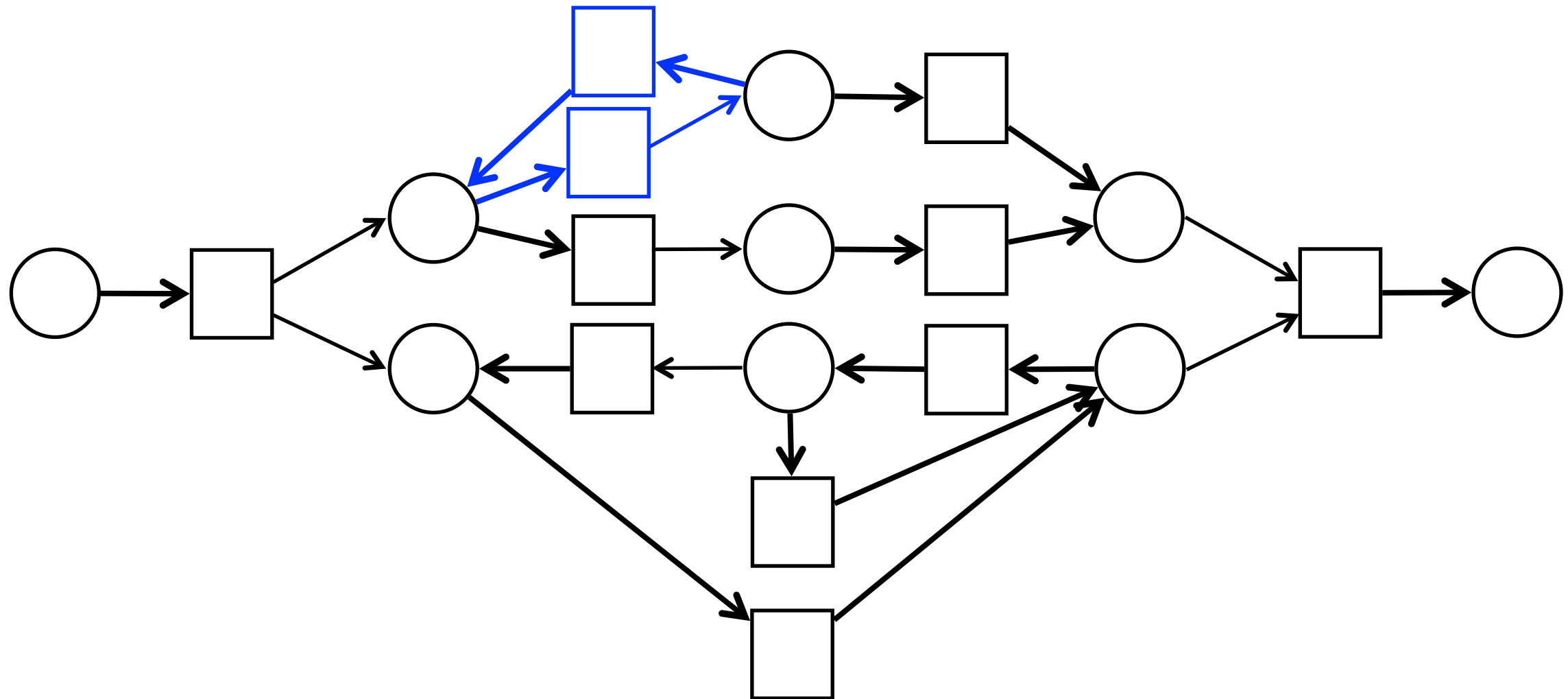
Exercise

Prove that the net below is a safe and sound workflow net (hint: "desugar" it)



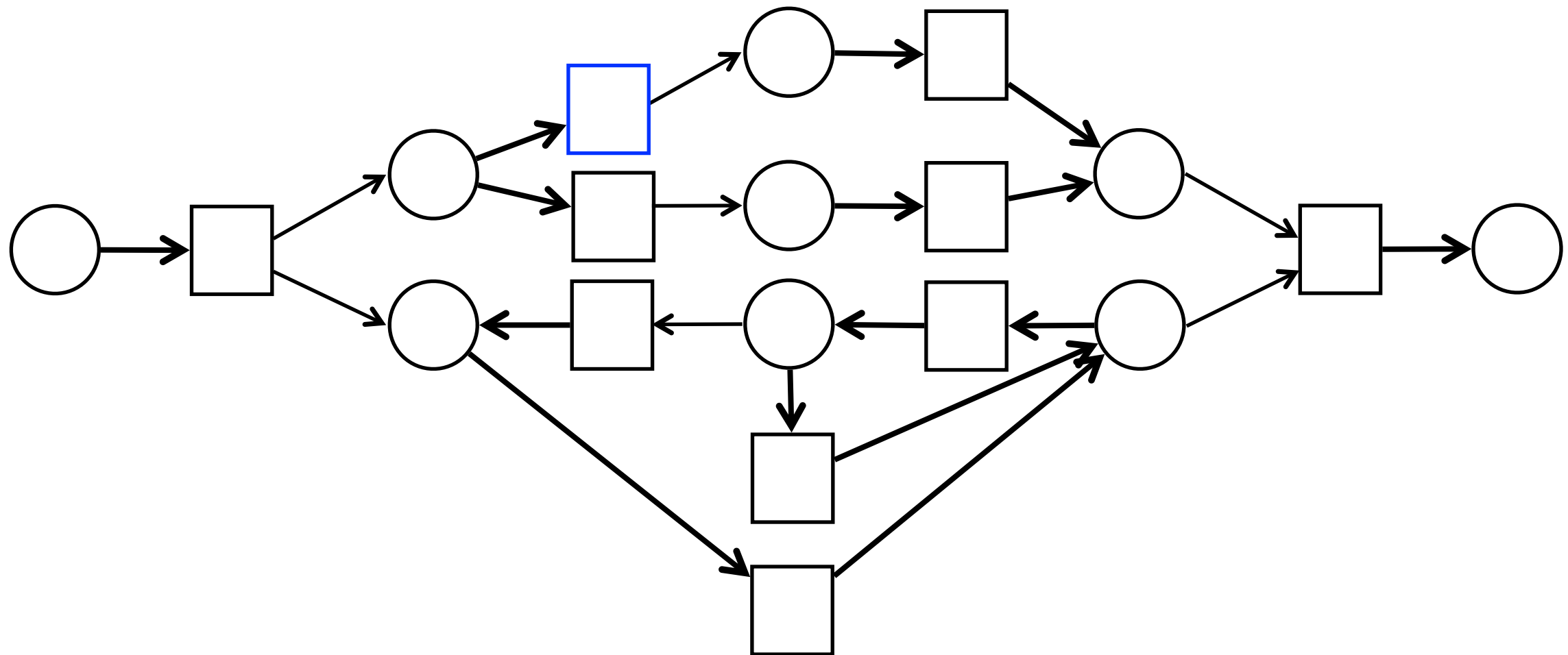
Exercise

Prove that the net below is a safe and sound workflow net (hint: "desugar" it)



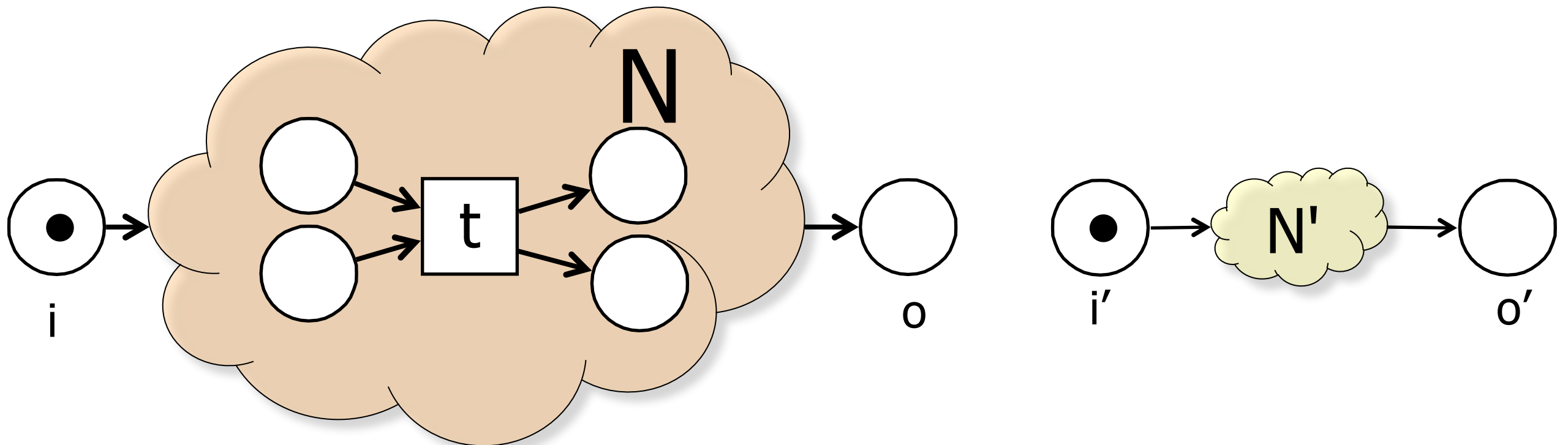
Exercise

This is the same net we have examined before

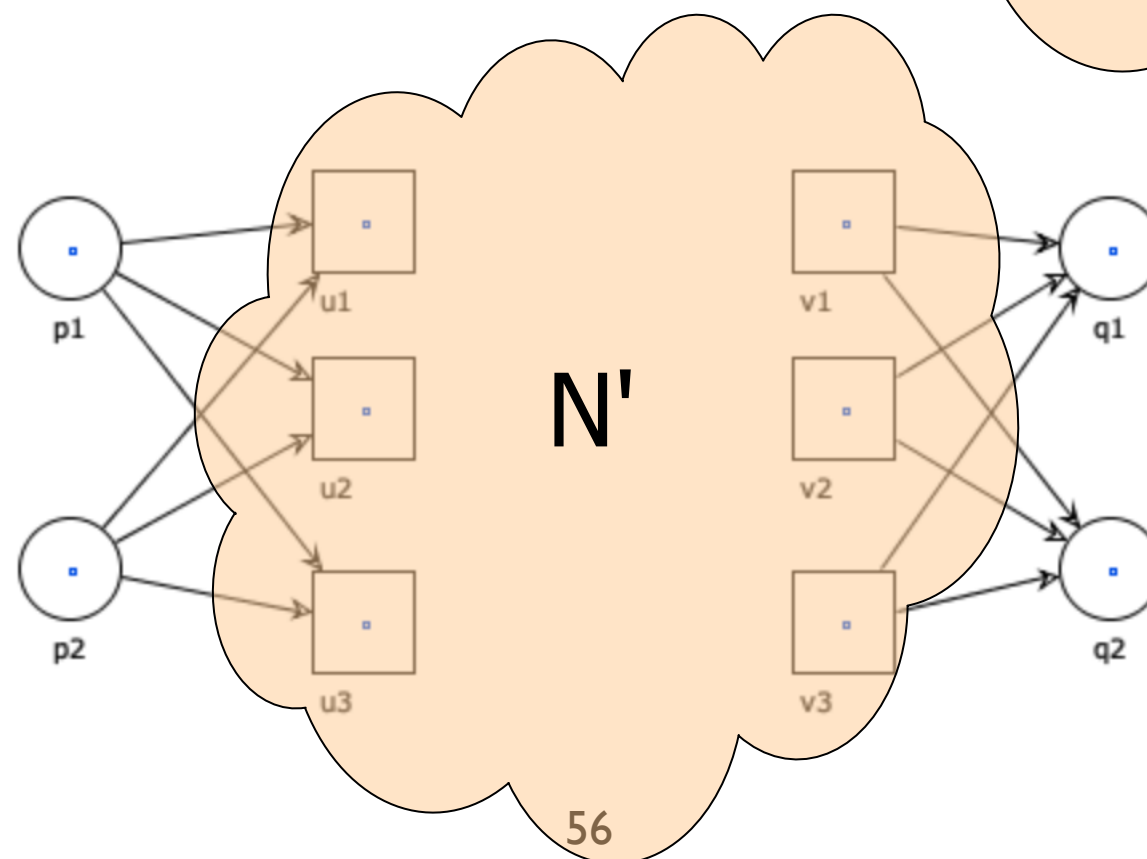
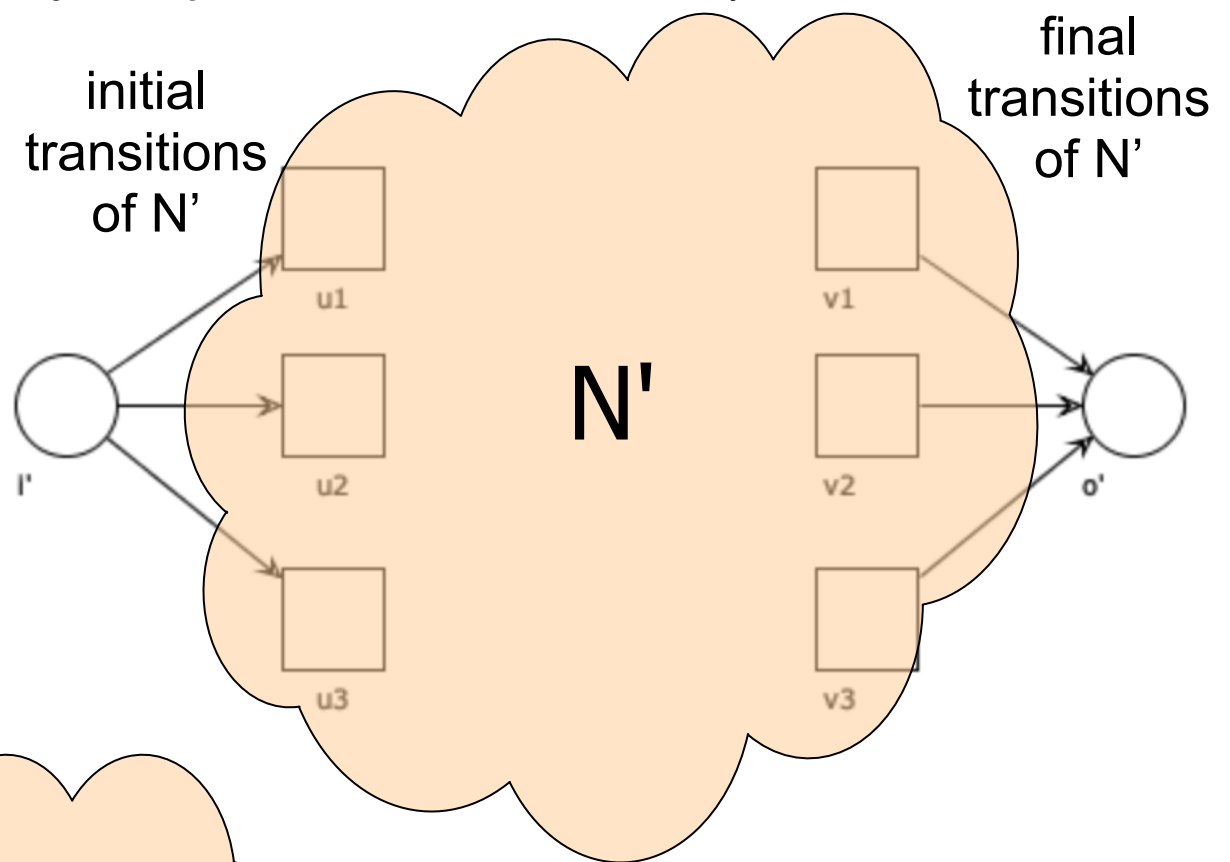
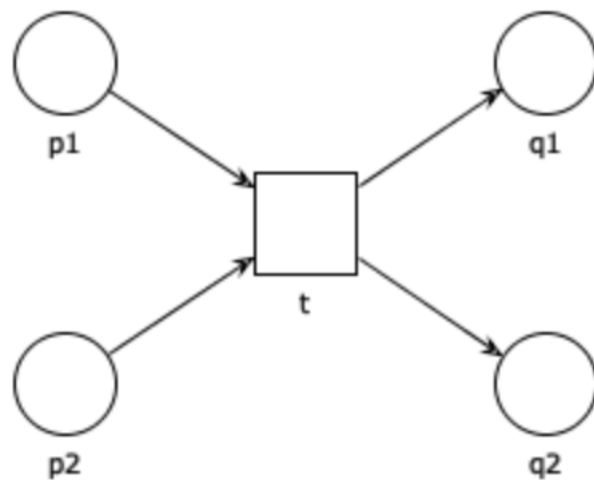


Generalization

We would like to progressively refine transitions with multiple incoming and outgoing arcs



Sketching the idea



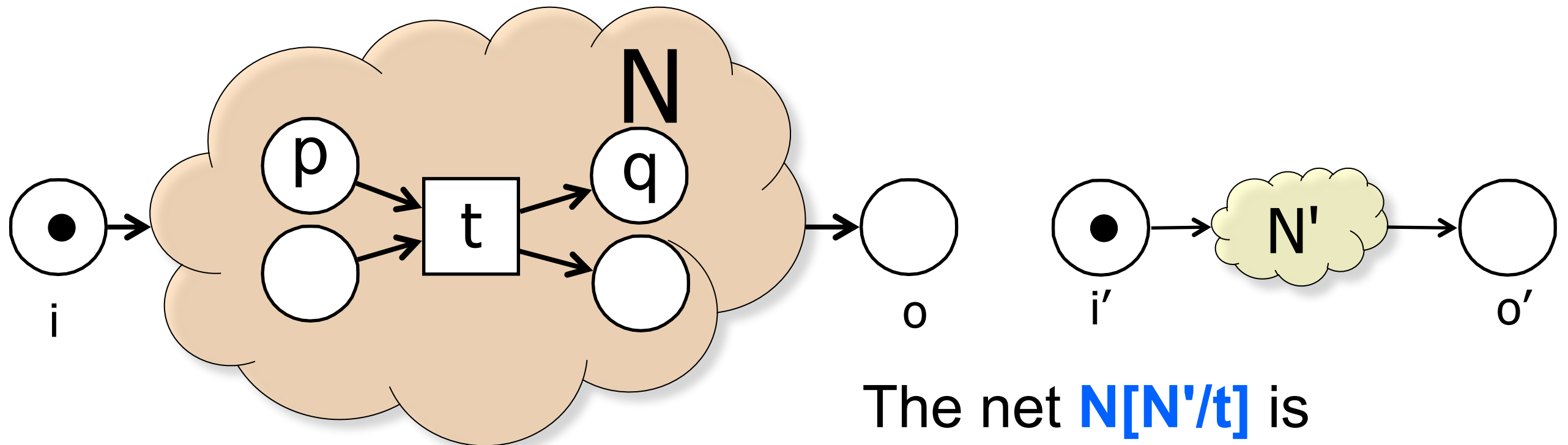
General replacement

Let $T_{i'} = \{ u \mid \bullet u = \{i'\} \}$. (initial transitions of N')

Let $T_{o'} = \{ v \mid v\bullet = \{o'\} \}$. (final transitions of N')

If $(p, t) \in F_N, u \in T_{i'}$ then $(p, u) \in F_{N[N'/t]}$

If $(t, q) \in F_N, v \in T_{o'}$ then $(v, q) \in F_{N[N'/t]}$



The net $N[N'/t]$ is
a **sound** and **safe workflow net**

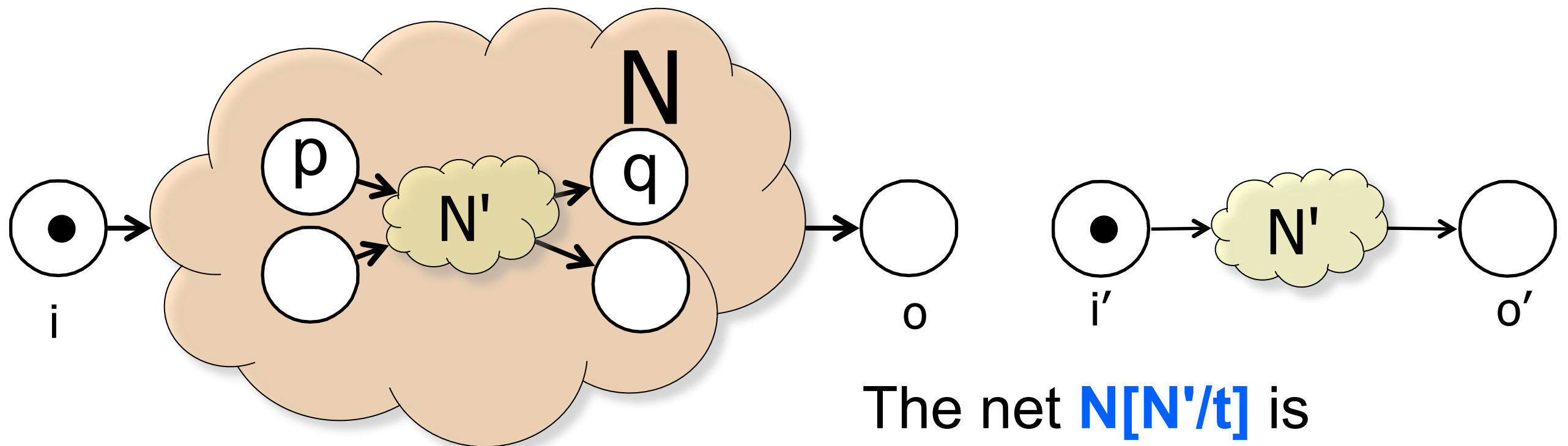
General replacement

Let $T_{i'} = \{ u \mid \bullet u = \{i'\} \}$. (initial transitions of N')

Let $T_{o'} = \{ v \mid v\bullet = \{o'\} \}$. (final transitions of N')

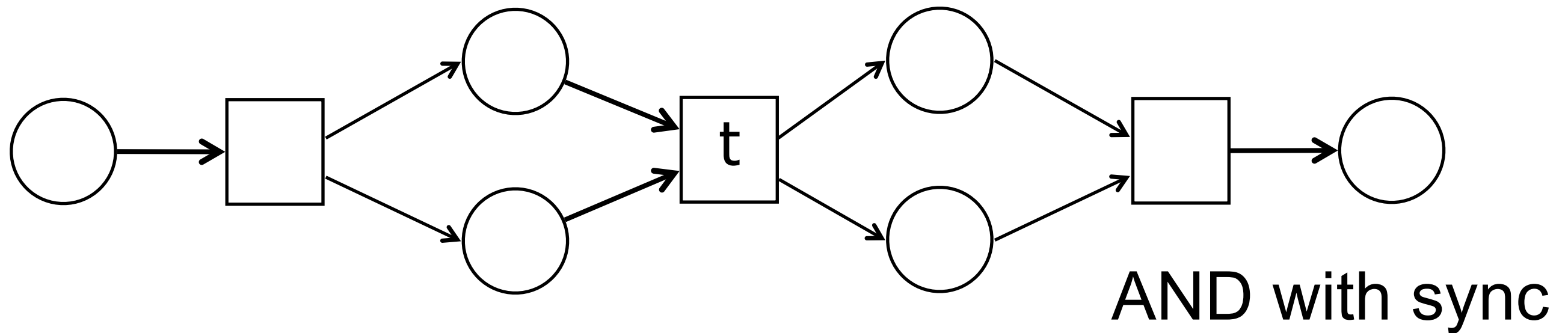
If $(p, t) \in F_N, u \in T_{i'}$ then $(p, u) \in F_{N[N'/t]}$

If $(t, q) \in F_N, v \in T_{o'}$ then $(v, q) \in F_{N[N'/t]}$



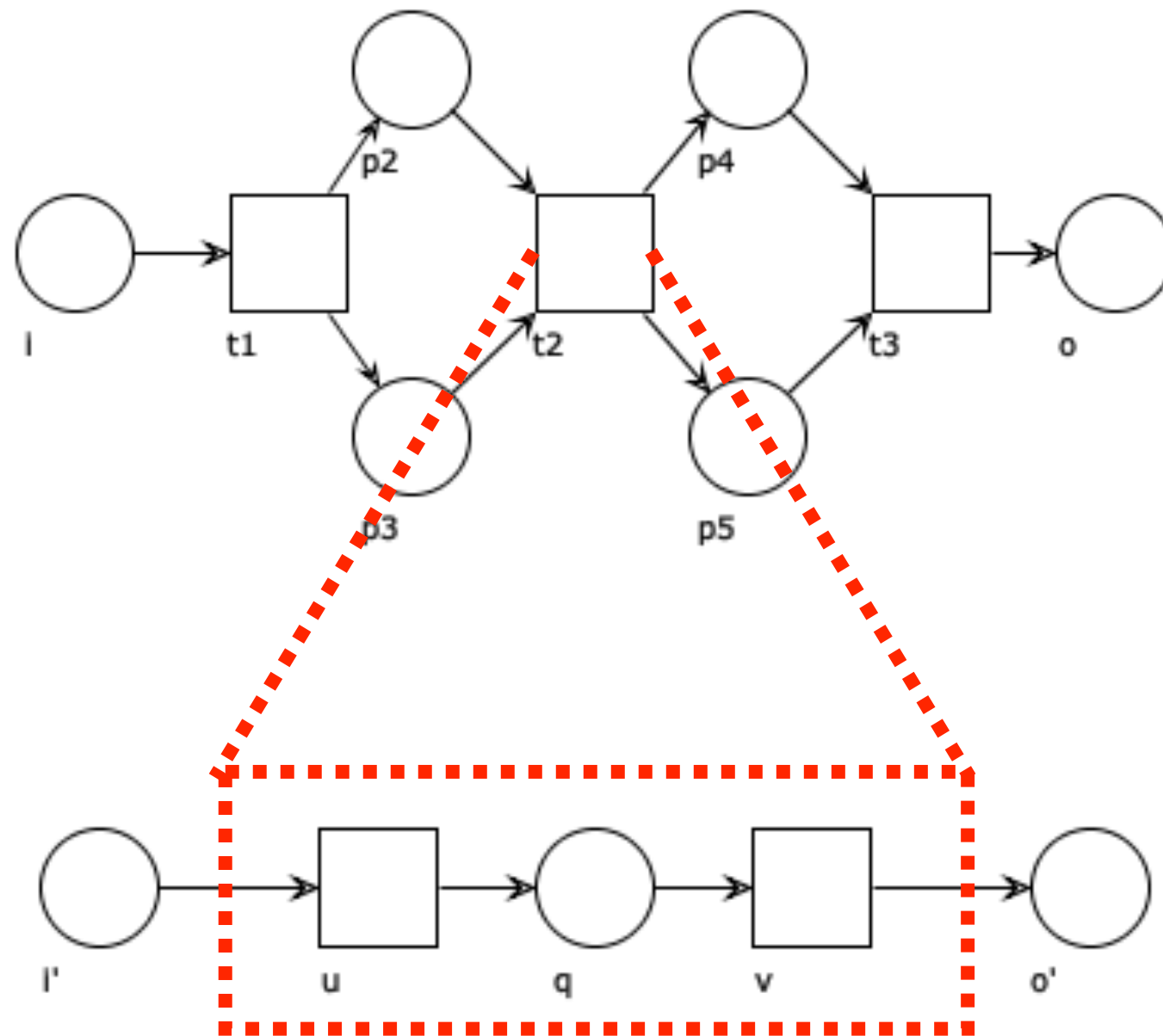
The net $N[N'/t]$ is
a **sound and safe workflow net**

Some Building Blocks 5

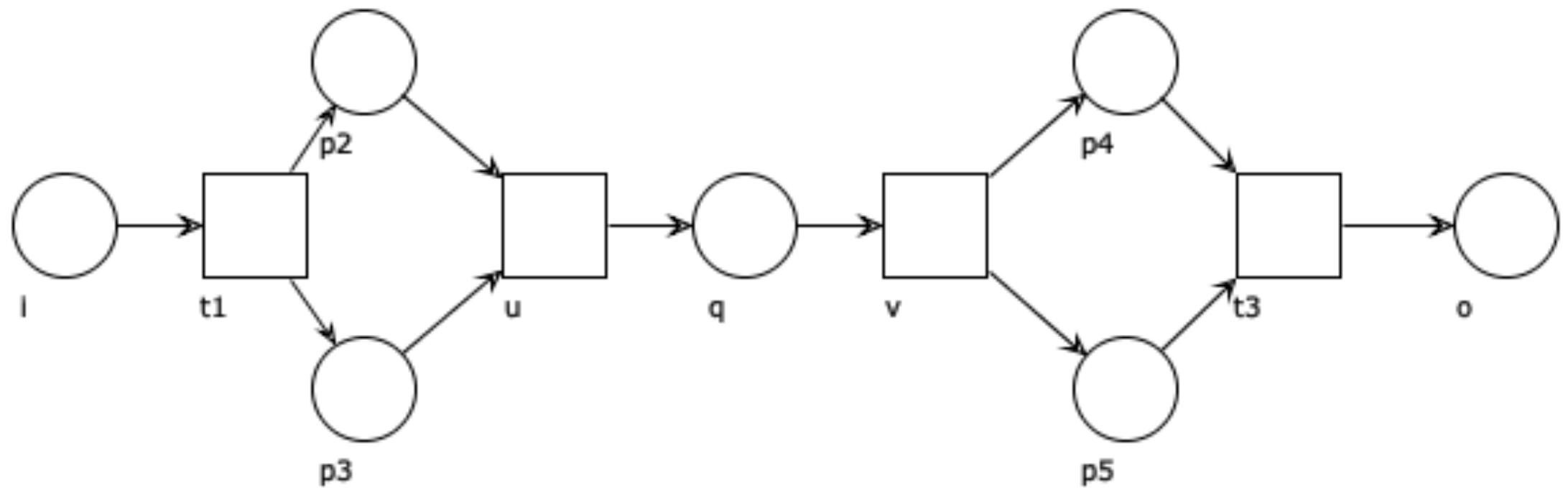


But you can define more blocks on your own

Example

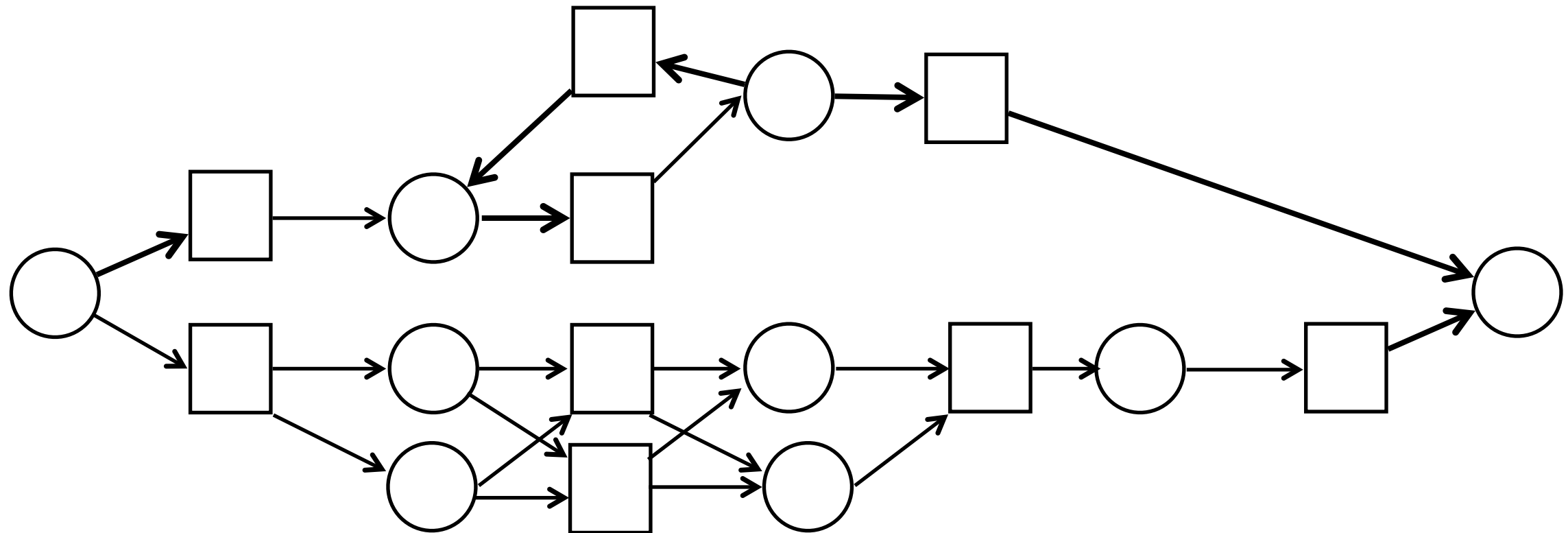


Example



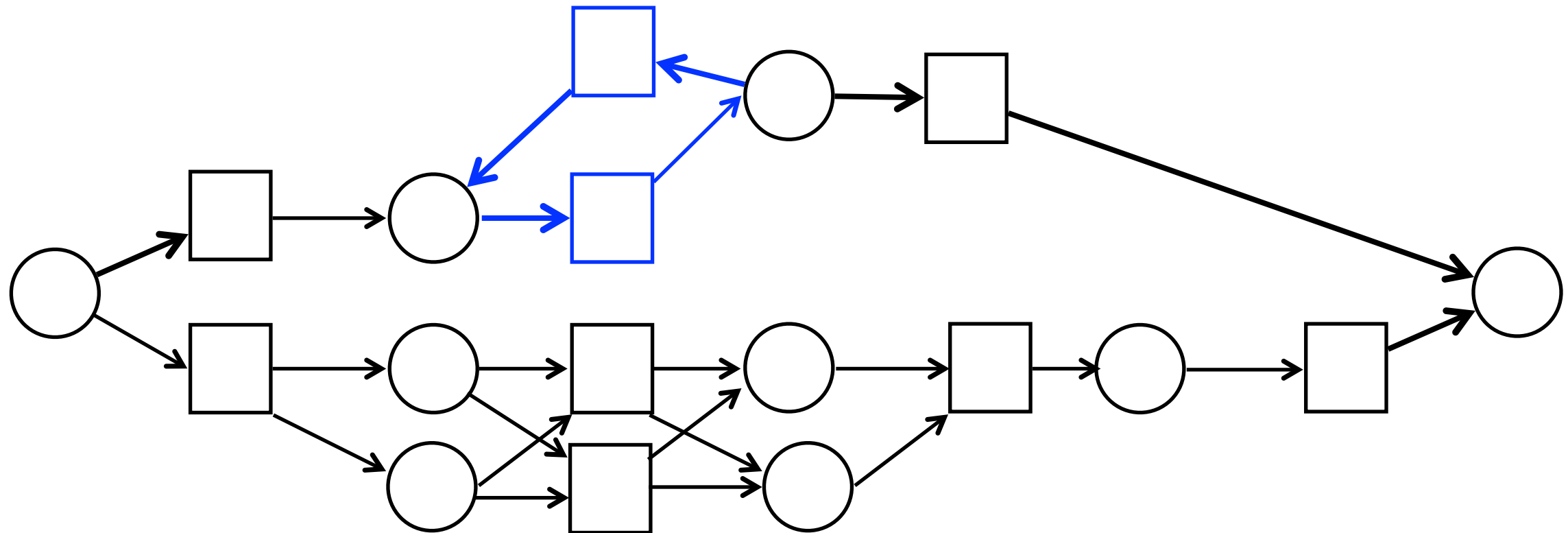
Exercise

Prove that the net below is
a safe and sound workflow net



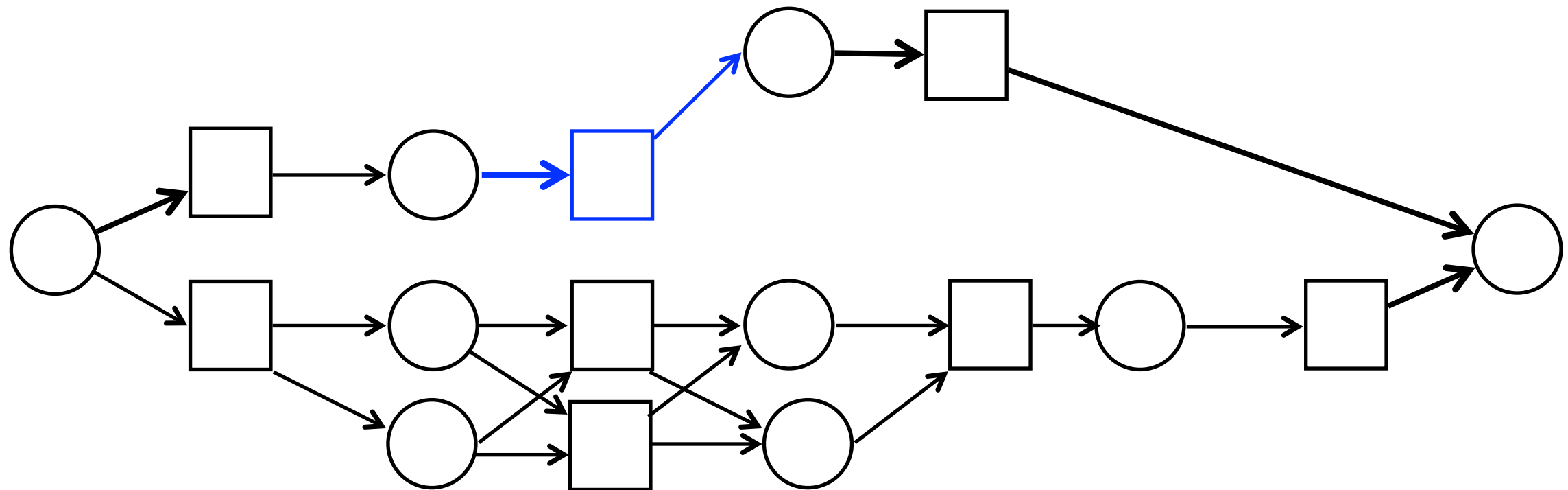
Exercise

Prove that the net below is
a safe and sound workflow net



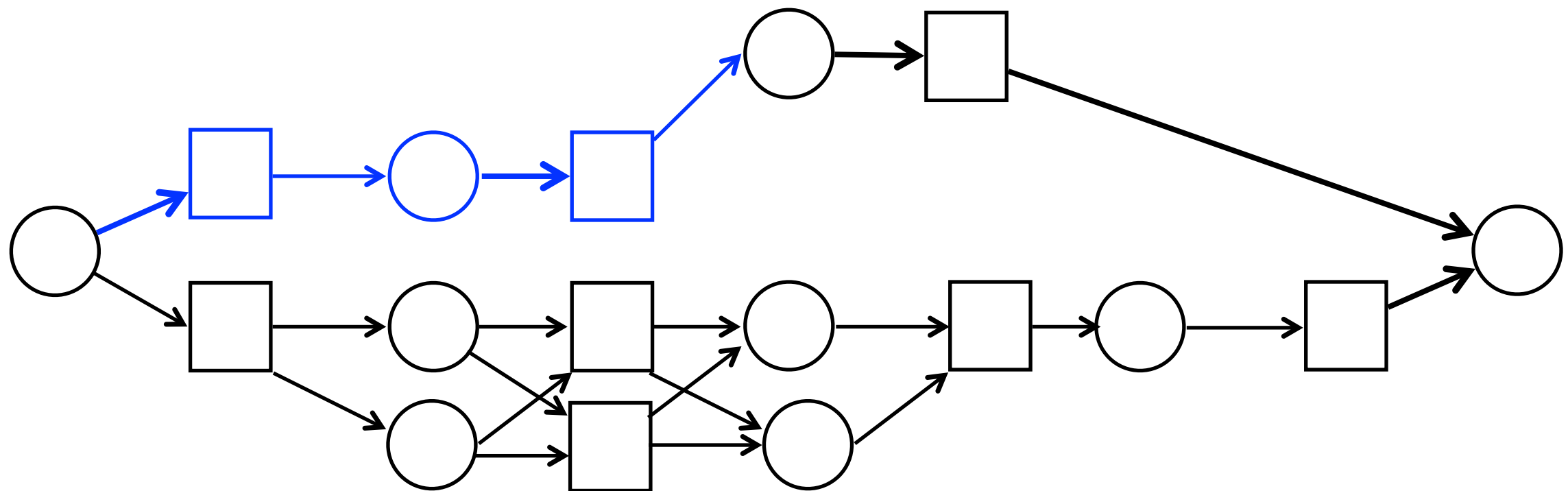
Exercise

Prove that the net below is
a safe and sound workflow net



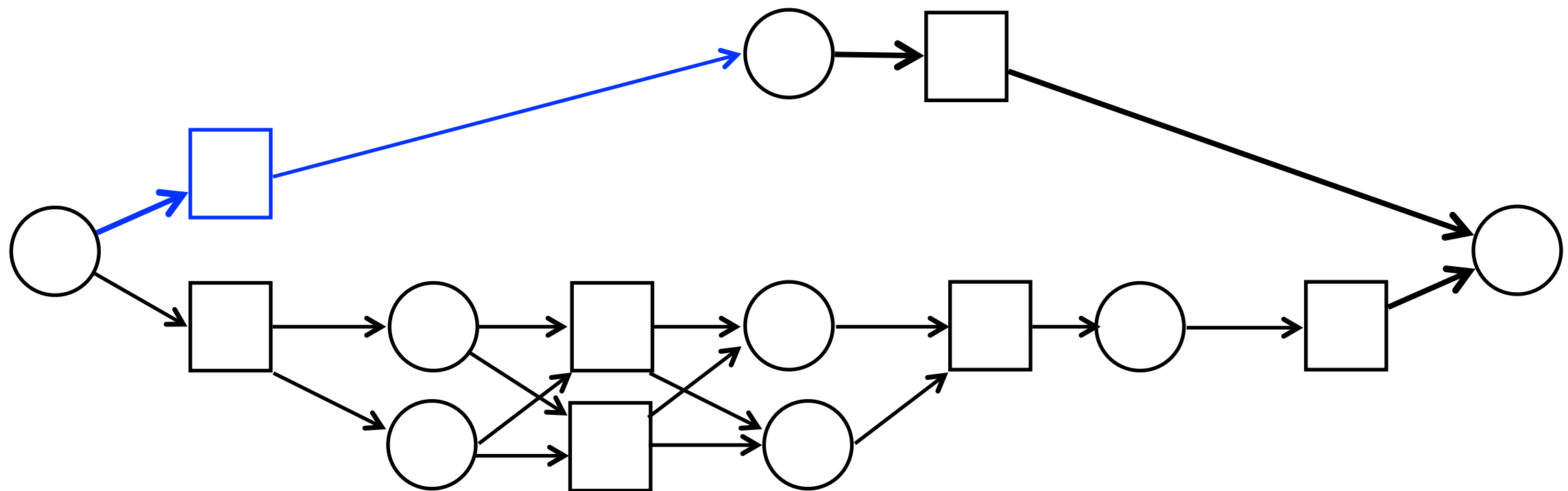
Exercise

Prove that the net below is
a safe and sound workflow net



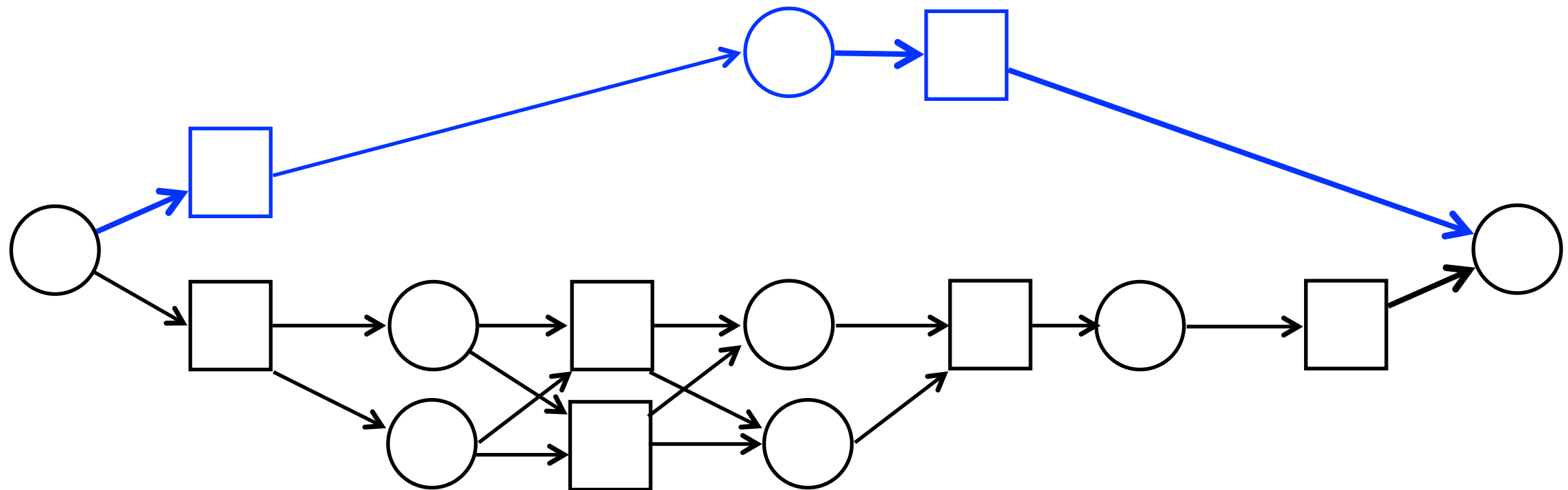
Exercise

Prove that the net below is
a safe and sound workflow net



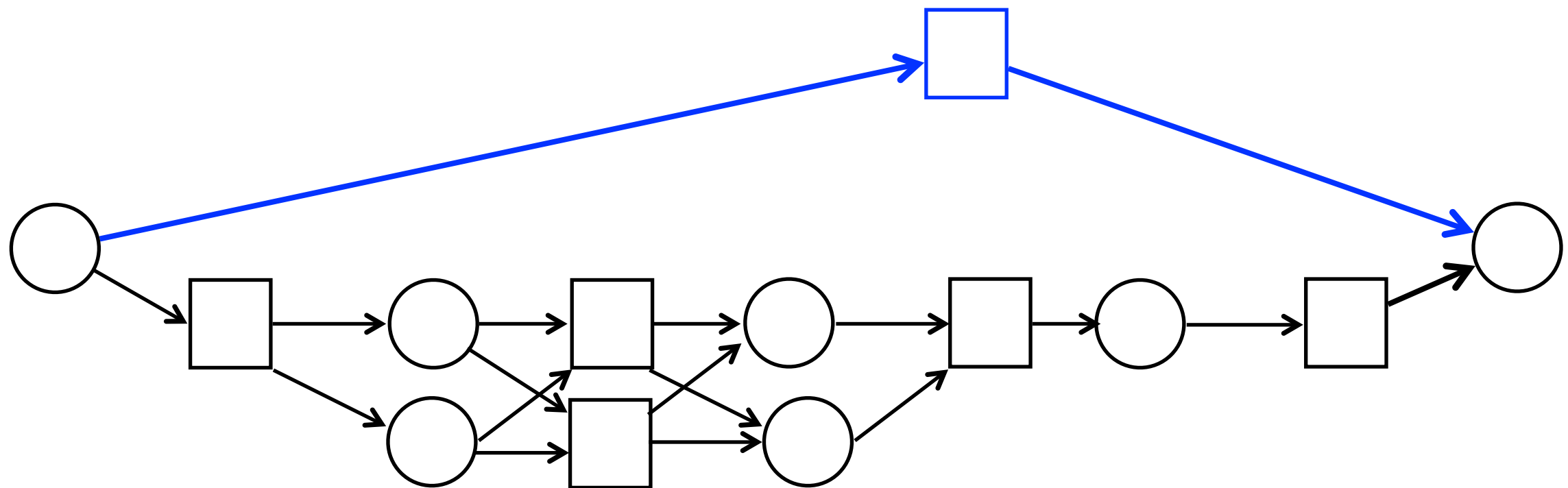
Exercise

Prove that the net below is
a safe and sound workflow net



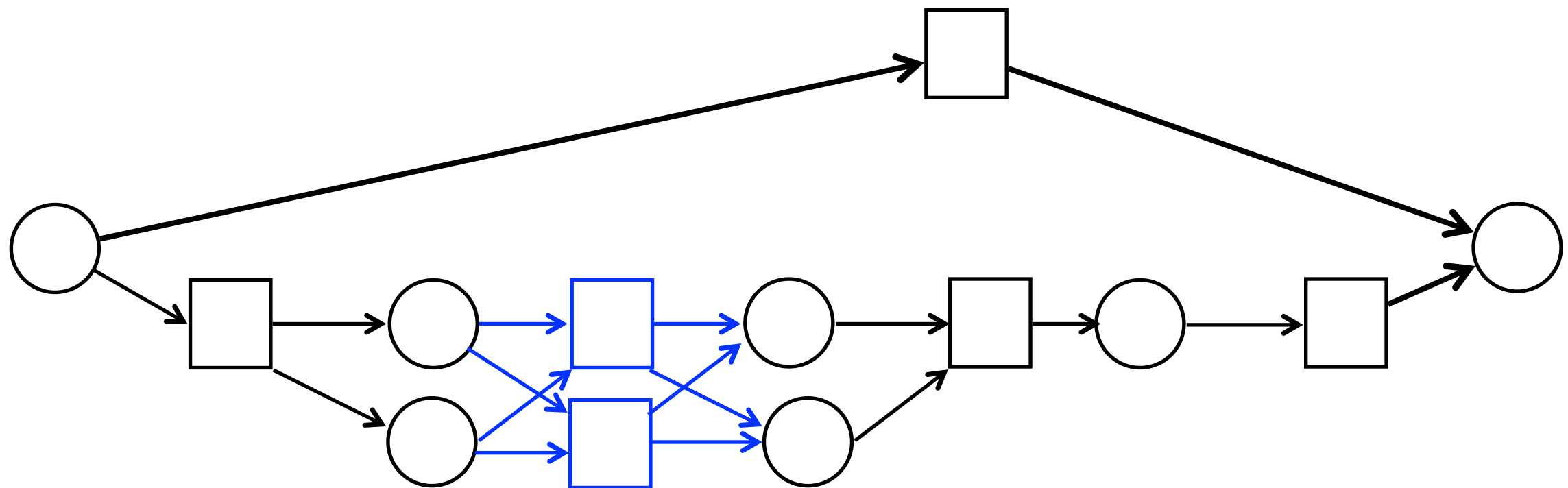
Exercise

Prove that the net below is
a safe and sound workflow net



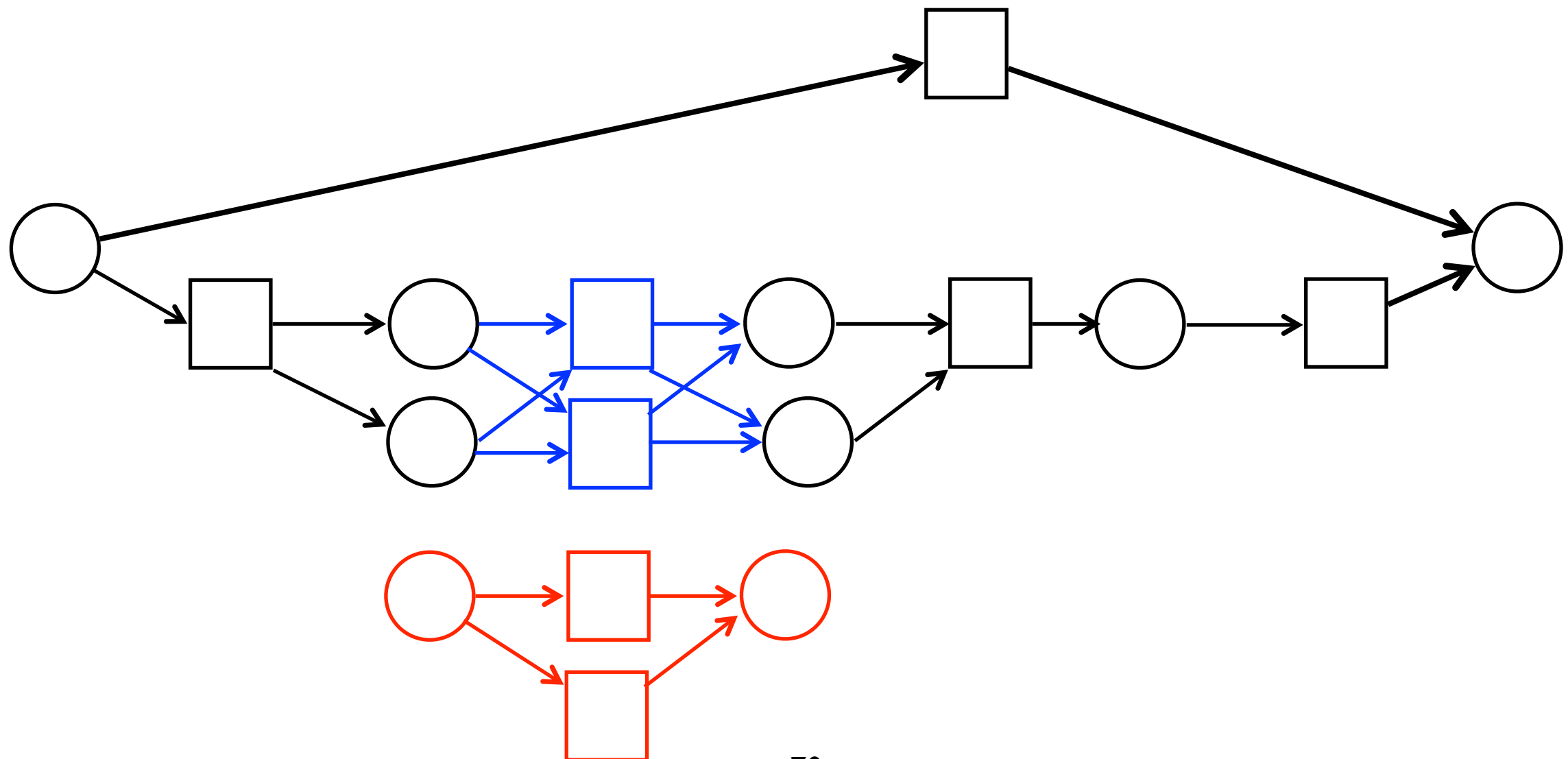
Exercise

Prove that the net below is
a safe and sound workflow net



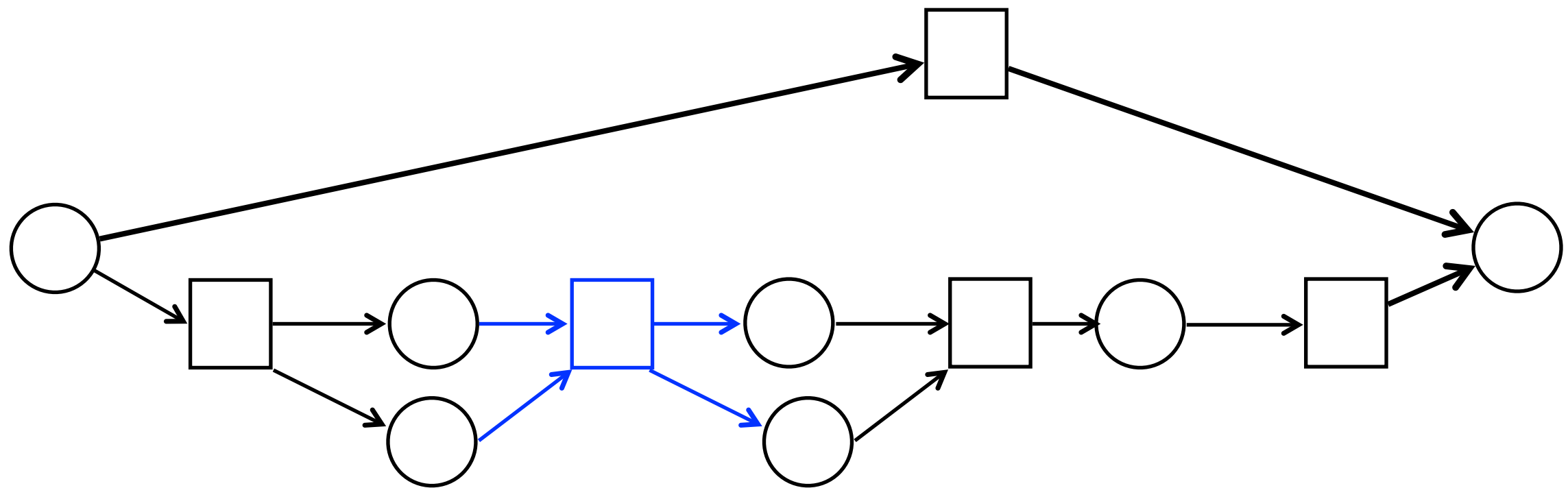
Exercise

Prove that the net below is
a safe and sound workflow net



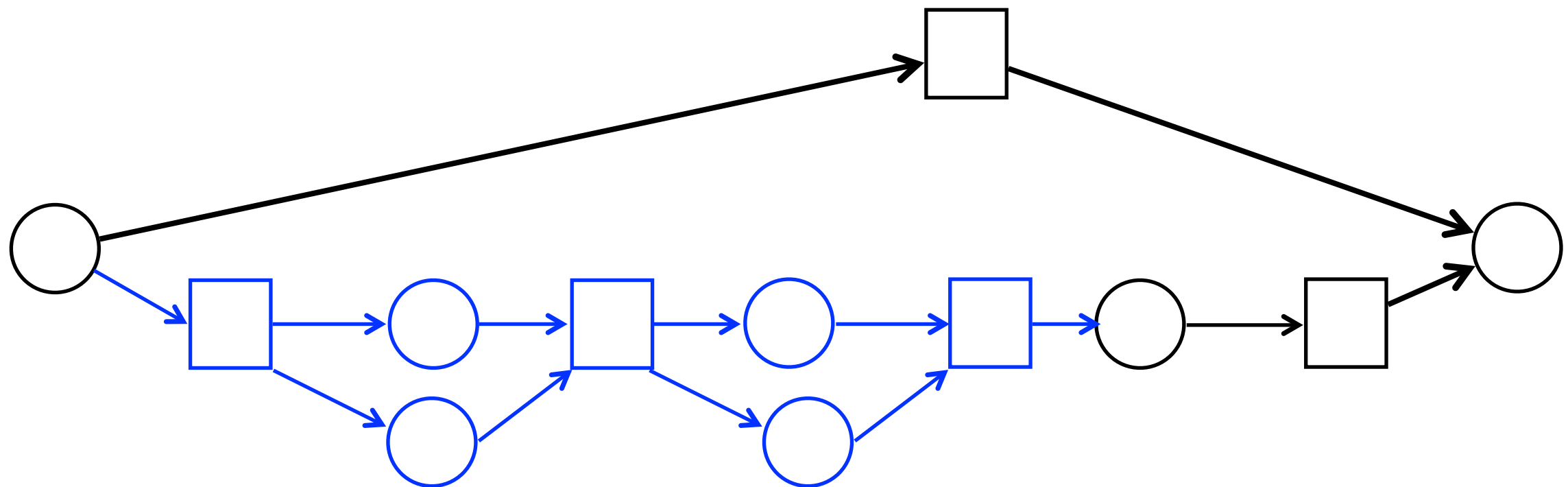
Exercise

Prove that the net below is
a safe and sound workflow net



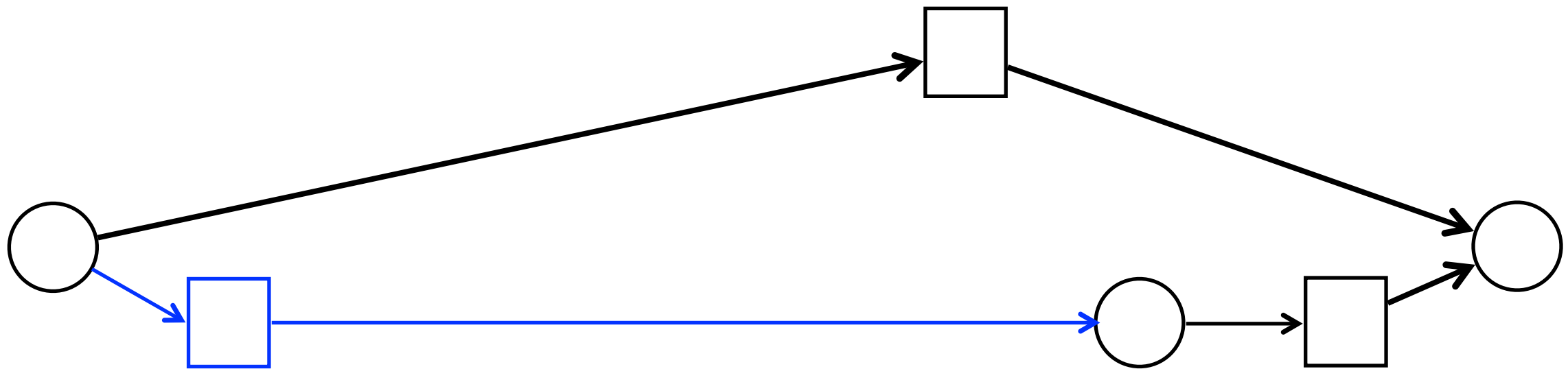
Exercise

Prove that the net below is
a safe and sound workflow net



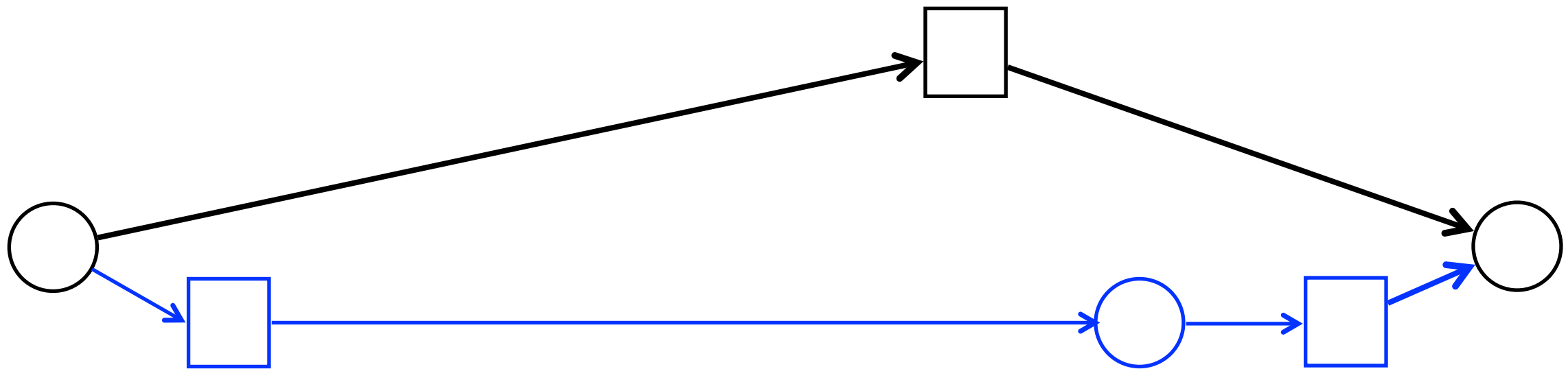
Exercise

Prove that the net below is
a safe and sound workflow net



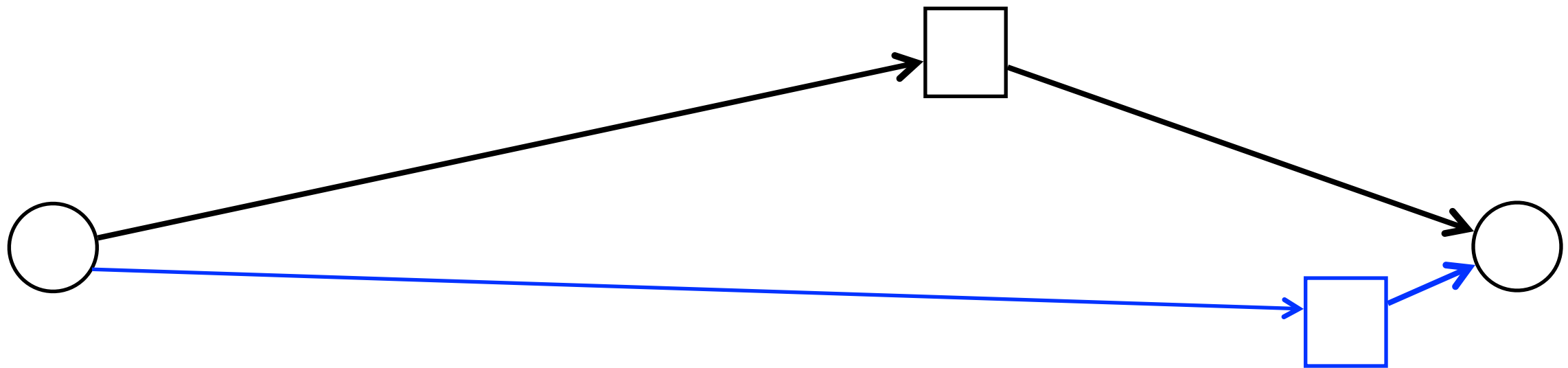
Exercise

Prove that the net below is
a safe and sound workflow net



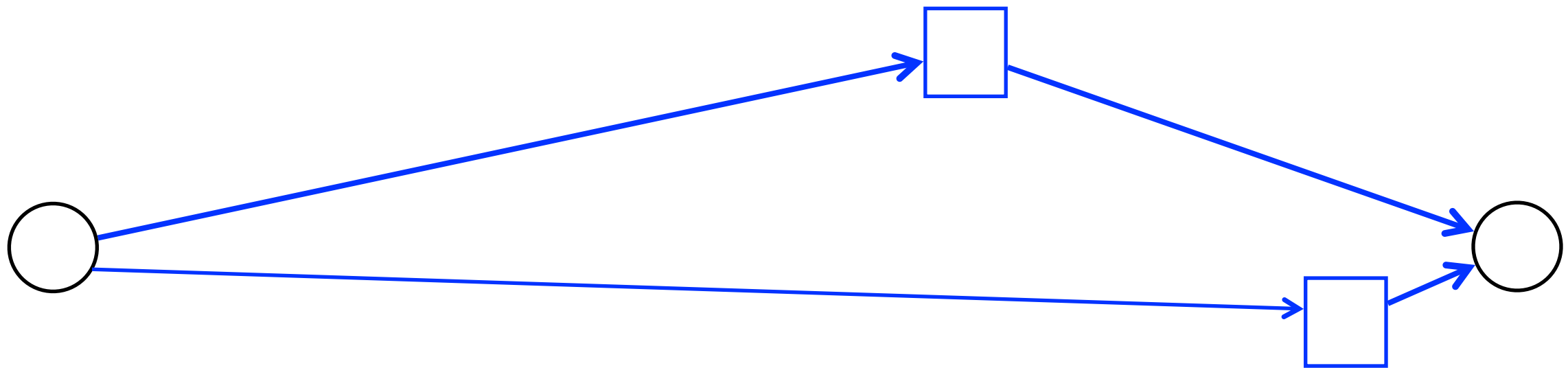
Exercise

Prove that the net below is
a safe and sound workflow net



Exercise

Prove that the net below is
a safe and sound workflow net



Exercise

Prove that the net below is
a safe and sound workflow net

