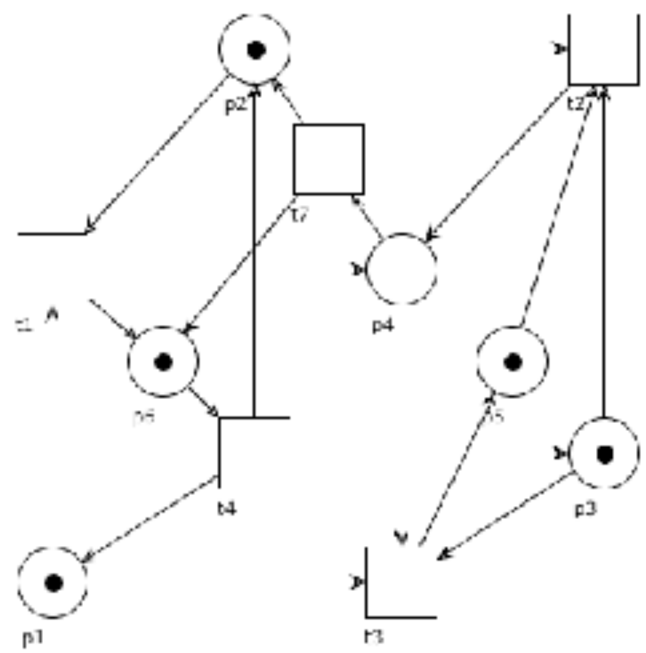
Business Processes Modelling

MPB (6 cfu, 295AA)

Roberto Bruni
http://www.di.unipi.it/~bruni

09 - Petri nets basics

Object



Formalization of the basic concepts of Petri nets

Free Choice Nets (book, optional reading)
https://www7.in.tum.de/~esparza/bookfc.html

Petri nets:
basic definitions

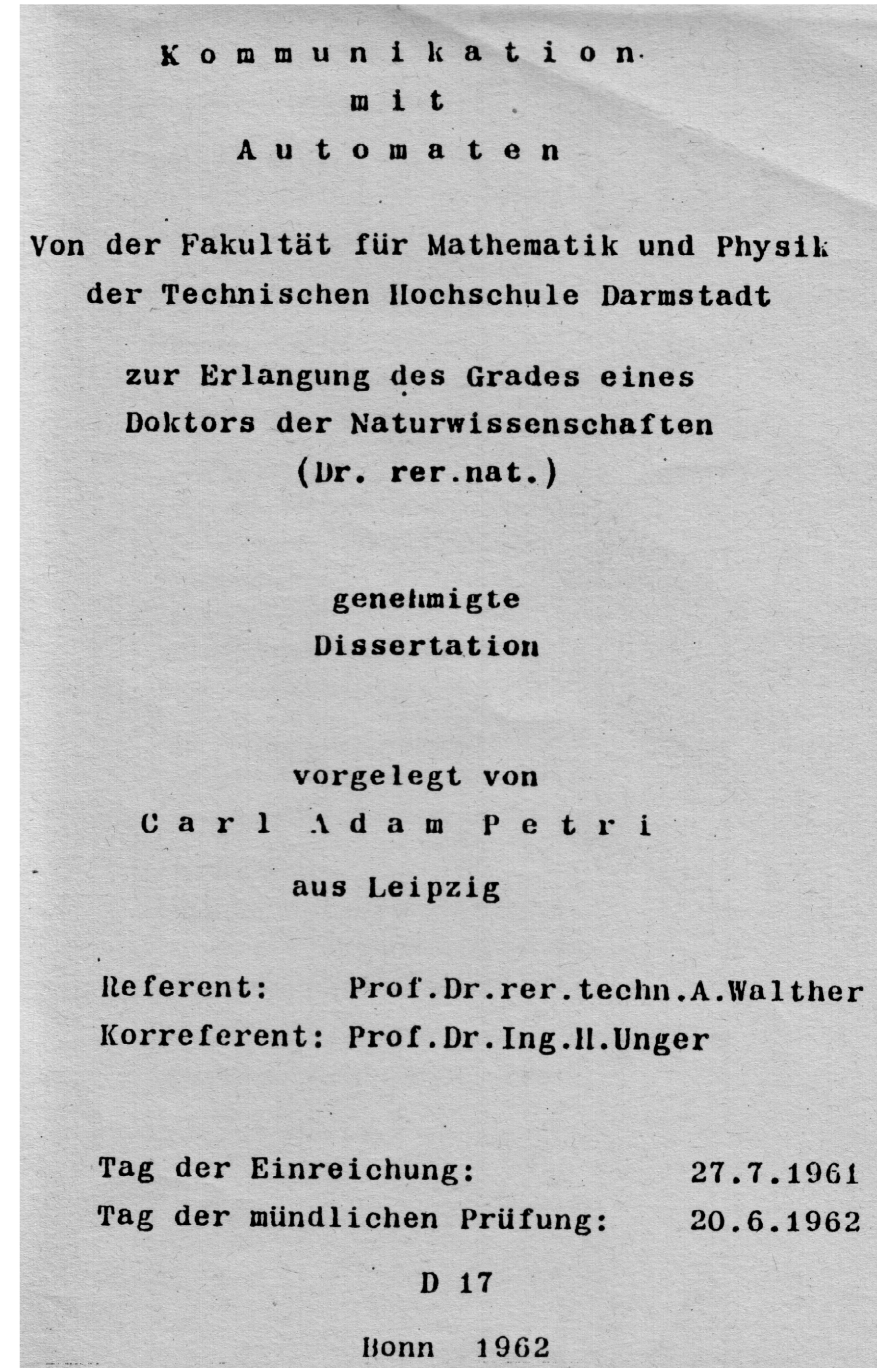
Carl Adam Petri

July 12, 1926 - July 2, 2010

http://www.informatik.uni-hamburg.de/TGI/mitarbeiter/profs/petri_eng.html

Introduced in 1962 (Petri’s PhD thesis)
60’s and 70’s main focus on theory
80‘s focus on tools and applications
Now applied in several fields

Success due to simple and clean graphical and conceptual representation



Kommunikation
mit
Automaten

Von der Fakultät für Mathematik und Physik
der Technischen Hochschule Darmstadt

zur Erlangung des Grades eines
Doktors der Naturwissenschaften
(Dr. rer.nat.)

genehmigte
Dissertation

vorgelegt von
Carl Adam Petri
aus Leipzig

Referent: Prof.Dr.rer.techn.A.Walther
Korreferent: Prof.Dr.Ing.H.Unger

Tag der Einreichung: 27.7.1961
Tag der mündlichen Prüfung: 20.6.1962

D 17

Bonn 1962

Petri nets for us

Formal and abstract business process specification

Formal: the semantics of process instances becomes well defined and not ambiguous

Abstract: execution environment is disregarded

(Remind about separation of concerns)

Places

A place can stand for
a state
a medium
a buffer
a condition
a repository of resources
a type
a memory location
...

Transitions

A transition can stand for
an operation
a calculation
an evaluation
a transformation
a transportation
a task
an activity
a decision
...

Tokens

A token can stand for
a physical object
a piece of data
a record
a resource
an activation mark
a message
a document
a case
a value
...

Notation: from sets…

Let S be a set.
Let $\wp(S)$ denote the set of sets over S.

Elements $A \in \wp(S)$ (i.e., $A \subseteq S$)
are in bijective correspondence with
functions $f : S \to \{0, 1\}$

$x \in A$ iff $f_A(x) = 1$

Sets vs Multisets

Set

Multiset

Order of elements does not matter

Each element appears at most once

Order of elements does not matter

Each element can appear multiple times

Notation: ... to multisets

Let $\mu(S)$ (or $S^{\oplus}$) denote the set of multisets over S.

Elements $B \in \mu(S)$ are in bijective correspondence with functions $M : S \to \mathbb{N}$

$M_B(x)$ is the number of instances of x in B
$x \in B$ iff $M_B(x) > 0$

Marking

A **marking** $M : P \to \mathbb{N}$ denotes the number of tokens in each place

The marking of a Petri net represents its state

$M(a) = 0$ denotes the absence of tokens in place a

Notation: sets

Empty set:
$\emptyset = \{ \}$ is such that $x \notin \emptyset$ for all $x \in S$

Set inclusion:
we write $A \subseteq B$ if $x \in A$ implies $x \in B$

Set strict inclusion:
we write $A \subset B$ if $A \subseteq B$ and $A \neq B$

Set union:
$A \cup B$ is the set s.t. $x \in (A \cup B)$ iff $x \in A$ or $x \in B$

Set difference:
$A - B$ is the set s.t. $x \in (A - B)$ iff $x \in A$ and $x \notin B$

Notation: multisets

Empty multiset:
$\emptyset$ is such that $\emptyset(x) = 0$ for all $x \in S$

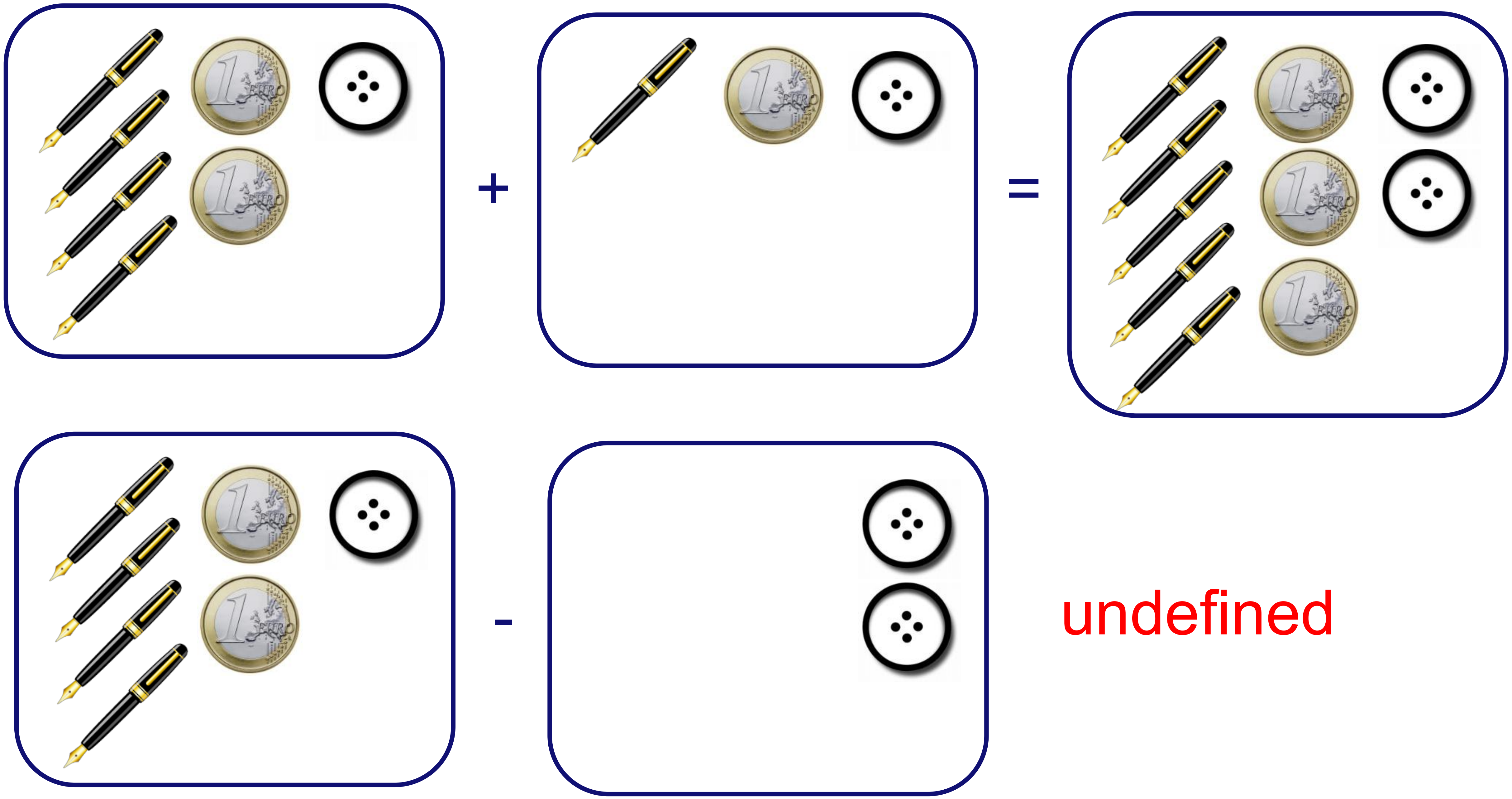
Multiset containment:
we write $M \subseteq M'$ if $M(x) \leq M'(x)$ for all $x \in S$

Multiset strict containment:
we write $M \subset M'$ if $M \subseteq M'$ and $M \neq M'$

Multiset union:
$M + M'$ is the multiset s.t. $(M + M')(x) = M(x) + M'(x)$ for all $x \in S$

Multiset difference (defined only if $M \supseteq M'$):
$M - M'$ is the multiset s.t. $(M - M')(x) = M(x) - M'(x)$ for all $x \in S$

Operations on Multisets



Notation: multisets

Multiset $M = \{ k_1 x_1, k_2 x_2, ..., k_n x_n \}$

element

multiplicity
(positive, omitted if 1)

or just

Notation: multisets

Multiset $M = \{\ k_1x_1, k_2x_2, ..., k_nx_n\}$ as formal sum:

$$k_1x_1 + k_2x_2 + ... + k_nx_n$$

3  +2  + 

$$\sum_{i=1}^{n} k_ix_i$$

As set is just a special case: multiplicity is either 0 or 1

$$x_1 + x_2 + \ldots + x_n$$

$$\sum_{i=1}^{n} x_i$$

Question time

$$3a + 2b \overset{?}{\subseteq} 2a + 3b + c$$

$$3a + 2b \overset{?}{\supseteq} 2a + 3b + c$$

$$a + 2b \overset{?}{\subset} 2a + 3b$$

$$(a + 2b) + (2a + c) = ?$$

$$(2a + 3b) - (2a + b) = ?$$

$$(2a + 2b) - (a + c) = ?$$

Question time

$$3a + 2b \overset{?}{\subseteq} 2a + 3b + c$$ No

$$3a + 2b \overset{?}{\supseteq} 2a + 3b + c$$ No

$$a + 2b \overset{?}{\subset} 2a + 3b$$ Yes

$$(a + 2b) + (2a + c) = ?$$ $3a + 2b + c$

$$(2a + 3b) - (2a + b) = ?$$ $2b$

$$(2a + 2b) - (a + c) = ?$$ Not defined

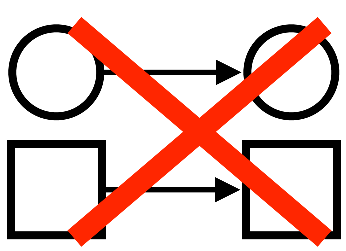
Petri nets

A **Petri net** is a tuple (P, T, F, M_0) where

- P is a finite set of **places**;

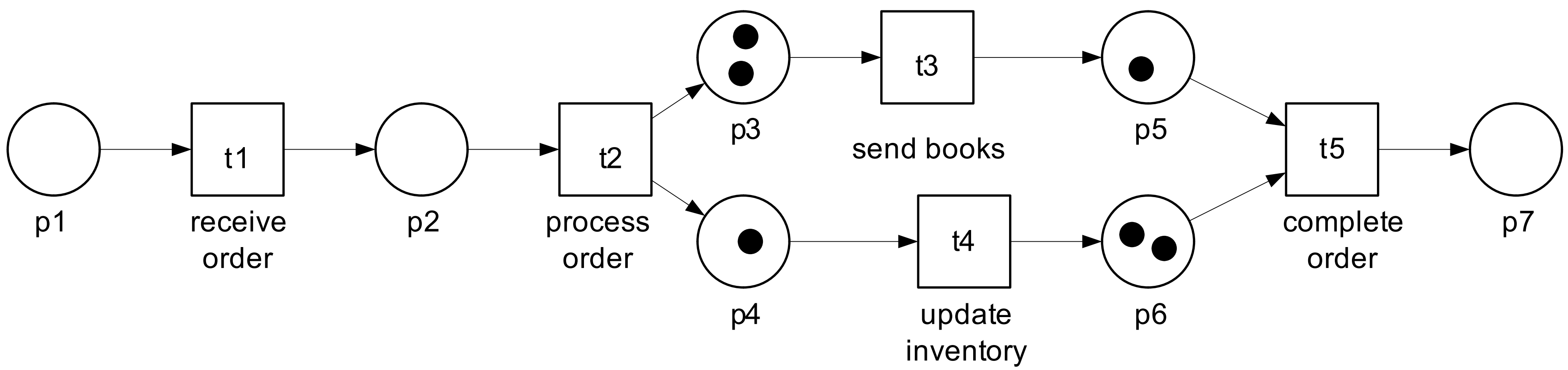
$P \cap T = \emptyset$

- T is a finite set of **transitions**;



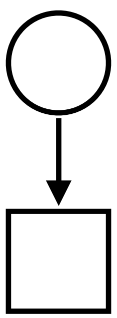
- $F \subseteq (P \times T) \cup (T \times P)$ is a **flow relation**;
- $M_0 : P \to \mathbb{N}$ is the **initial marking**.
  (i.e. $M_0 \in \mu(P)$)

Example

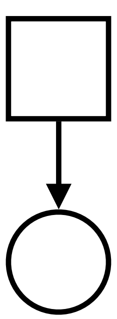


$$P = \{p_1, p_2, p_3, p_4, p_5, p_6, p_7\}$$
$$T = \{t_1, t_2, t_3, t_4, t_5\}$$
$$F = \{(p_1, t_1), (t_1, p_2), \ldots ?\}$$
$$M_0 = 2p_3 + \ldots ?$$

Pre-set and post-set

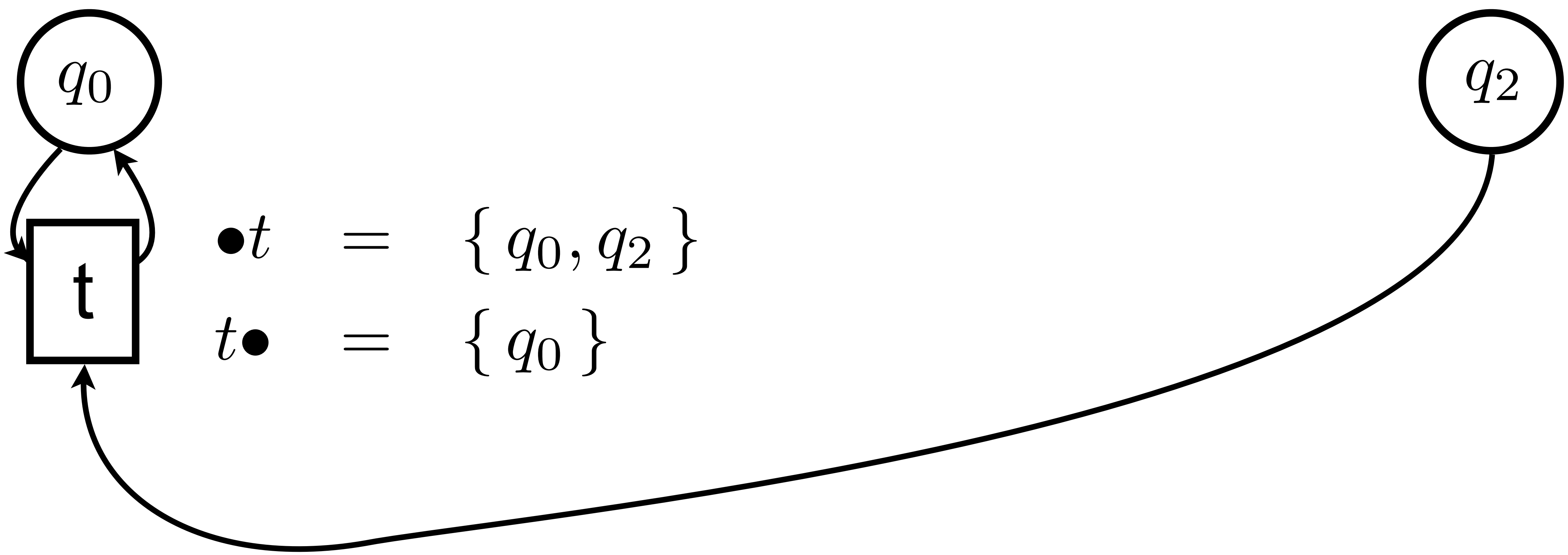


A place p is an input place for transition t iff
$(p, t) \in F$
We let $\bullet t$ denote the set of input places of t.
(pre-set of t)



A place p is an output place for transition t iff
$(t, p) \in F$
We let $t\bullet$ denote the set of output places of t.
(post-set of t)

Example: pre and post



$$
\begin{aligned}
\bullet t &= \{ q_0, q_2 \} \\
t \bullet &= \{ q_0 \}
\end{aligned}
$$

Pre-set and post-set

Analogously, we let

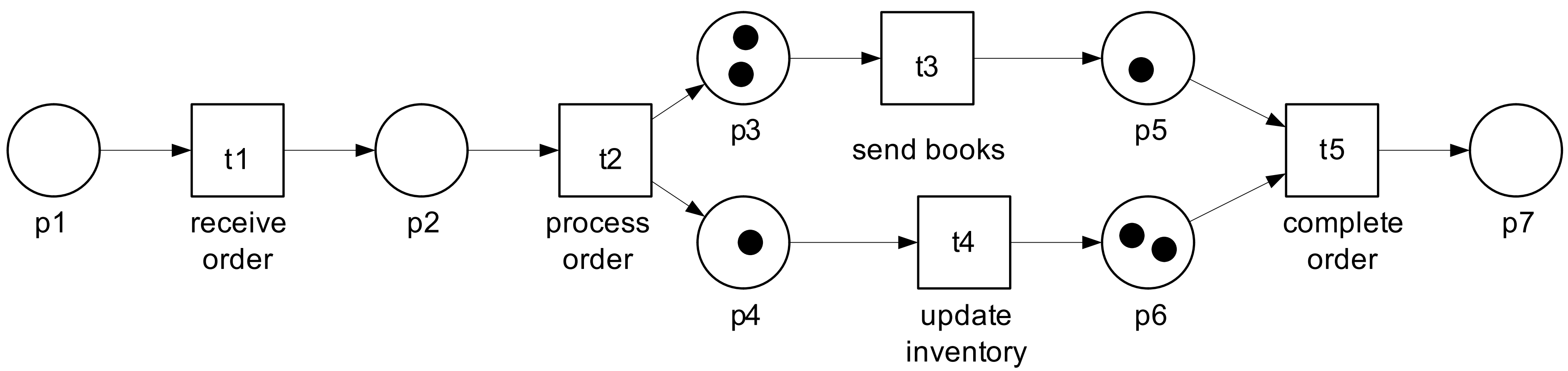
$\bullet p$ denote the set of transitions that share p as output place

$p\bullet$ denote the set of transitions that share p as input place

Formally:

$$\bullet x = \{\ y \ \mid\ (y,x) \in F\ \} \quad \text{pre-set}$$
$$x\bullet = \{\ y \ \mid\ (x,y) \in F\ \} \quad \text{post-set}$$

Question time



$\bullet t_1 = ?$

$\bullet t_2 = ?$

$\bullet t_3 = ?$

$\bullet t_4 = ?$

$\bullet t_5 = ?$

$t_1 \bullet = ?$

$t_2 \bullet = ?$

$t_3 \bullet = ?$

$t_4 \bullet = ?$

$t_5 \bullet = ?$

$\bullet p_1 = ?$

$\bullet p_2 = ?$

$\bullet p_3 = ?$

$\bullet p_4 = ?$

$\bullet p_5 = ?$

$\bullet p_6 = ?$

$\bullet p_7 = ?$

$p_1 \bullet = ?$

$p_2 \bullet = ?$

$p_3 \bullet = ?$

$p_4 \bullet = ?$

$p_5 \bullet = ?$

$p_6 \bullet = ?$

$p_7 \bullet = ?$

Petri nets: enabling and firing

Enabling M[t⟩

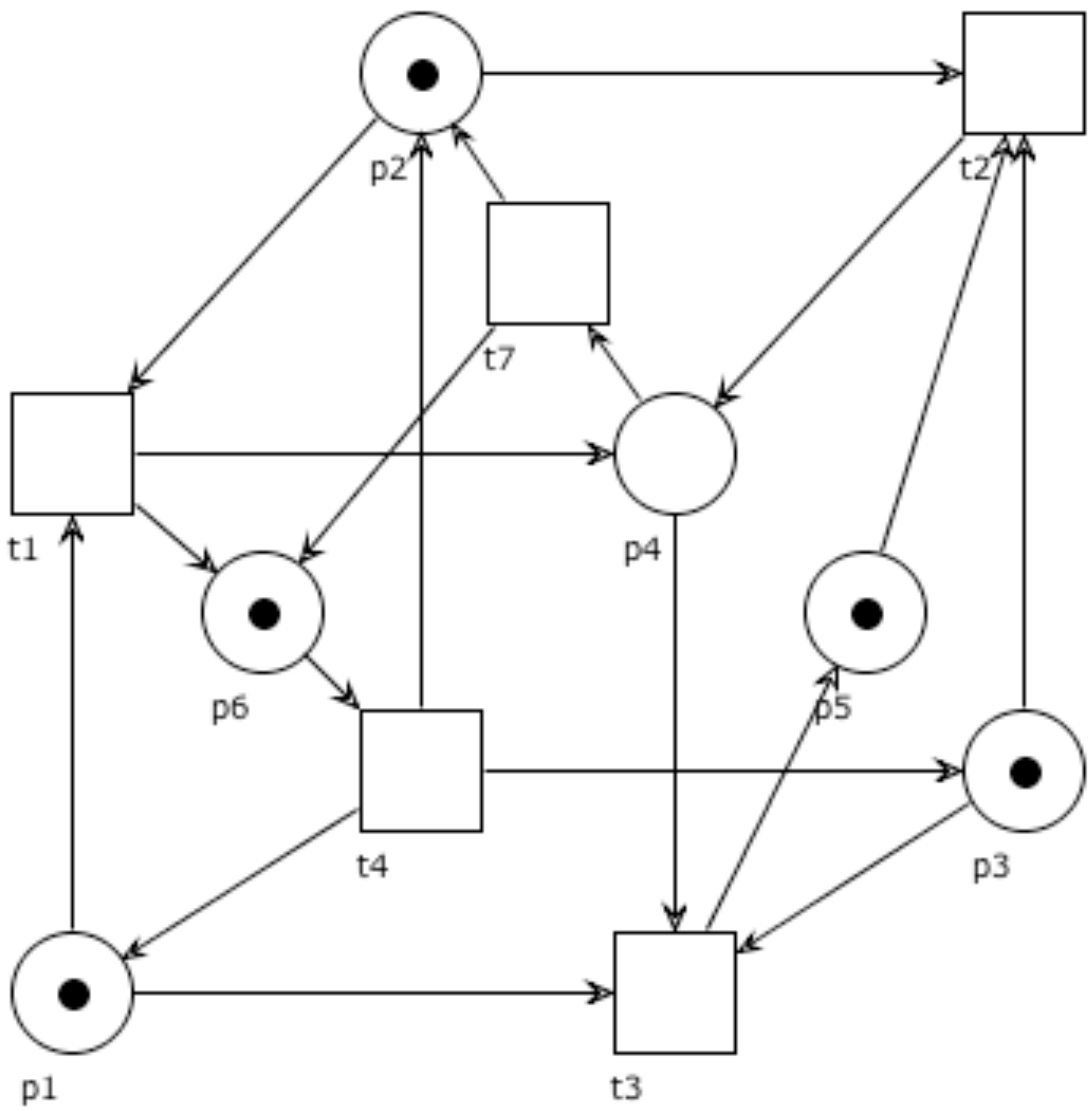
(a set can be seen as a multiset whose elements have multiplicity 1)

A transition t is **enabled** at marking M iff $\bullet t \subseteq M$
and we write $M \xrightarrow{t}$ (also $M\,[t\rangle$)

A transition is enabled if each of its input places contains at least one token

Question time

$$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$$

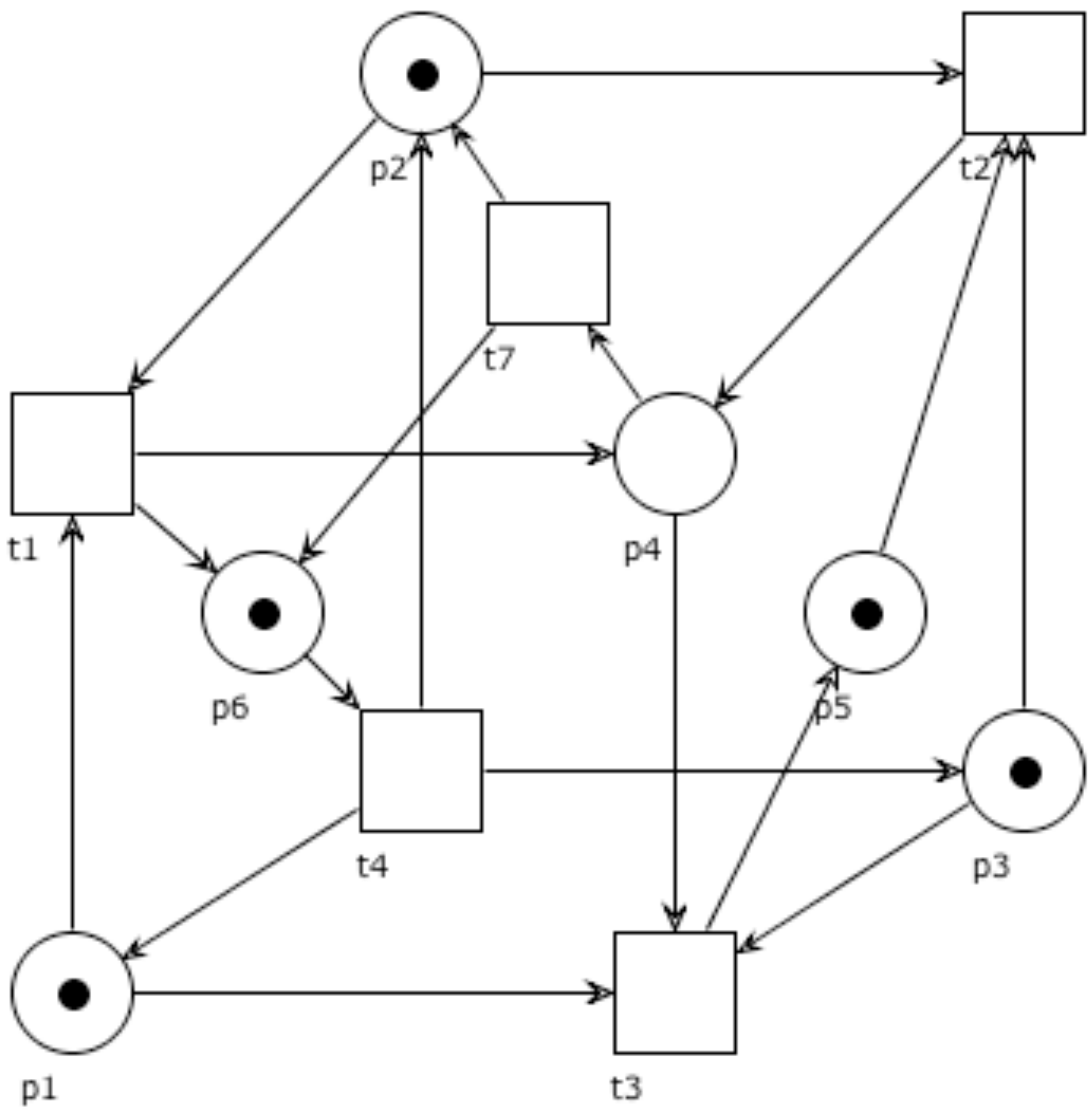


Which of the following holds true?

- $M_0 \xrightarrow{t_1}$
- $M_0 \xrightarrow{t_2}$
- $M_0 \xrightarrow{t_3}$
- $M_0 \xrightarrow{t_7}$

Question time

$$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$$



Which of the following holds true?

- $M_0 \xrightarrow{t_1}$ Yes
- $M_0 \xrightarrow{t_2}$ Yes
- $M_0 \xrightarrow{t_3}$ No (no token in p_4)
- $M_0 \xrightarrow{t_7}$ No (no token in p_4)

Firing M[t>M'

A transition t that is enabled at M can **fire**.
The **firing** of t at M changes the state to

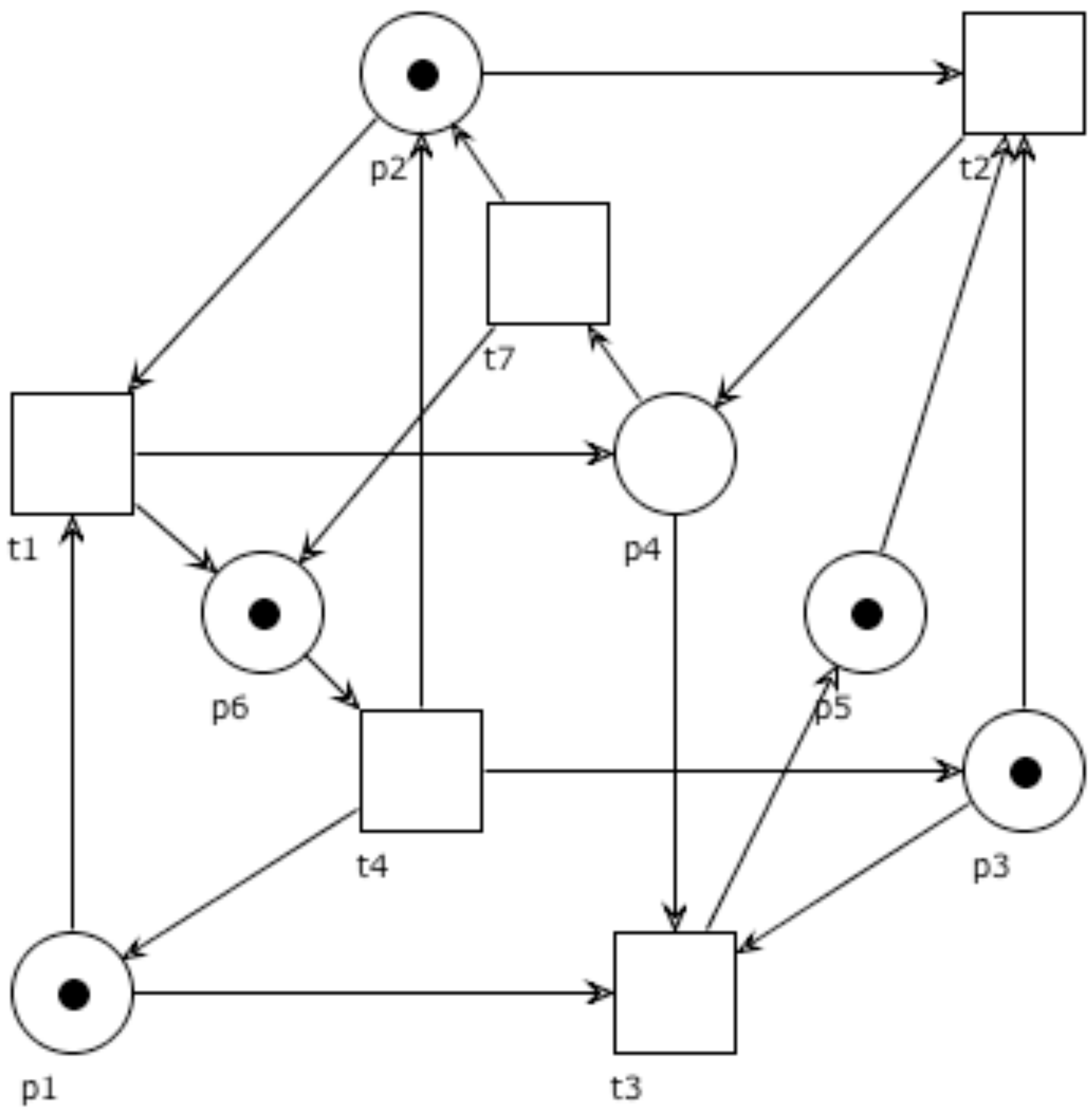
$$M' = M - \bullet t + t \bullet$$

and we write $M \xrightarrow{t} M'$ (also $M\,[t\rangle\,M'$)

When a transition fires
it consumes a token from each input place
it produces a token into each output place

Question time

$$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$$

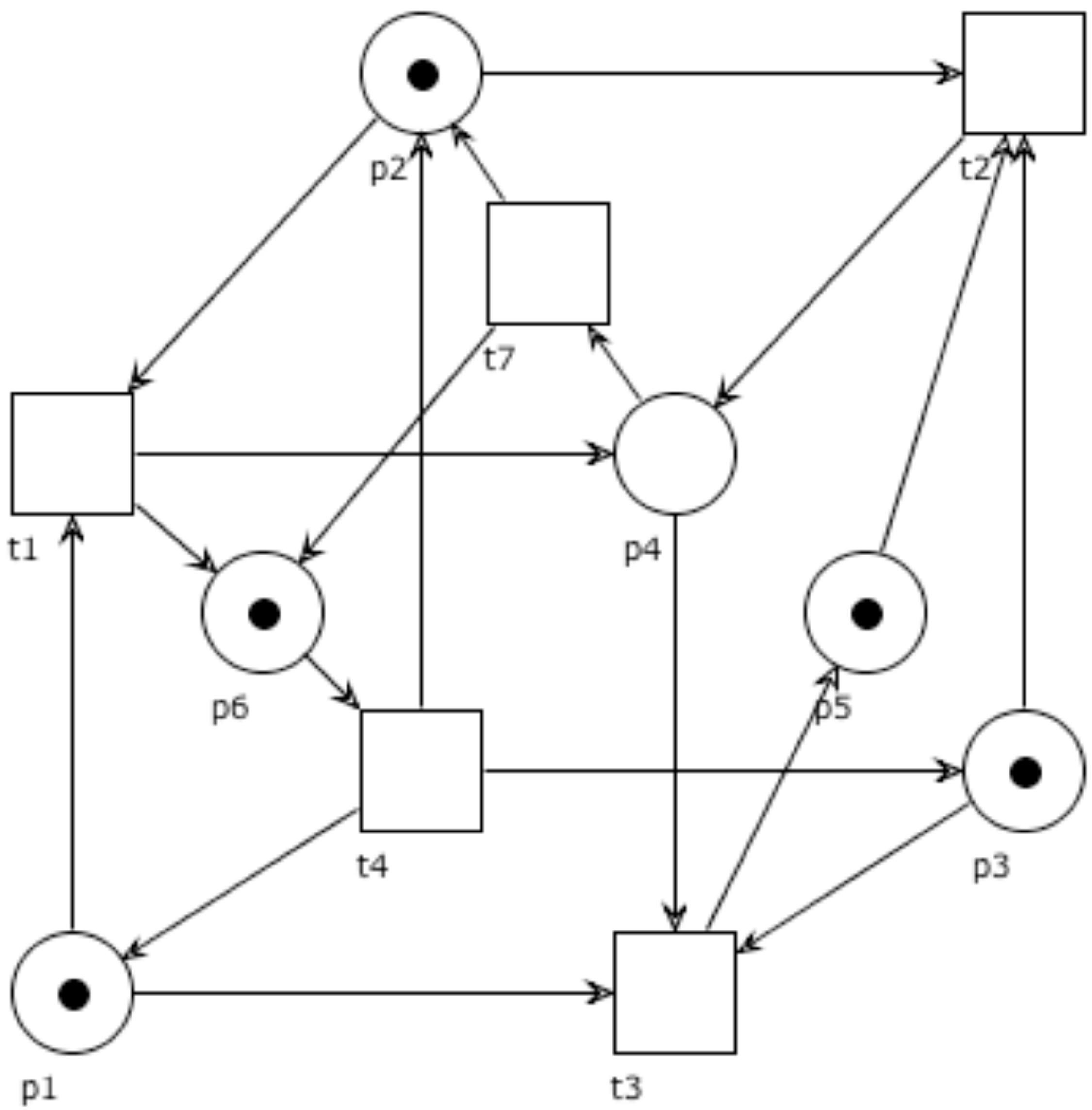


Which of the following holds true?

- $M_0 \xrightarrow{t_1} p_3 + p_4 + p_5 + p_6$
- $M_0 \xrightarrow{t_2} p_1 + p_4 + p_6$
- $M_0 \xrightarrow{t_4} 2p_1 + 2p_2 + 2p_3 + p_5$

Question time

$$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$$



Which of the following holds true?

- $M_0 \xrightarrow{t_1} p_3 + p_4 + p_5 + p_6$ No ($2p_6$)
- $M_0 \xrightarrow{t_2} p_1 + p_4 + p_6$ Yes
- $M_0 \xrightarrow{t_4} 2p_1 + 2p_2 + 2p_3 + p_5$ Yes

Some remarks

Firing is an atomic action

Our semantics is interleaving:
multiple transitions may be enabled,
but only one fires at a time

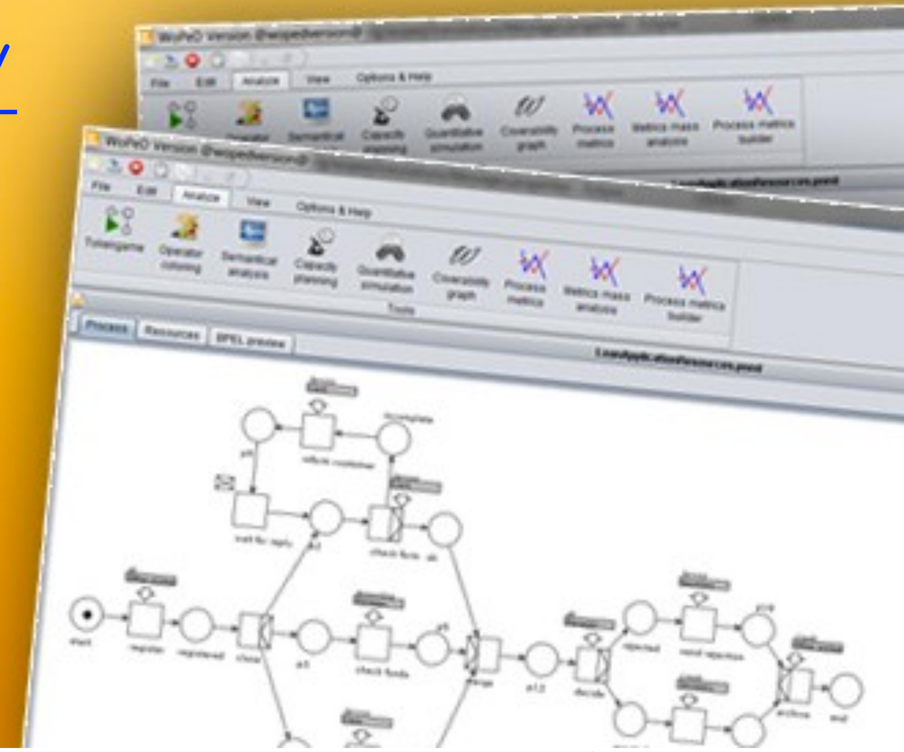
The network is static, but
the overall number of tokens may vary over time
(if transitions are fired for which the number of input places is not equal to the number of output places)

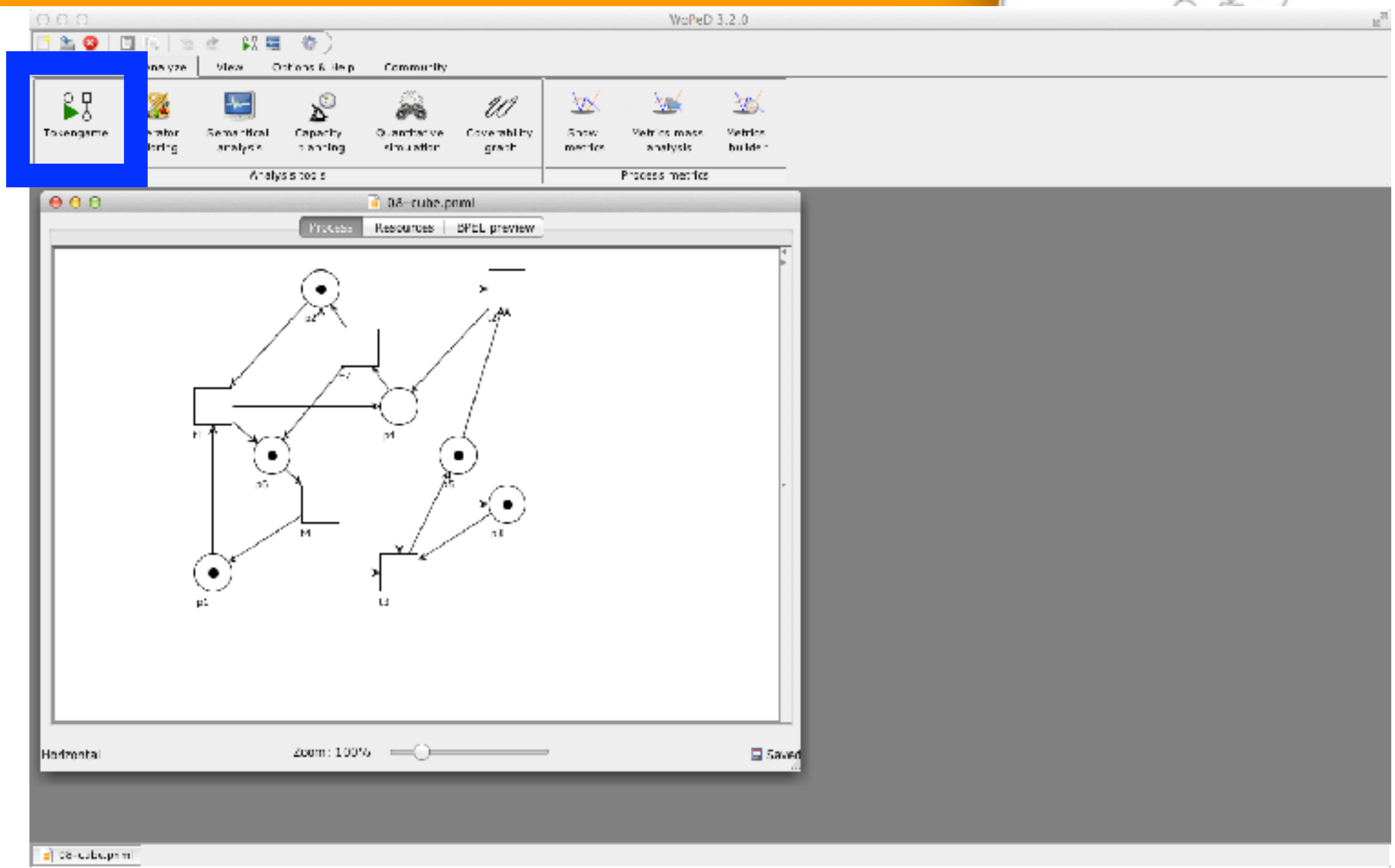
http://woped.dhbw-karlsruhe.de/woped/

WoPeD

Workflow Petri Net Designer

Download WoPeD at sourceforge!





Notation

We write $M \rightarrow$ if $M \xrightarrow{t}$ for some transition t

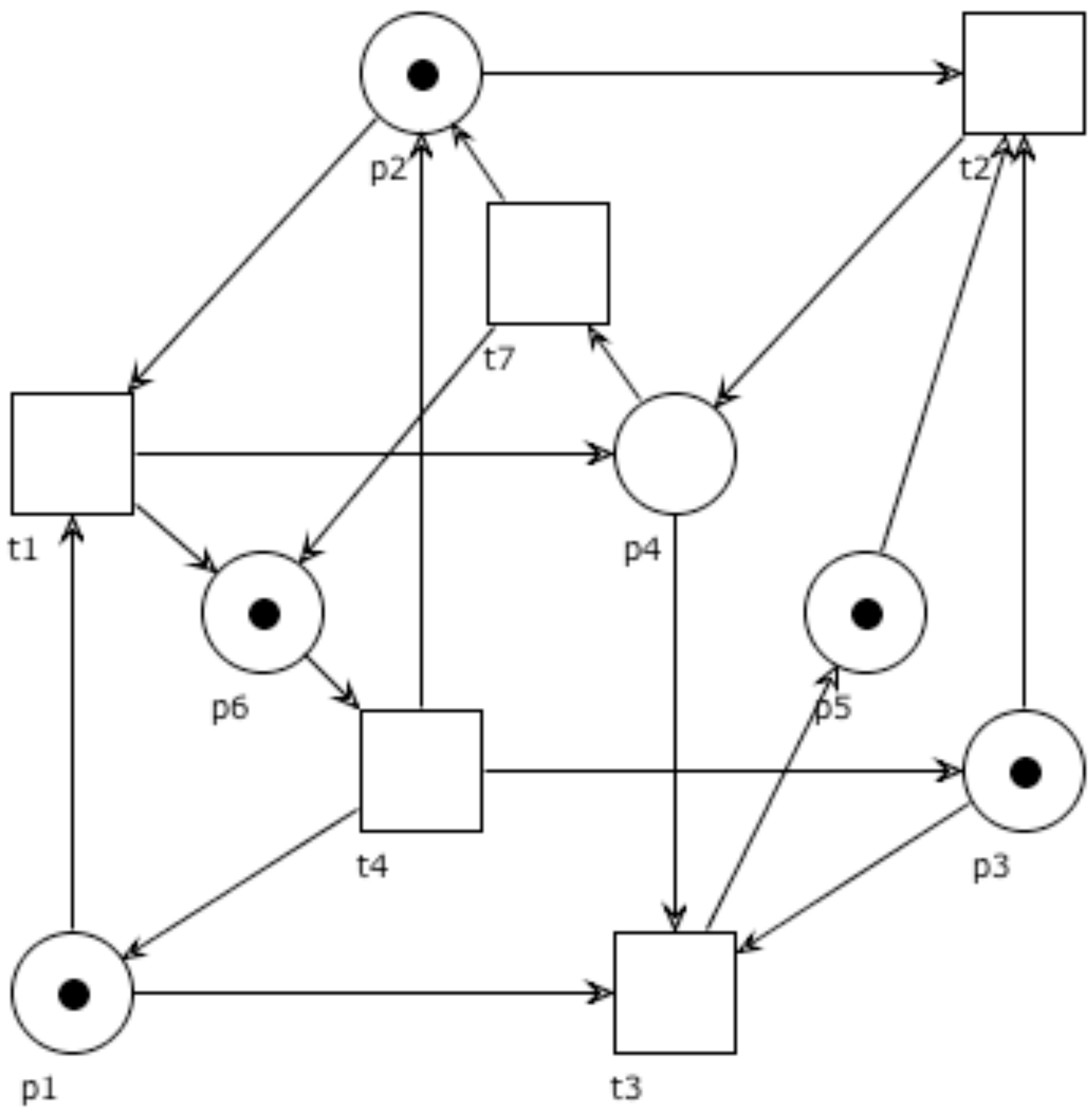
We write $M \rightarrow M'$ if $M \xrightarrow{t} M'$ for some transition t

We write $M \stackrel{t}{\not\rightarrow}$ if transition t is not enabled at M

We write $M \not\rightarrow$ if no transition is enabled at M

Example

$$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$$



We can write that

- $M_0 \longrightarrow$
- $M_0 \longrightarrow p_1 + p_4 + p_6$ (by firing t2)
- $M_0 \overset{t_7}{\not\longrightarrow}$
- $p_1 + p_5 \not\longrightarrow$

Firing sequence

Let $\sigma = t_1 t_2 ... t_{n-1} \in T^*$ be a sequence of transitions.

We write $M \xrightarrow{\sigma} M'$ (and $M \xrightarrow{\sigma}$) if:

there is a sequence of markings $M_1, ..., M_n$

with $M = M_1$ and $M' = M_n$

and $M_i \xrightarrow{t_i} M_{i+1}$ for $1 \leq i < n$

(i.e. $M = M_1 \xrightarrow{t_1} M_2 \xrightarrow{t_2} ... \xrightarrow{t_{n-1}} M_n = M'$)

Reachable markings [M⟩

We write $M \xrightarrow{*} M'$ if $M \xrightarrow{\sigma} M'$ for some $\sigma \in T^*$

A marking M' is **reachable from** M if $M \xrightarrow{*} M'$

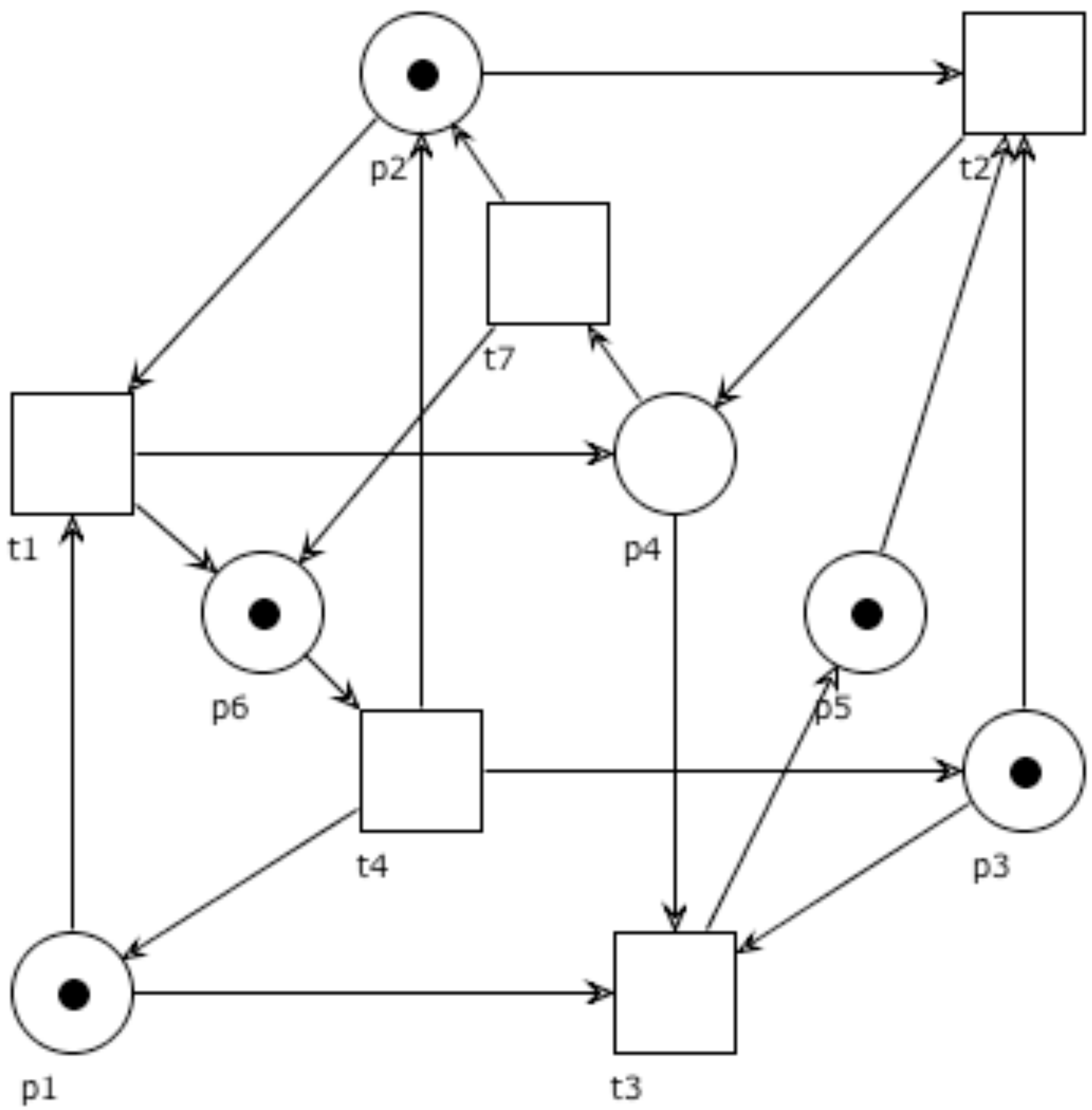
Note that $M \xrightarrow{\epsilon} M$ for ϵ the empty sequence

The set of markings reachable from M is often denoted:

$$reach(M) \text{ or also } [M\rangle$$

Question time

$$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$$

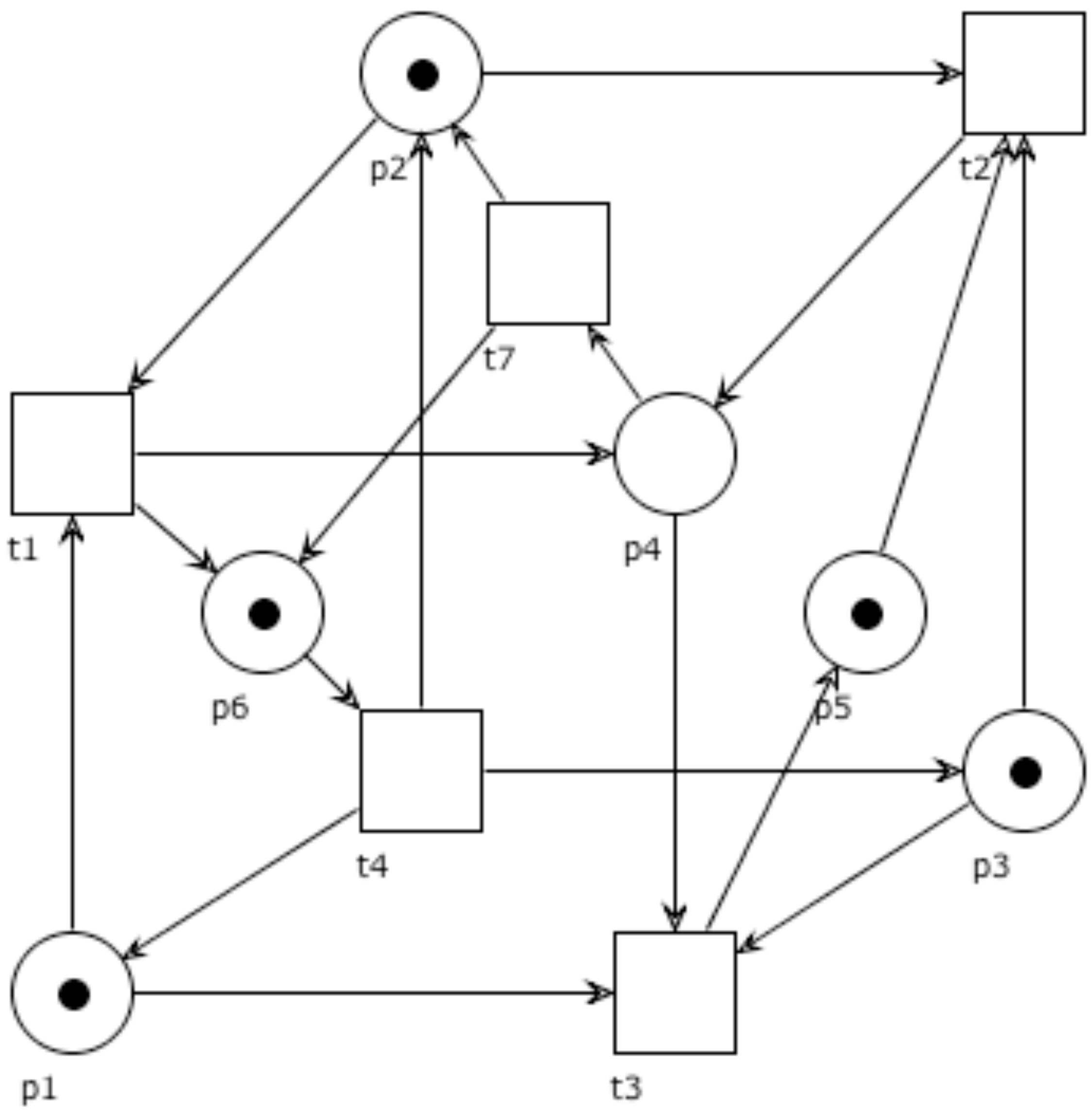


Which of the following holds true?

- $M_0 \xrightarrow{t_1 t_4 t_2 t_3}$
- $M_0 \xrightarrow{t_2 t_7 t_4}$
- $M_0 \xrightarrow{t_1 t_2 t_7}$
- $M_0 \xrightarrow{t_1 t_4 t_2 t_1}$

Question time

$$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$$

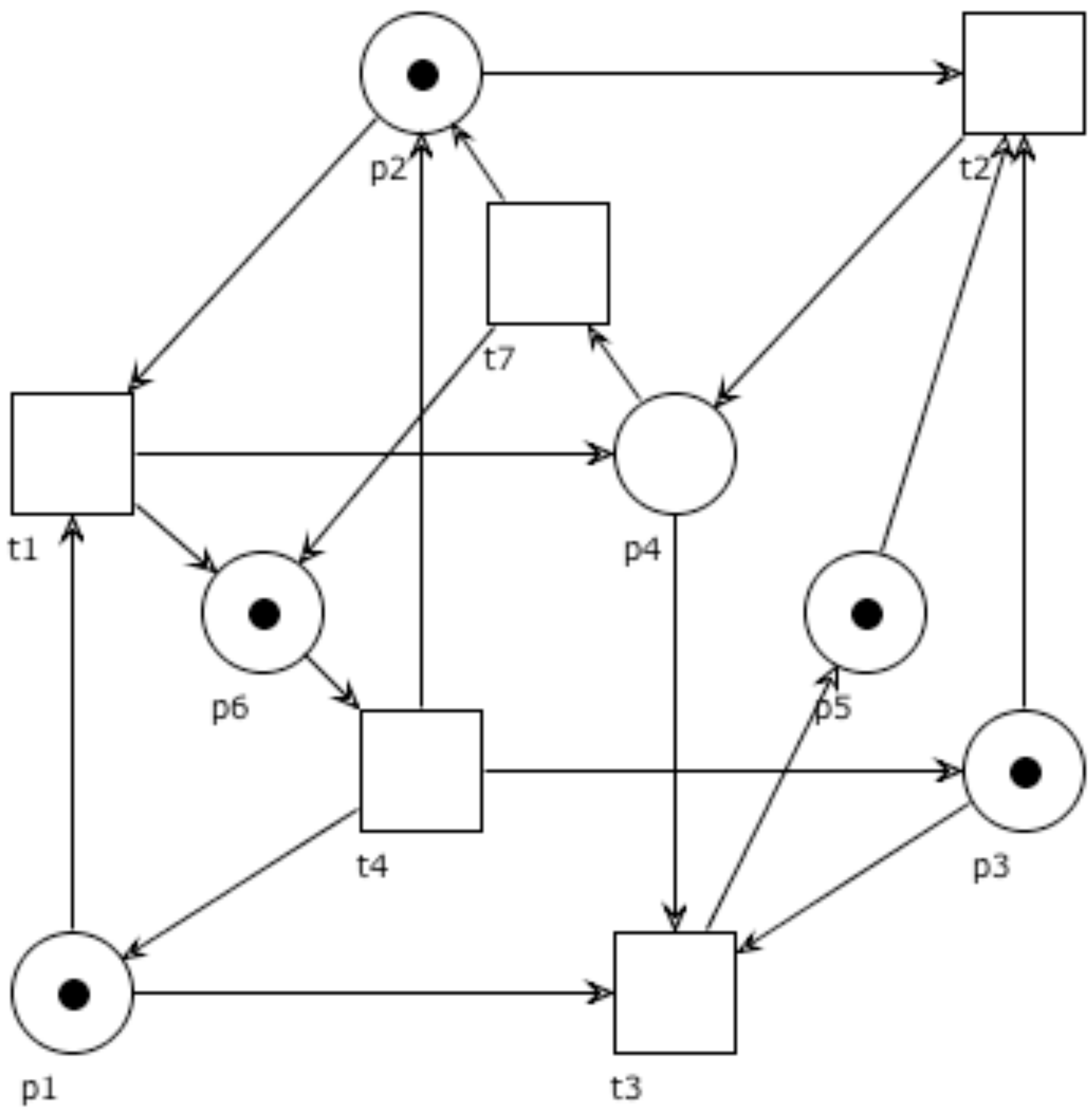


Which of the following holds true?

- $M_0 \xrightarrow{t_1 t_4 t_2 t_3}$ Yes
- $M_0 \xrightarrow{t_2 t_7 t_4}$ Yes
- $M_0 \xrightarrow{t_1 t_2 t_7}$ No (t_2 not enabled)
- $M_0 \xrightarrow{t_1 t_4 t_2 t_1}$ No (t_1 not enabled)

Example

$$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$$



We have that

- $M_0 \xrightarrow{t_1 t_4 t_2 t_3} p_4 + p_5 + p_6$
- $M_0 \xrightarrow{t_2 t_7 t_4} 2p_1 + 2p_2 + p_3 + p_6$
- $M_0 \xrightarrow{t_1 t_4 t_3 t_2 t_7} p_2 + p_5 + 2p_6$

Infinite sequence

Let $\sigma = t_1 t_2 ... \in T^\omega$ be an infinite sequence of transitions.

We write $M \xrightarrow{\sigma}$ if:

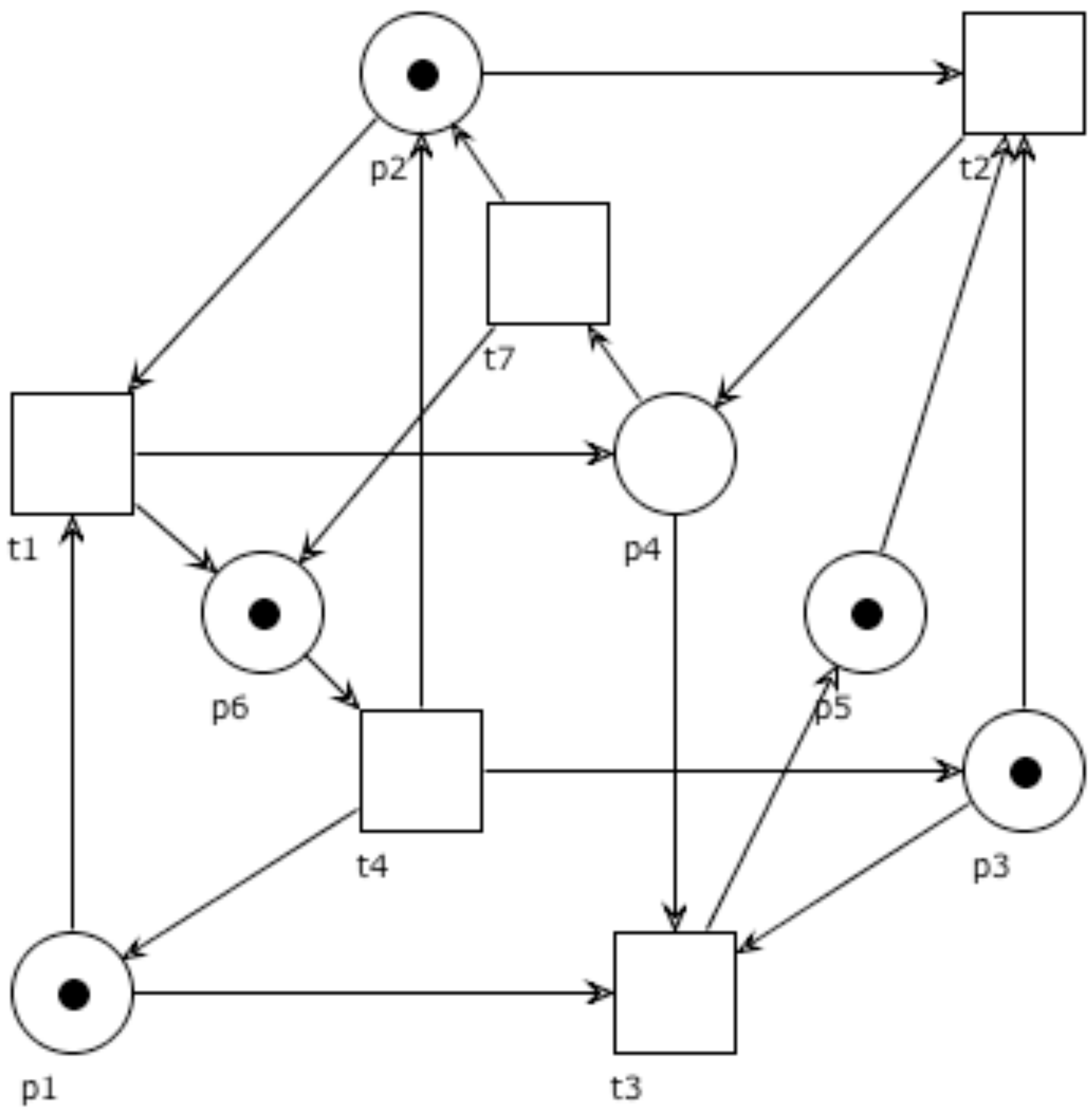
there is an infinite sequence of markings $M_1, M_2, ...$

with $M = M_1$ and $M_i \xrightarrow{t_i} M_{i+1}$ for $1 \leq i$

(i.e. $M = M_1 \xrightarrow{t_1} M_2 \xrightarrow{t_2} ...$)

Example

$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$



We have that

- $M_0 \xrightarrow{t_1 t_4 t_1 t_4 t_1 t_4 \cdots}$
- $M_0 \xrightarrow{t_1 t_4 t_7 t_1 t_4 t_7 t_1 t_4 t_7 \cdots}$

Enabled sequence

We say that an occurrence sequence σ is **enabled** if $M \xrightarrow{\sigma}$

(σ can be finite or infinite)

Note that an infinite sequence can be represented as a map $\sigma : \mathbb{N} \to T$, where $\sigma(i) = t_i$

More on sequences: concatenation & prefix

Concatenation:

finite + finite = finite

for $\sigma_1 = a_1 ... a_n$ and $\sigma_2 = b_1 ... b_m$, we let $\sigma_1 \sigma_2 = a_1 ... a_n b_1 ... b_m$

for $\sigma_1 = a_1 ... a_n$ and $\sigma_2 = b_1 b_2 ...$, we let $\sigma_1 \sigma_2 = a_1 ... a_n b_1 b_2 ...$

finite + infinite = infinite

σ is a **prefix** of σ' if $\sigma = \sigma'$ or $\sigma\sigma'' = \sigma'$ for some $\sigma'' \neq \epsilon$

σ is a **proper prefix** of σ' if $\sigma\sigma'' = \sigma'$ for some $\sigma'' \neq \epsilon$

Example: prefixes

$t_1 t_4 t_2 t_3$	sequence	$t_1 t_4 t_7 t_1 t_4 t_7 t_1 t_4 t_7 \cdots$
ϵ		ϵ
t_1	prefixes	t_1
$t_1 t_4$		$t_1 t_4$
$t_1 t_4 t_2$		$t_1 t_4 t_7$
$t_1 t_4 t_2 t_3$		$t_1 t_4 t_7 t_1$
		$t_1 t_4 t_7 t_1 t_4$
		$\cdots$

Enabledness

Proposition: $M \xrightarrow{\sigma}$ iff $M \xrightarrow{\sigma'}$ for every prefix σ' of σ

($\Rightarrow$) immediate from definition

($\Leftarrow$) trivial if σ is finite (σ itself is a prefix of σ)

When σ is infinite: taken any $i \in \mathbb{N}$ we need to prove that $t_i = \sigma(i)$ is enabled after the firing of the prefix $\sigma' = t_1 t_2 ... t_{i-1}$ of σ.

But this is obvious, because

$$M \xrightarrow{t_1} M_1 \xrightarrow{t_2} ... \xrightarrow{t_{i-1}} M_{i-1} \xrightarrow{t_i} M_i$$

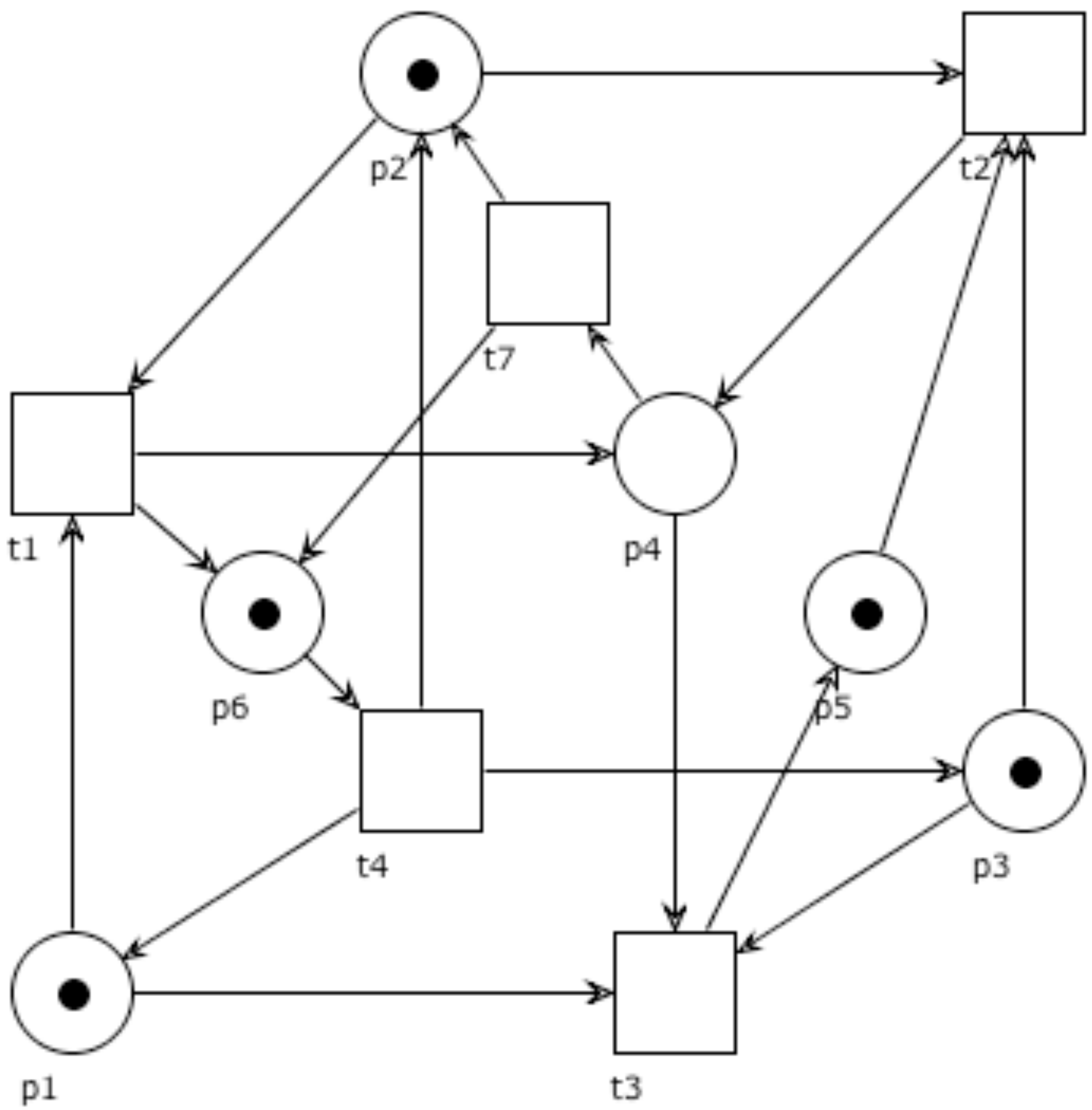
is also a finite prefix of σ and therefore $M_{i-1} \xrightarrow{t_i}$

More on sequences: projection

Restriction: (also extraction / projection)
given $T' \subseteq T$ we inductively define $\sigma_{|T'}$ as:

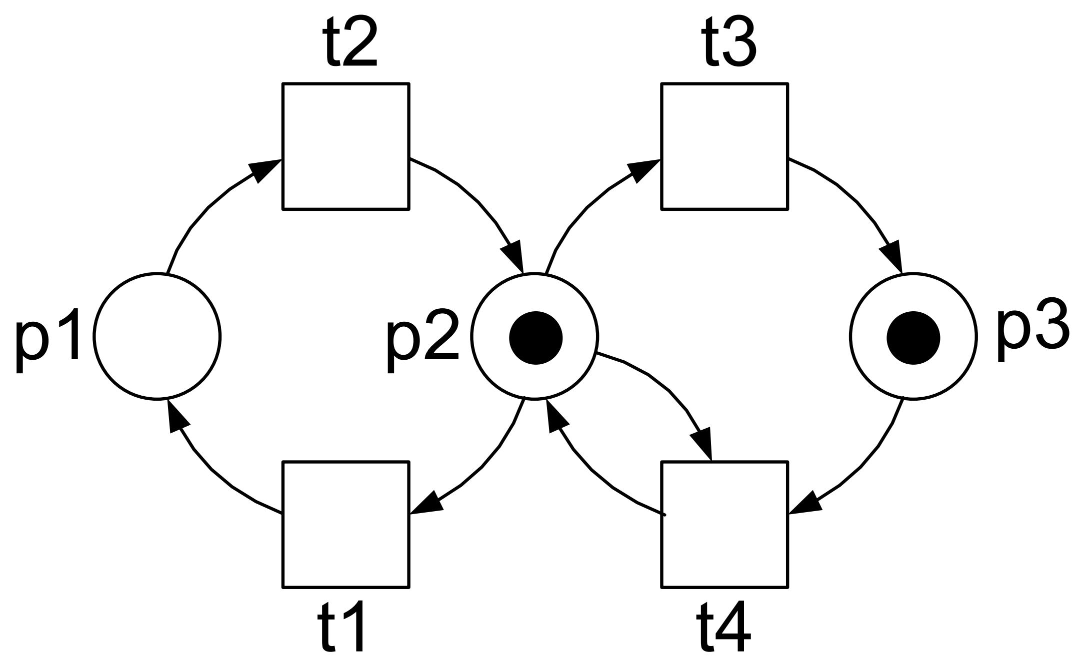
$$\epsilon_{|T'} = \epsilon \qquad (t\sigma)_{|T'} = \begin{cases} t(\sigma_{|T'}) & \text{if } t \in T' \\ \sigma_{|T'} & \text{if } t \notin T' \end{cases}$$

Example



$$
\begin{aligned}
(t_1 t_4 t_7 t_1 t_4 t_7)_{|\{t_1, t_4\}} &= t_1 (t_4 t_7 t_1 t_4 t_7)_{|\{t_1, t_4\}} \\
&= t_1 t_4 (t_7 t_1 t_4 t_7)_{|\{t_1, t_4\}} \\
&= t_1 t_4 (t_1 t_4 t_7)_{|\{t_1, t_4\}} \\
&= t_1 t_4 t_1 (t_4 t_7)_{|\{t_1, t_4\}} \\
&= t_1 t_4 t_1 t_4 (t_7)_{|\{t_1, t_4\}} \\
&= t_1 t_4 t_1 t_4 (t_7 \epsilon)_{|\{t_1, t_4\}} \\
&= t_1 t_4 t_1 t_4 (\epsilon)_{|\{t_1, t_4\}} \\
&= t_1 t_4 t_1 t_4 \epsilon \\
&= t_1 t_4 t_1 t_4
\end{aligned}
$$

Exercises

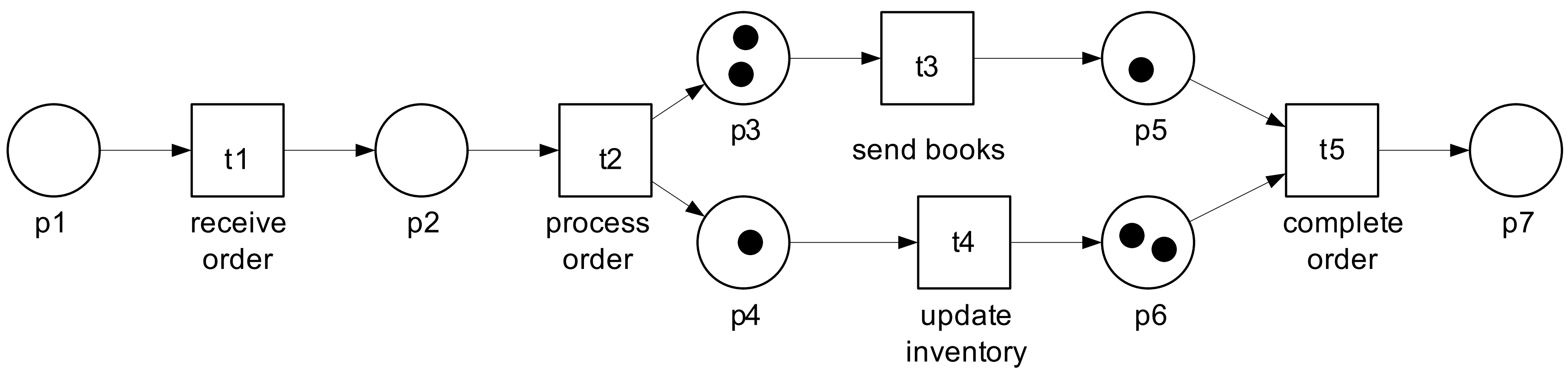


Determine the pre- and post-set of each element

Which are the currently enabled transitions?
For each of them, which state would the firing lead to?

What are the reachable states?

Exercises



Which are the currently enabled transitions?

For each of them, which state would the firing lead to?

What are the reachable states?

Petri nets:
occurrence graph

Occurrence graph (aka Reachability graph)

The reachability graph is a graph that represents all possible occurrence sequences of a net

Nodes of the graphs = reachable markings
Arcs of the graphs = firings

Formally, $OG(N) = ([M_0\rangle, A)$ where $A \subseteq [M_0\rangle \times T \times [M_0\rangle$ s.t.

$$(M, t, M') \in A \quad \text{iff} \quad M \xrightarrow{t} M'$$

How to compute OG(N)

Adding one arc at the time:

1. Initially $R = \{ M_0 \}$ and $A = \emptyset$
2. Take a marking $M \in R$ and a transition $t \in T$ such that
    1. M enables t and there is no arc labelled t leaving from M
3. Let $M' = M - {}^\bullet t + t^\bullet$
4. Add M' to R and (M,t,M') to A
5. Repeat steps 2,3,4 until no new arc can be added

How to compute OG(N)

Adding all exiting arcs each time:

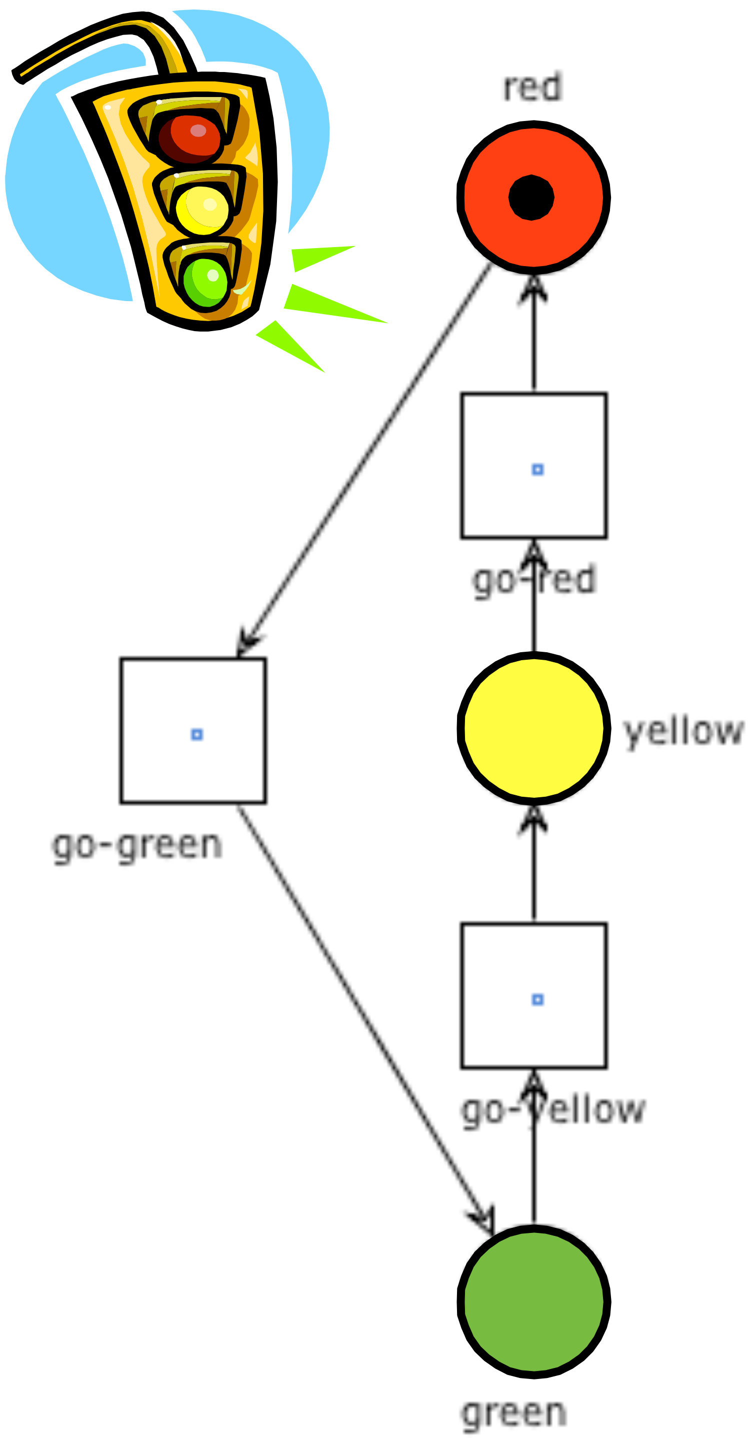
markings to explore

1. $Nodes = \{\},\ Arcs = \{\},\ Todo = \{M_0\}$
2. $M = next(Todo)$ select one marking to explore
3. $Nodes = Nodes \cup \{M\},\ Todo = Todo \setminus \{M\}$ update nodes

   collect all firings from M
4. $Firings = \{(M,t,M') \mid \exists t \in T, \exists M' \in \mu(P),\ M \xrightarrow{t} M'\}$

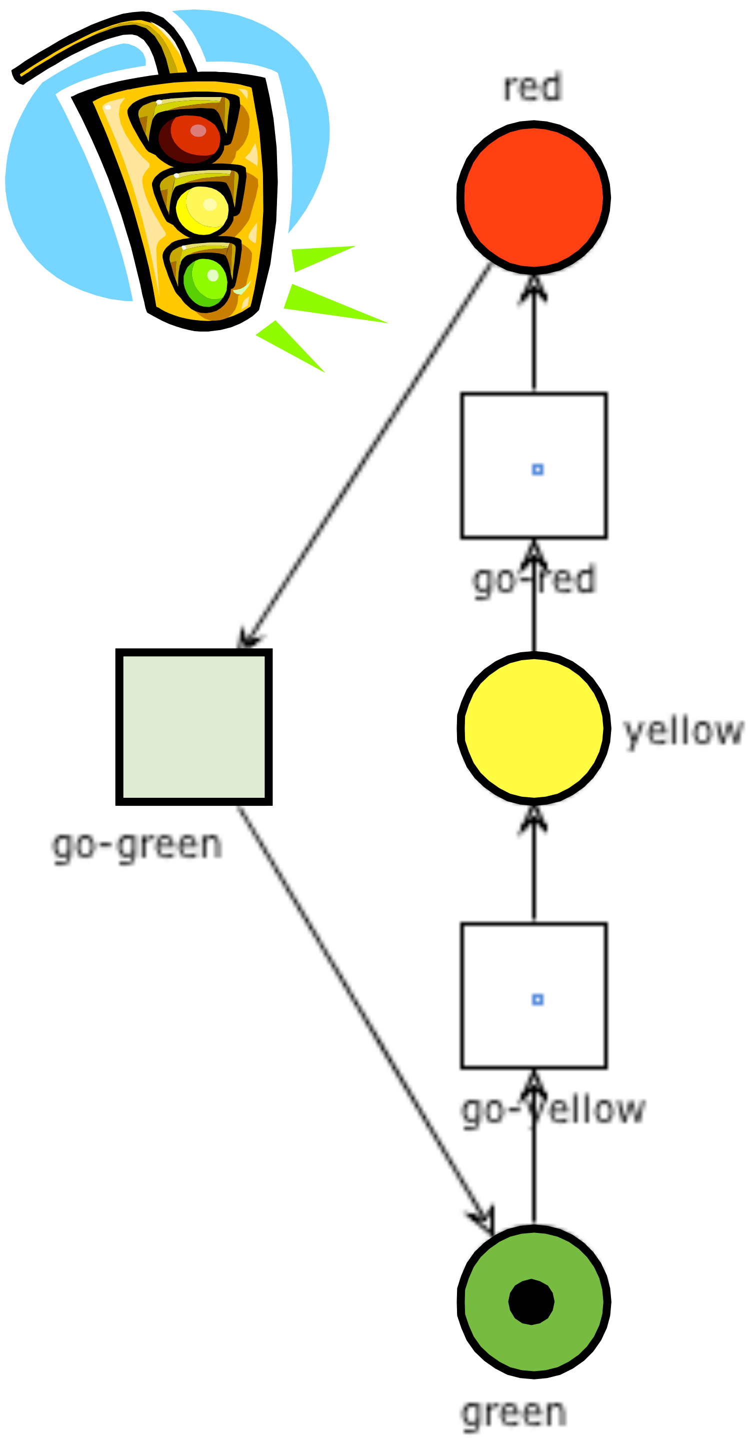
   find new markings to explore
5. $New = \{M' \mid (M,t,M') \in Firings\} \setminus (Nodes \cup Todo)$
6. $Todo = Todo \cup New,\ Arcs = Arcs \cup Firings$ update nodes and arcs
7. $isEmpty(Todo)\ ?\ \textbf{stop}\ :\ \textbf{goto}\ 2$ repeat if there are still markings to be explored

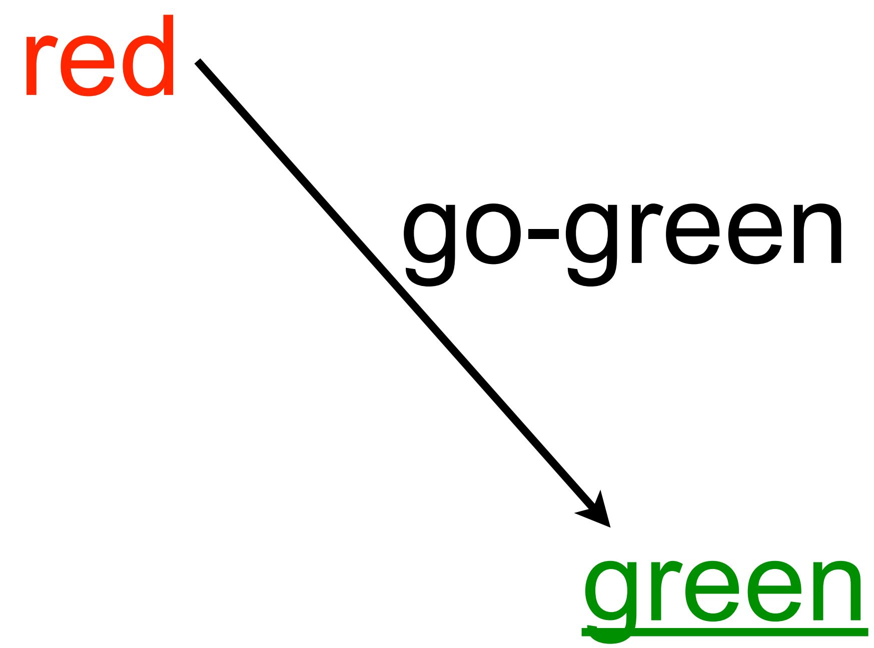
Example: traffic light



red

Example: traffic light





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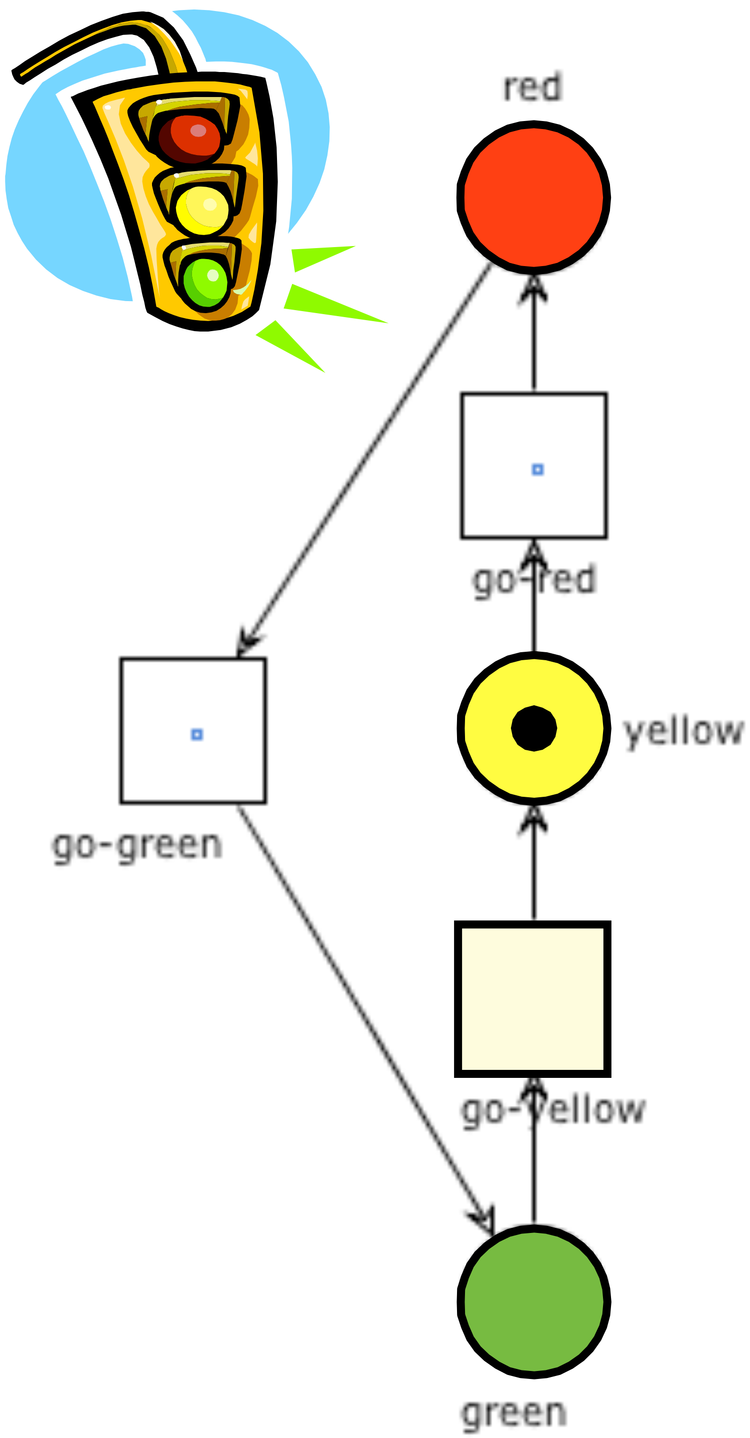

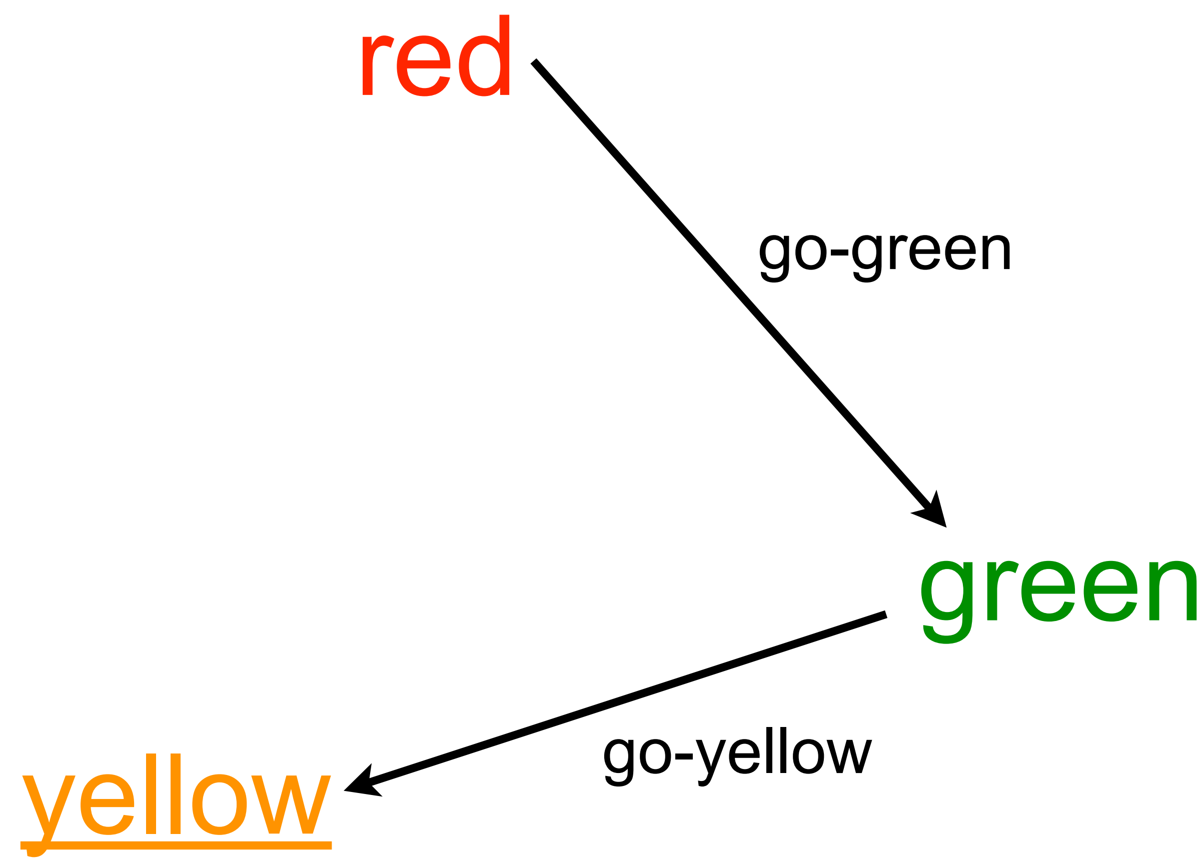

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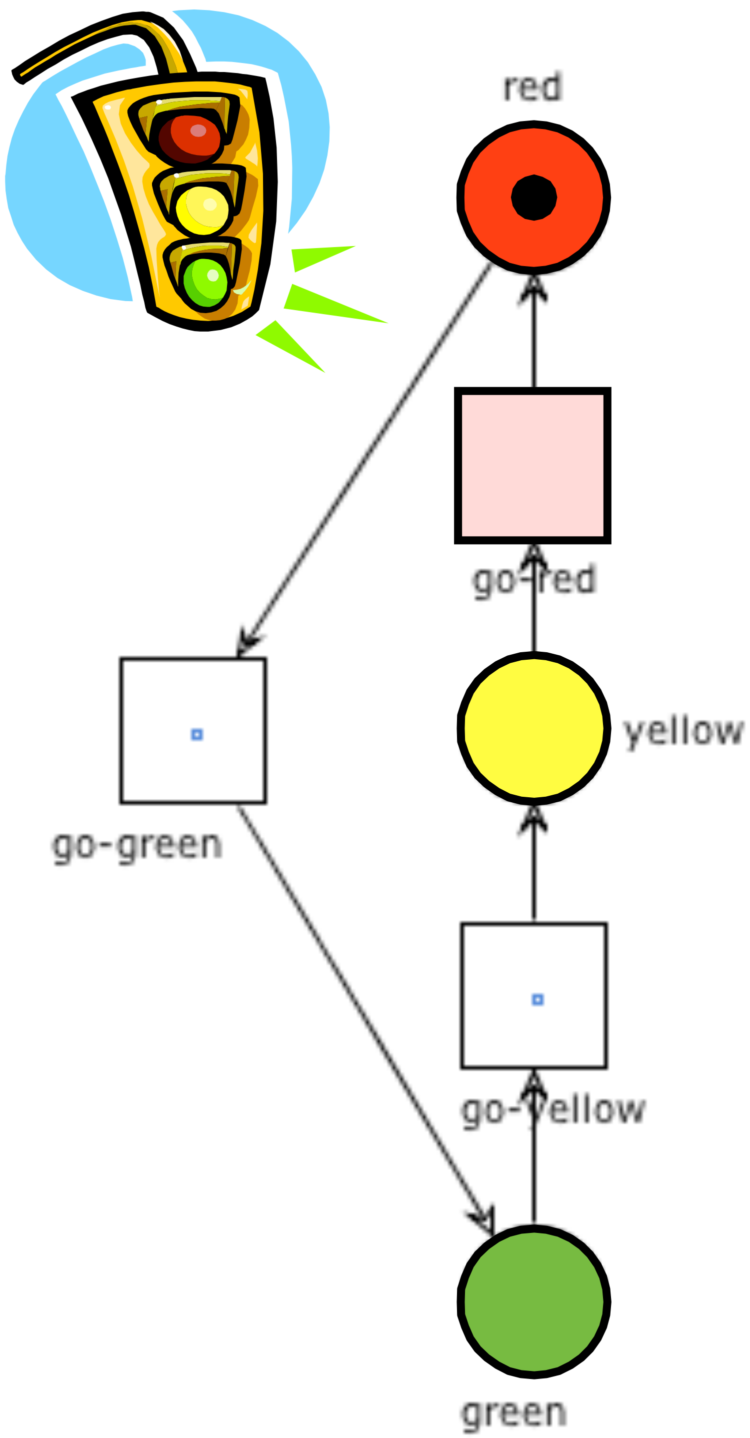

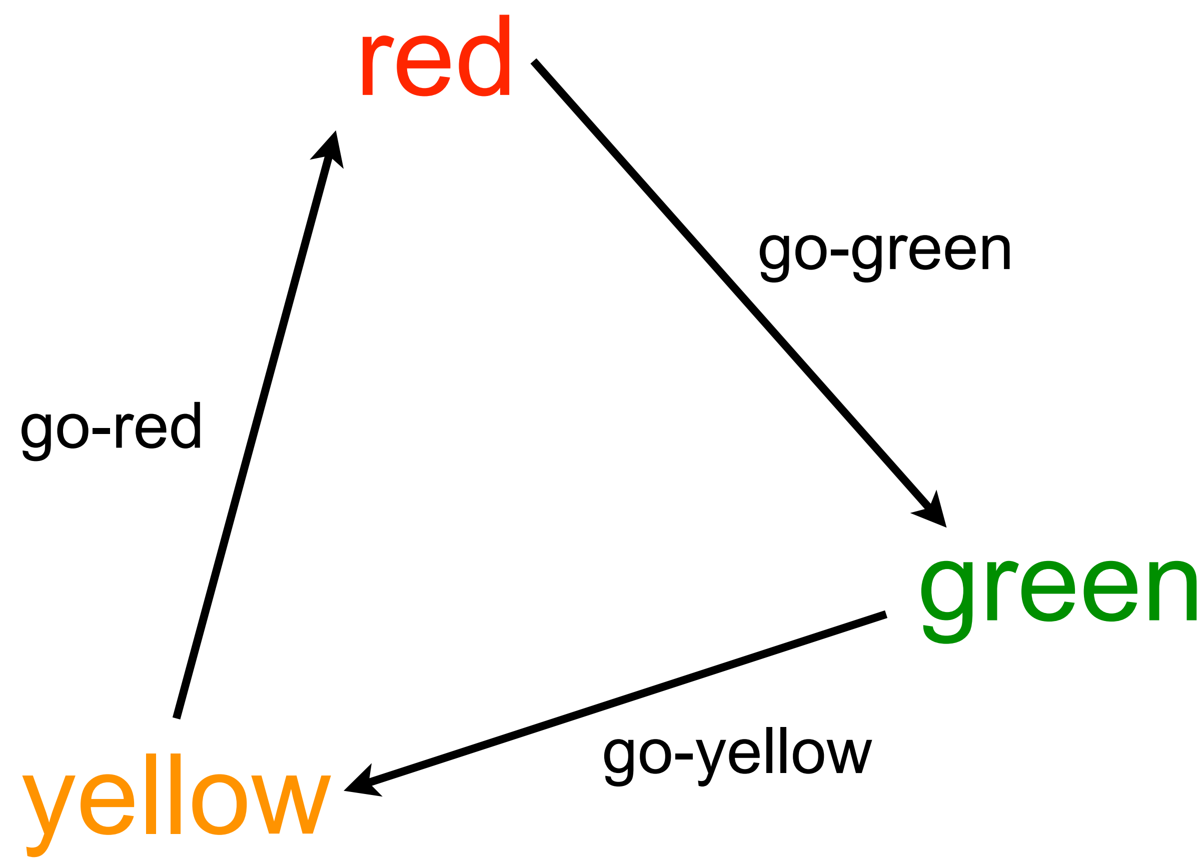

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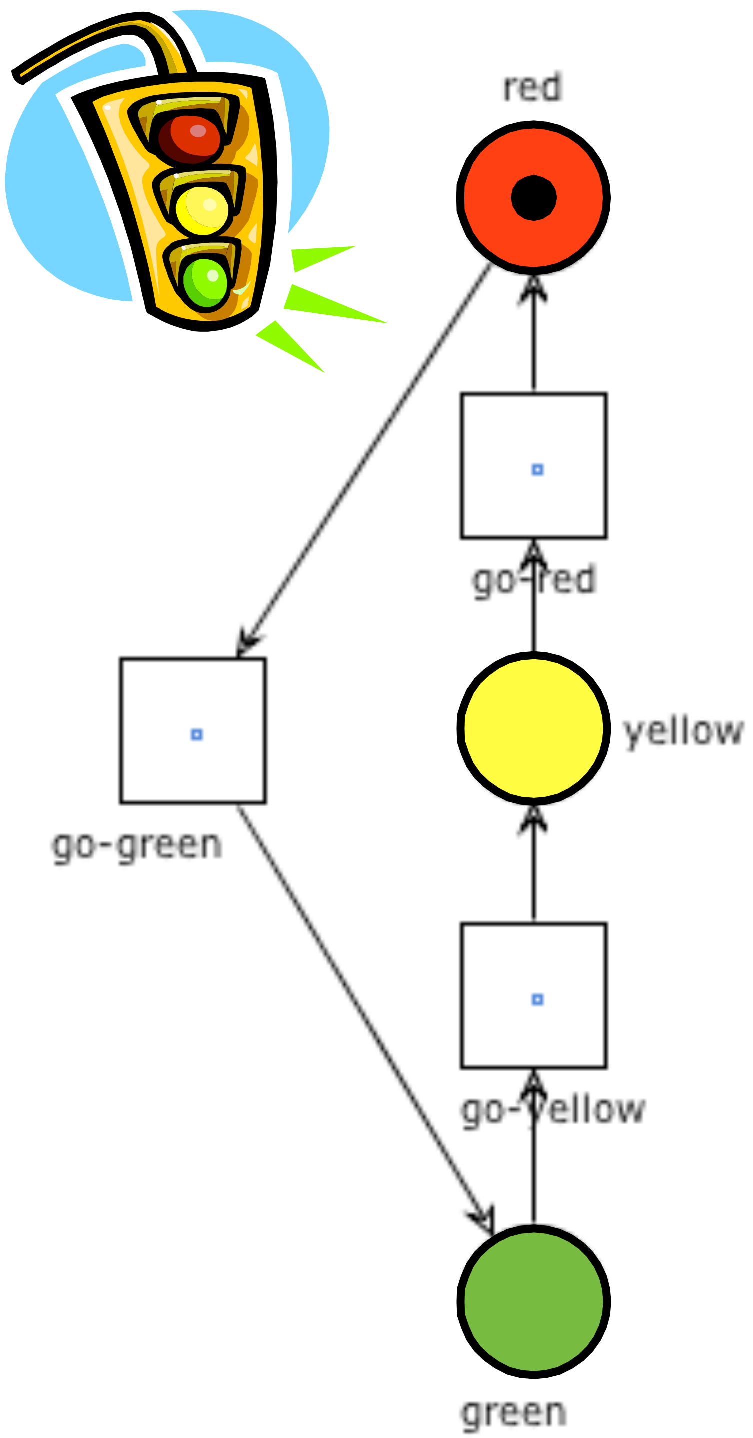
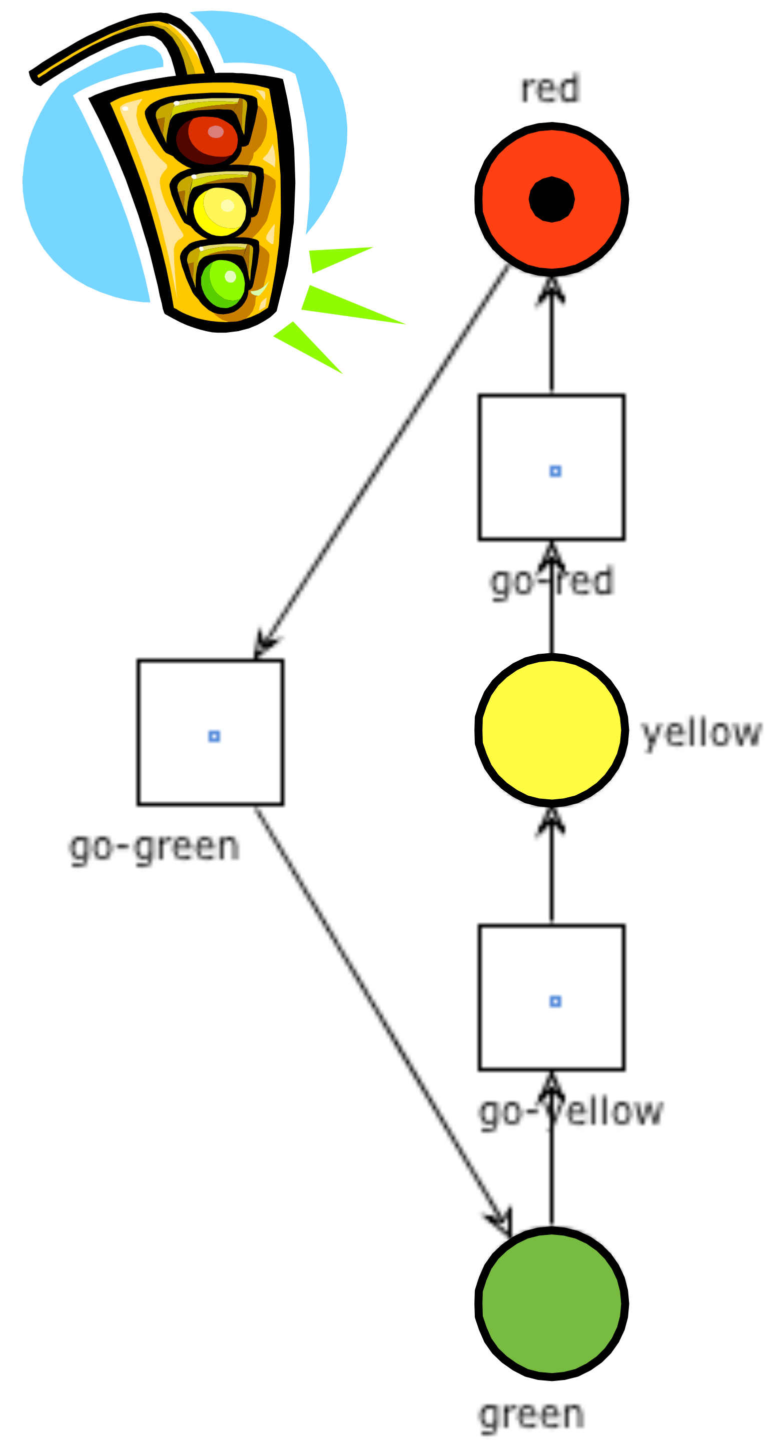


red + red’



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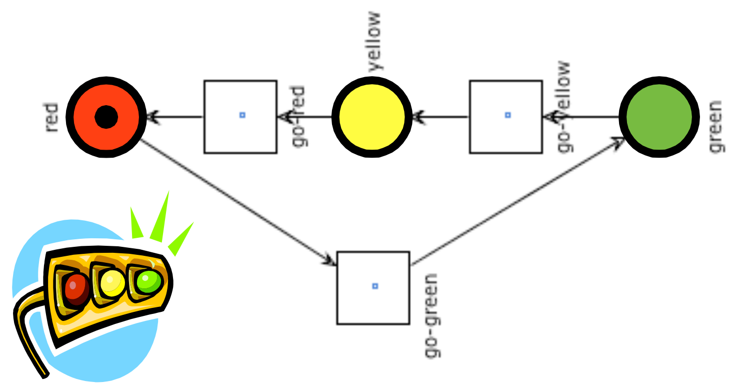
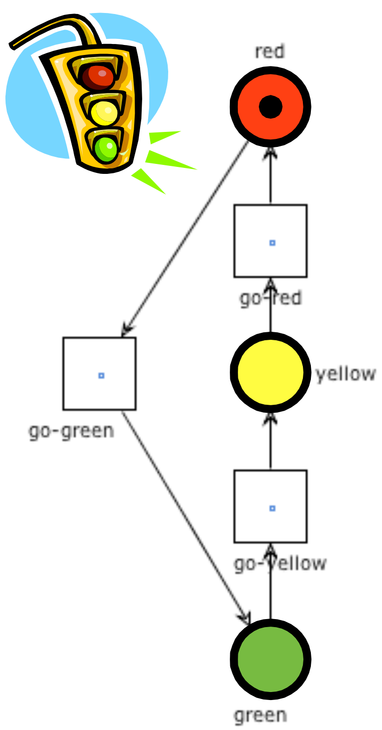
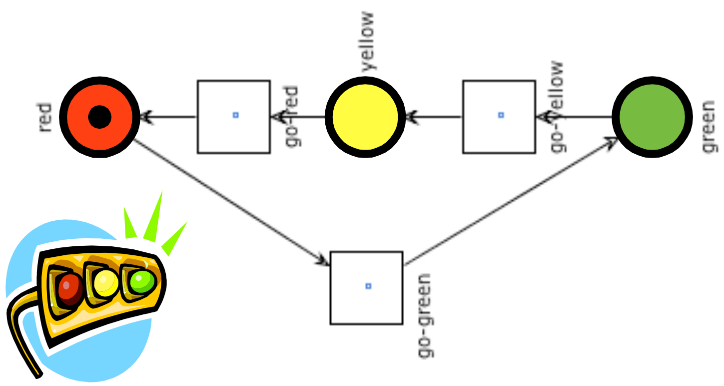


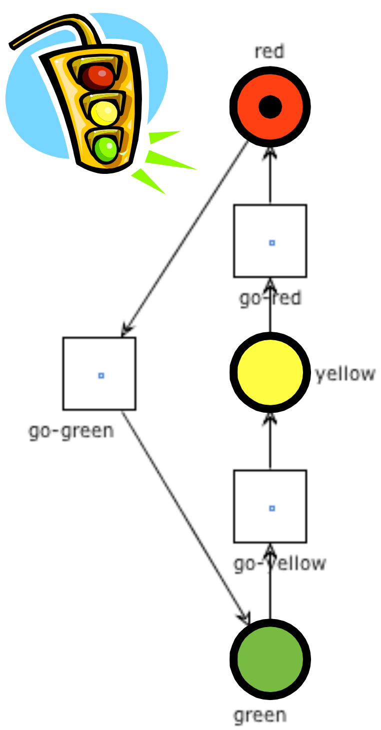


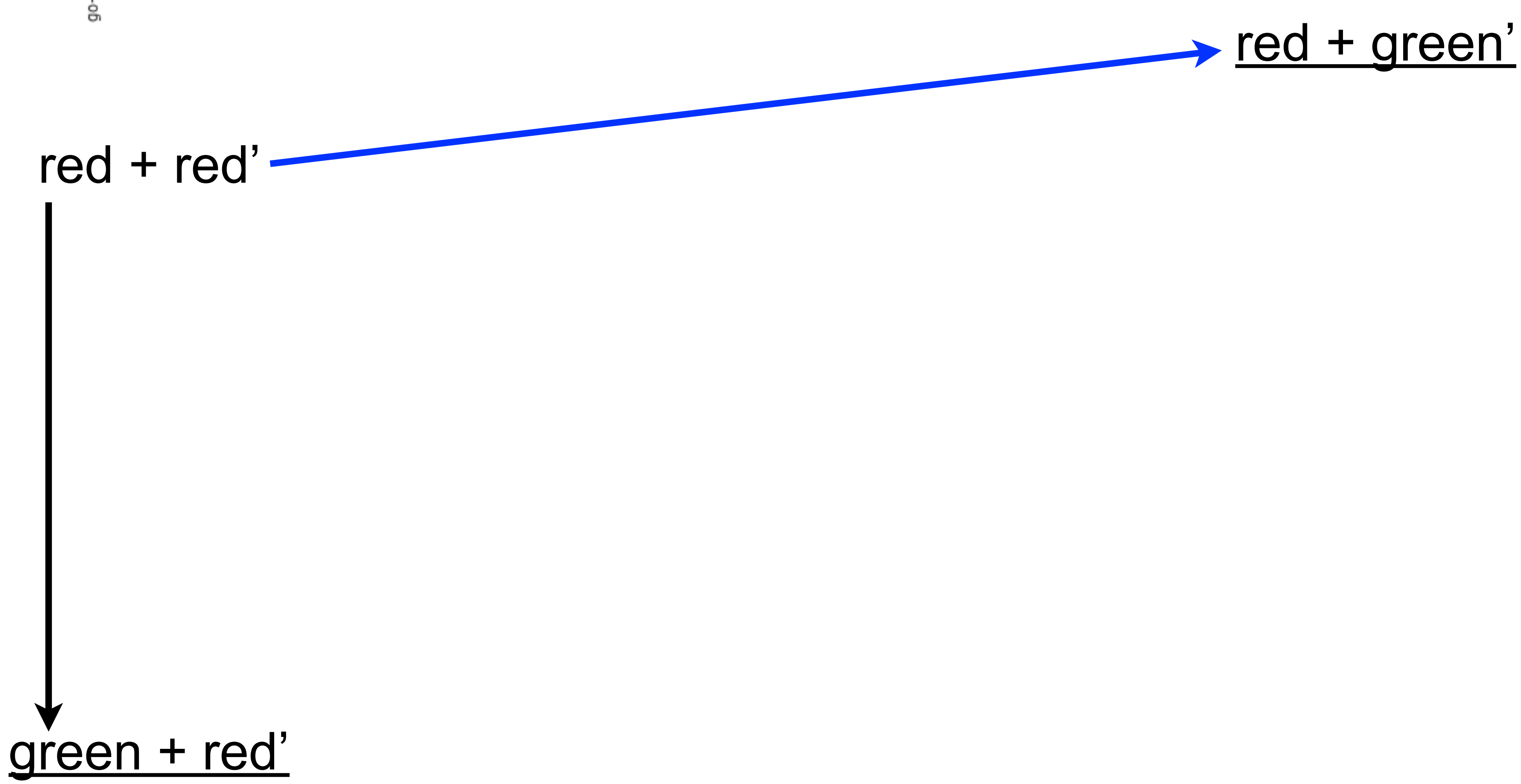
<u>red + red’</u>

(we omit arc labels
for readability issues)

Example: two traffic lights







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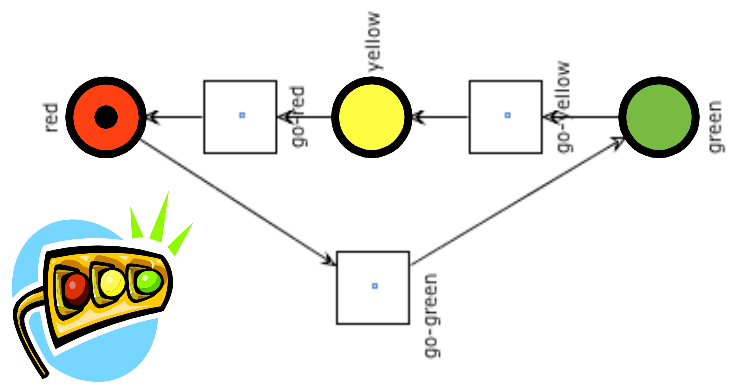

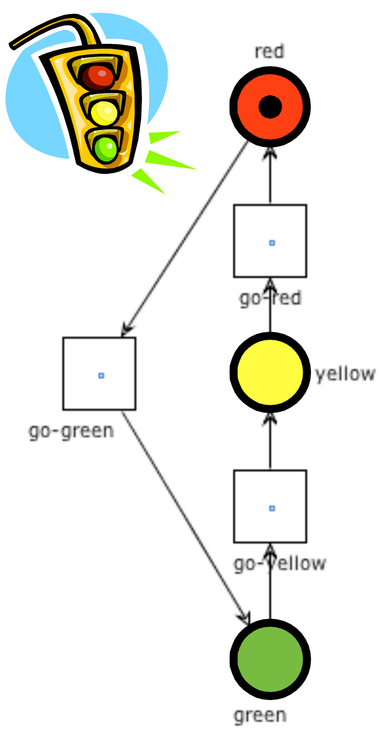

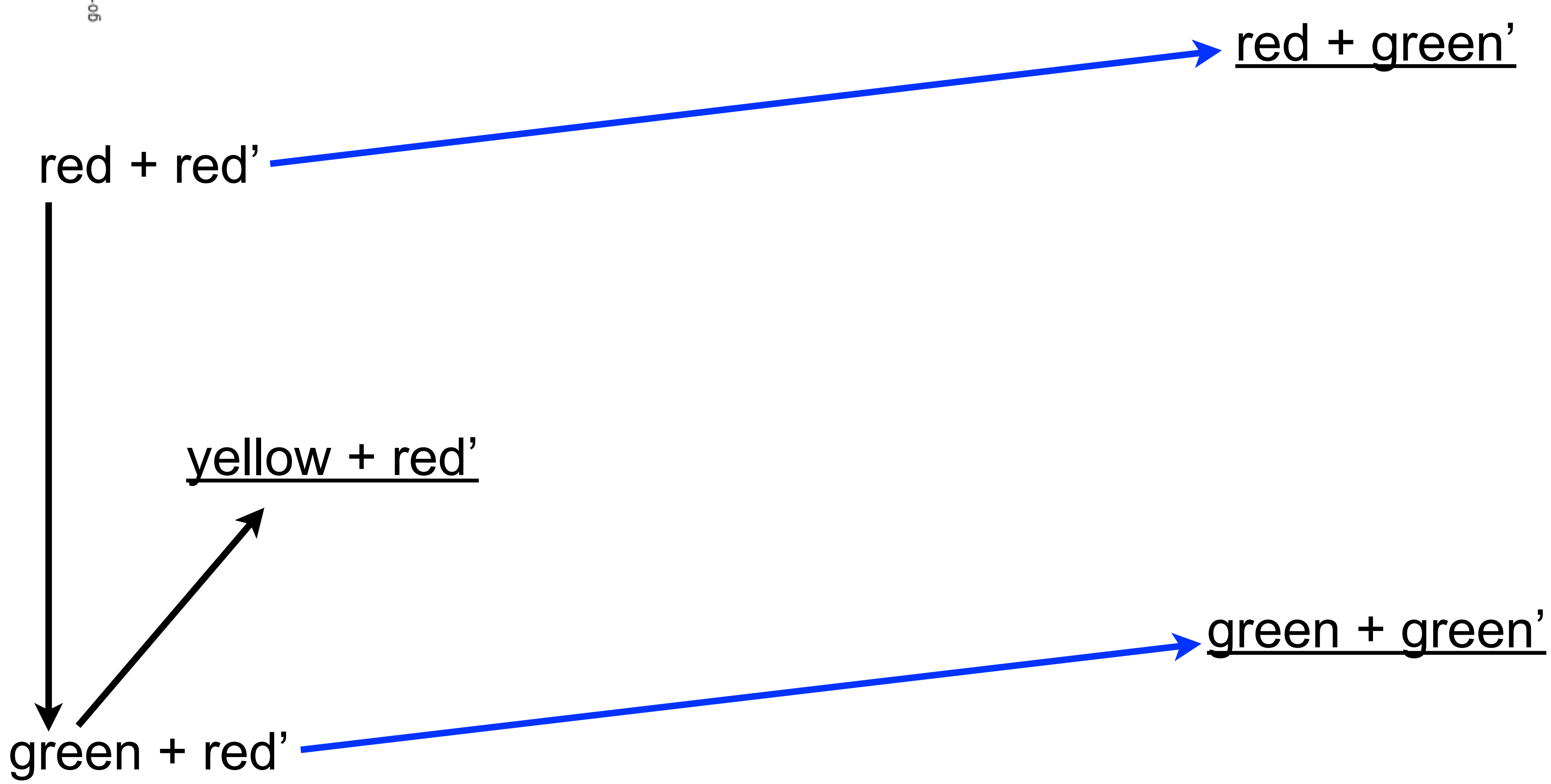

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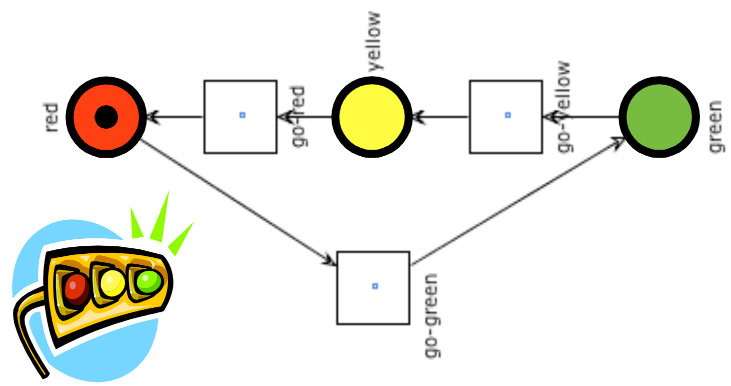

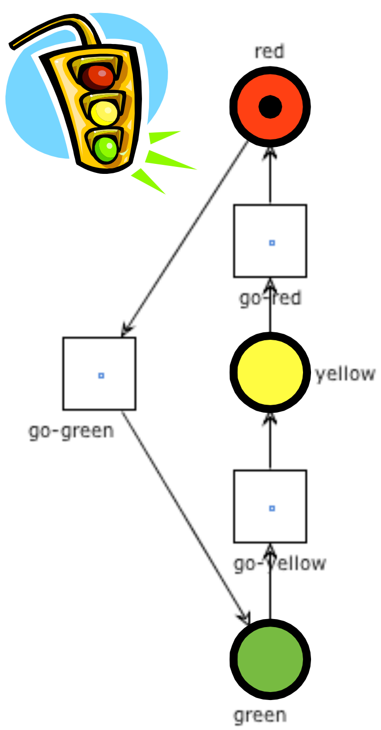

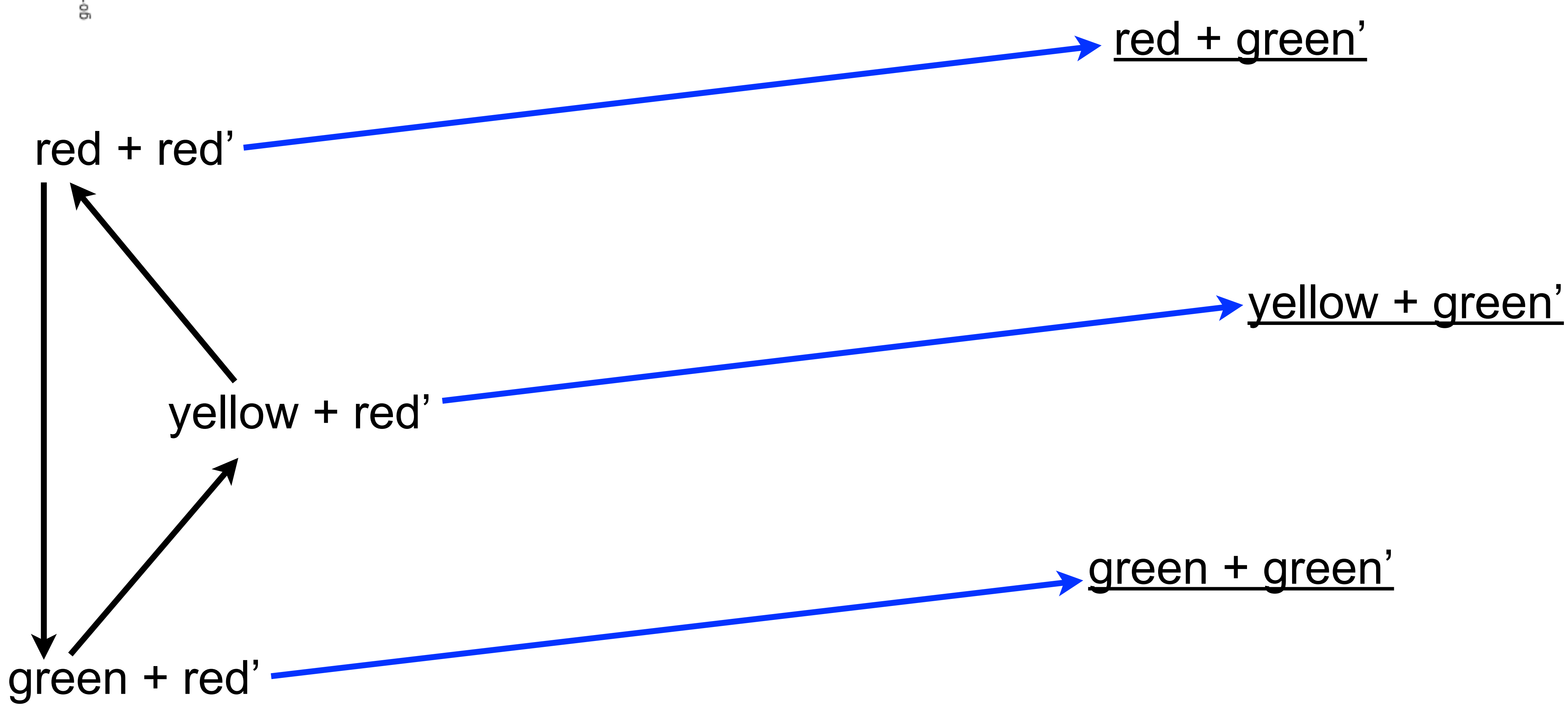

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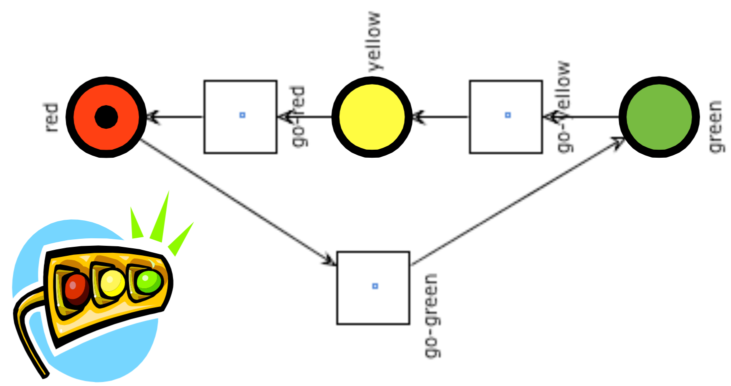

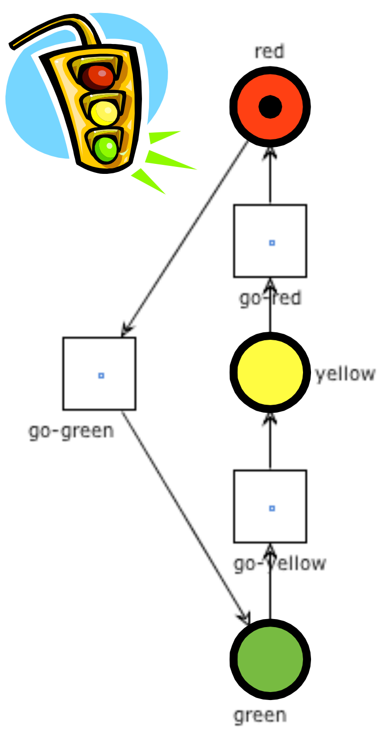

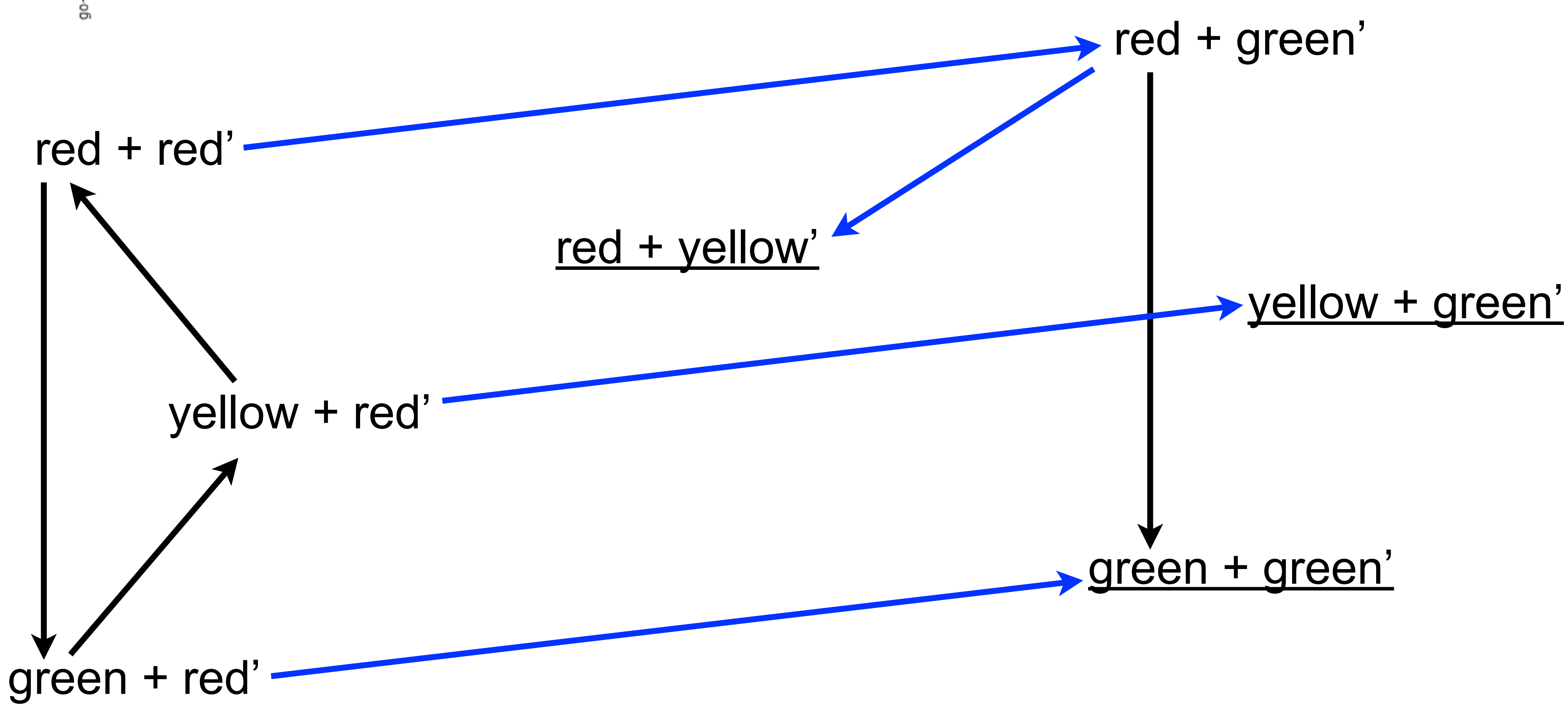

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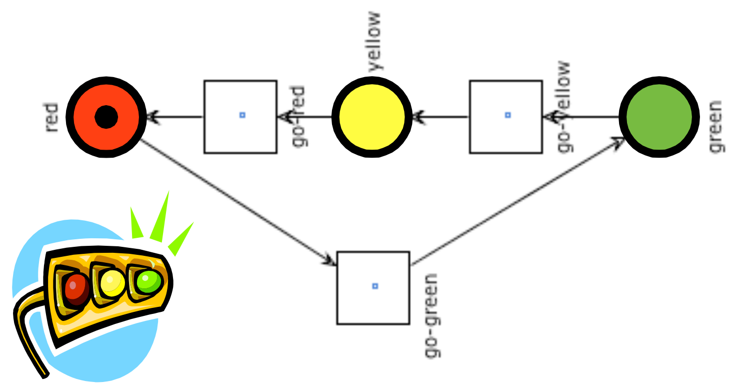

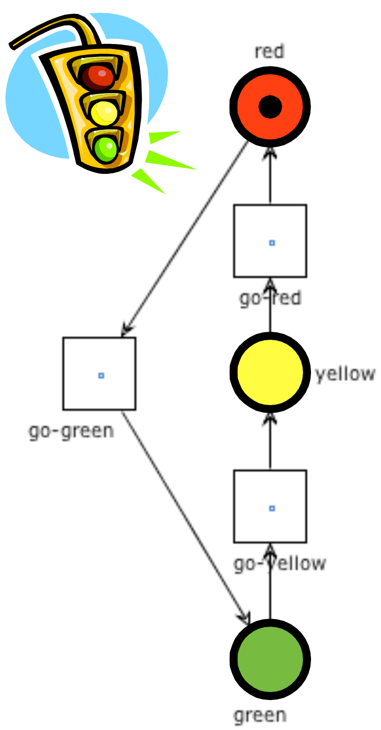

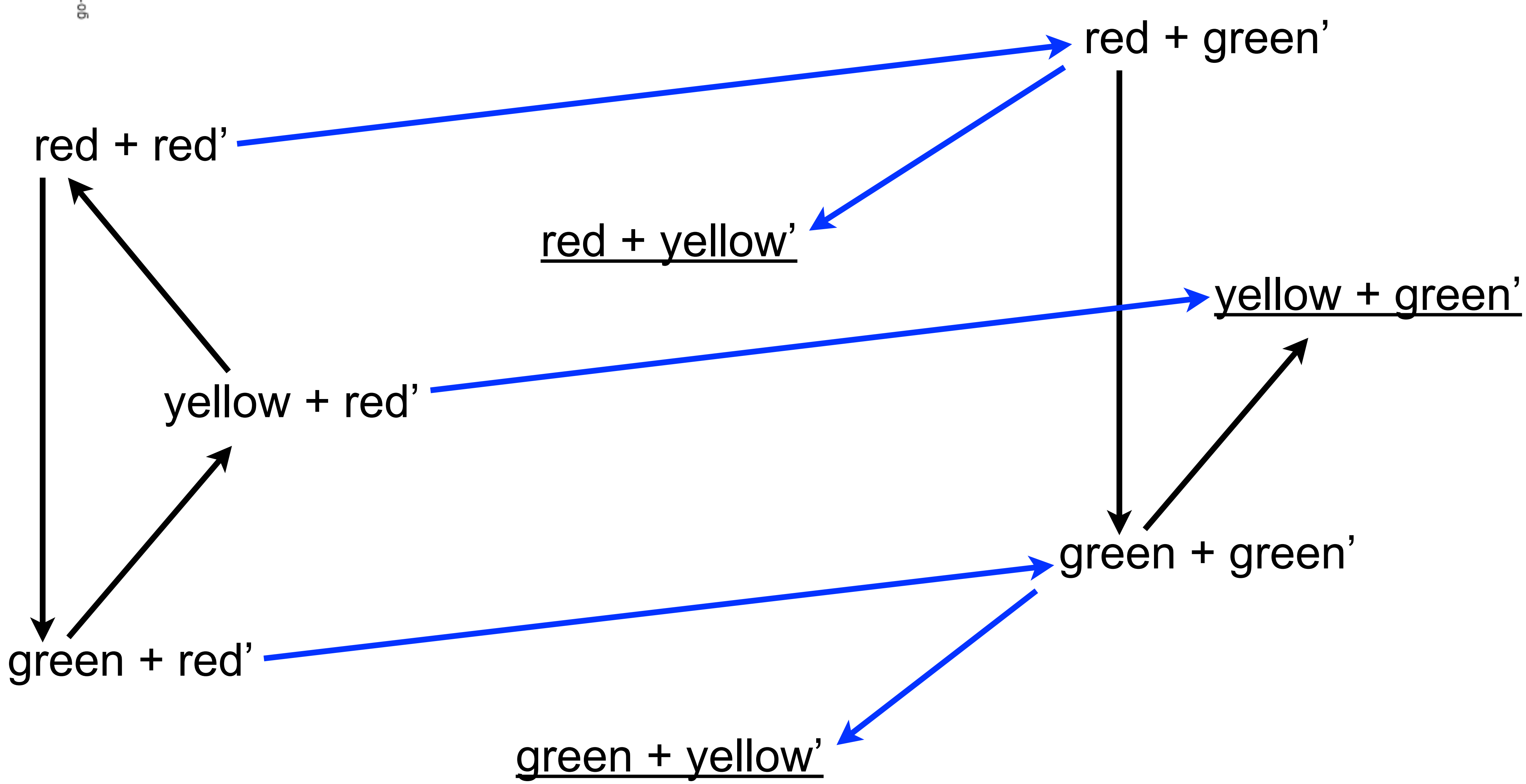

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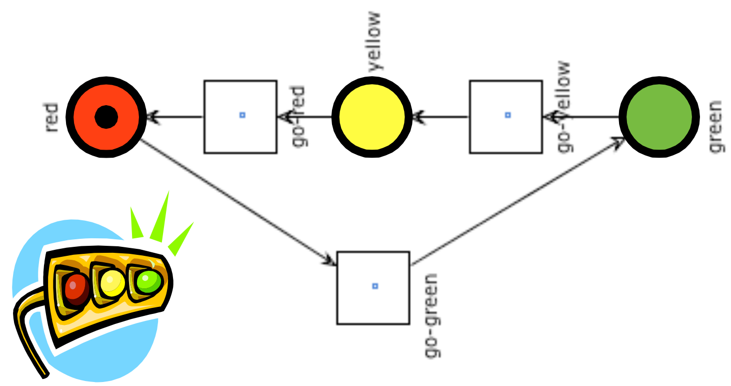

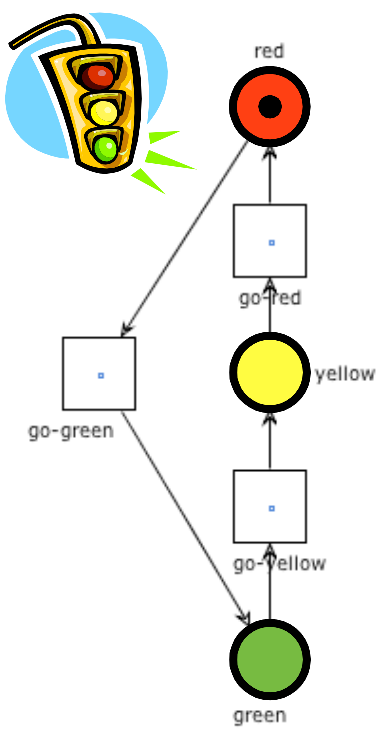

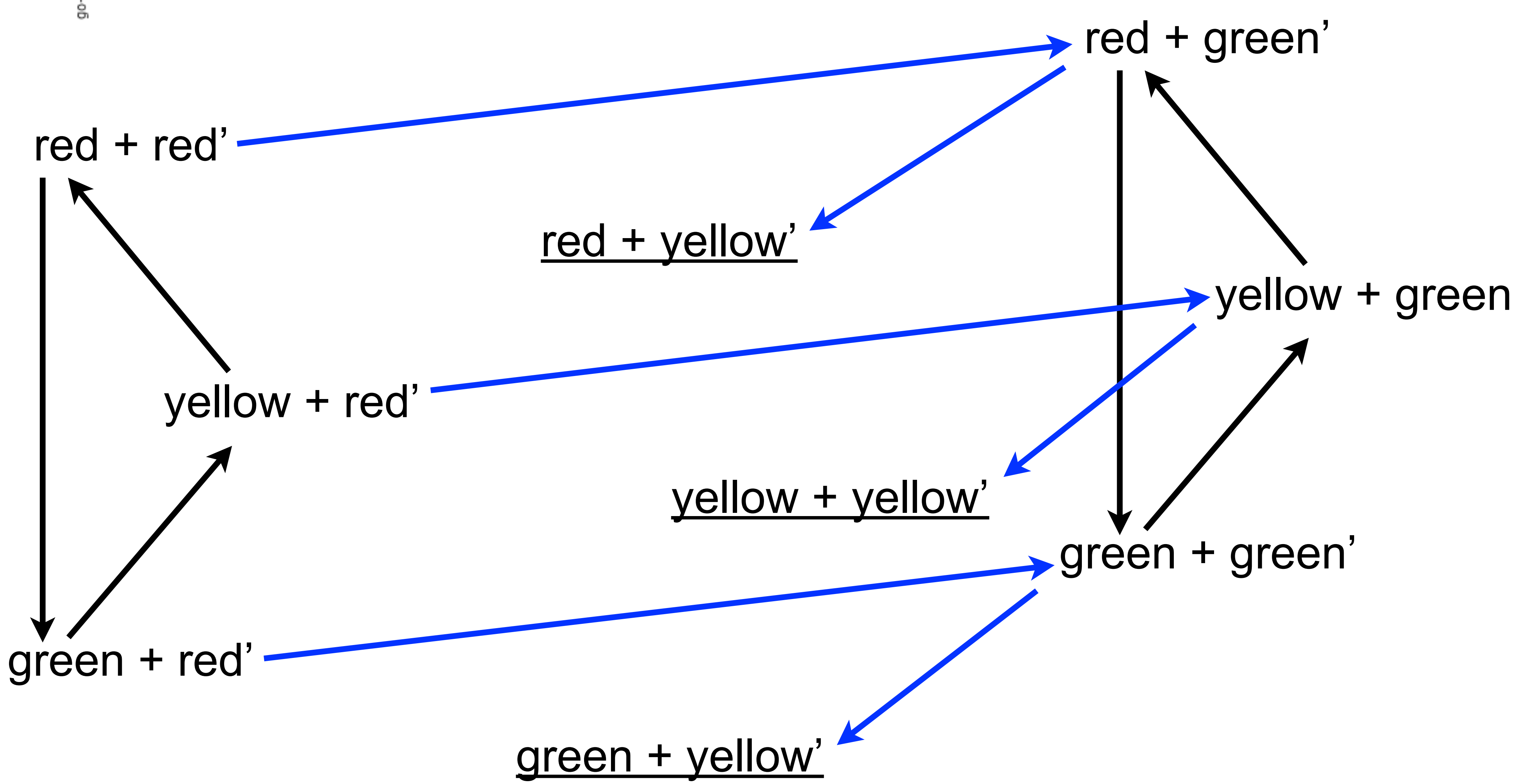

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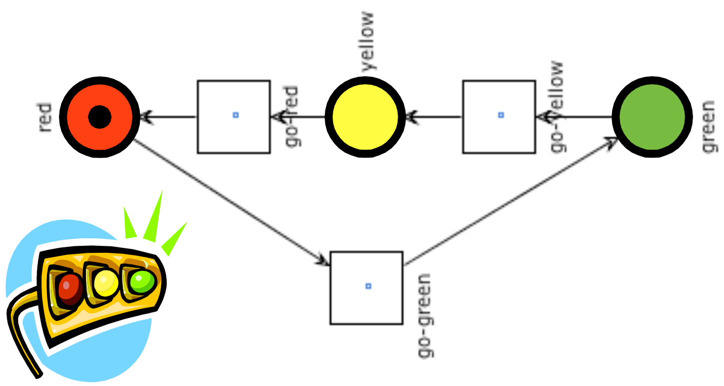

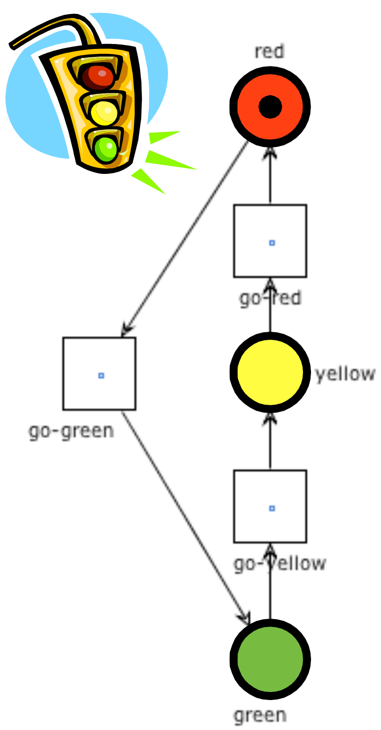

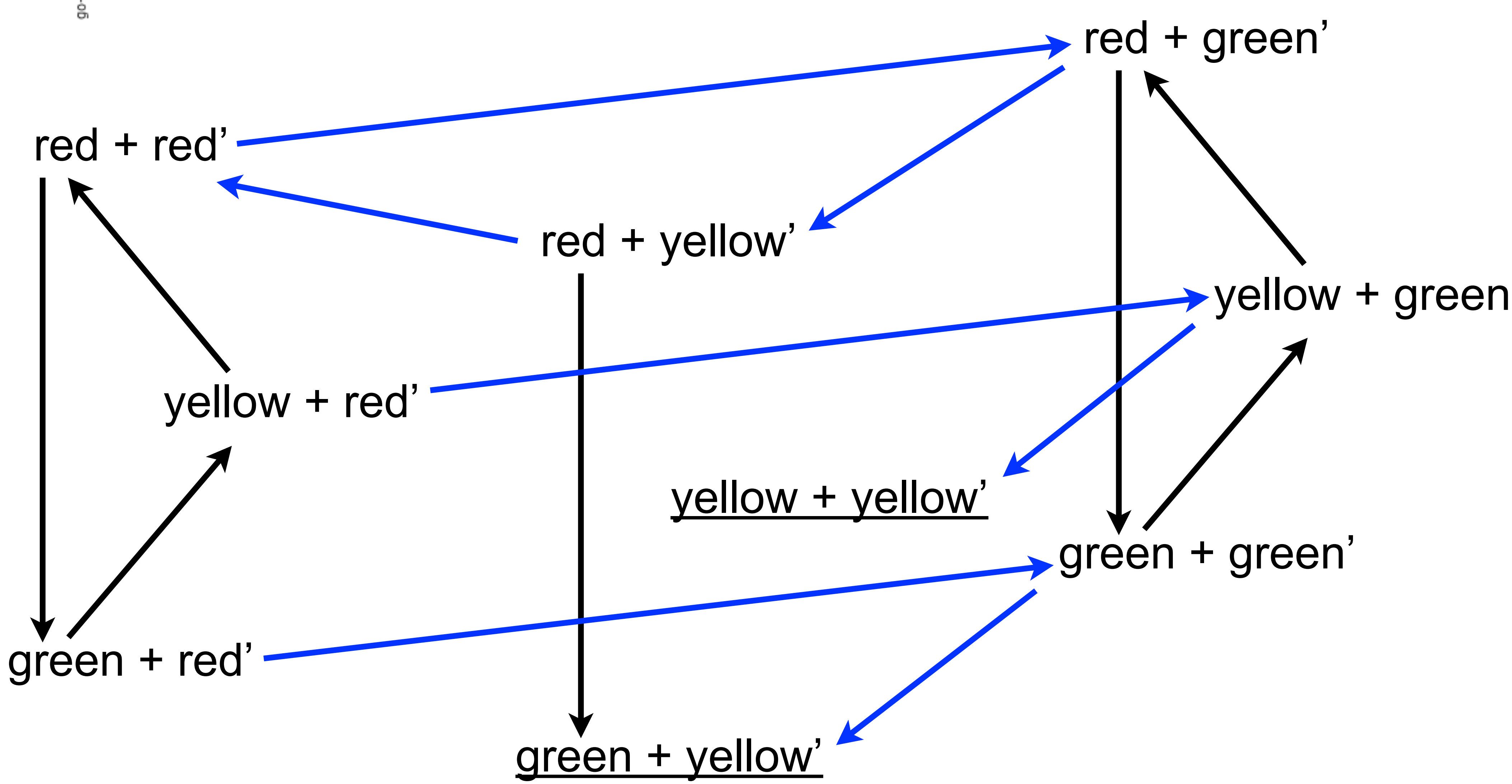

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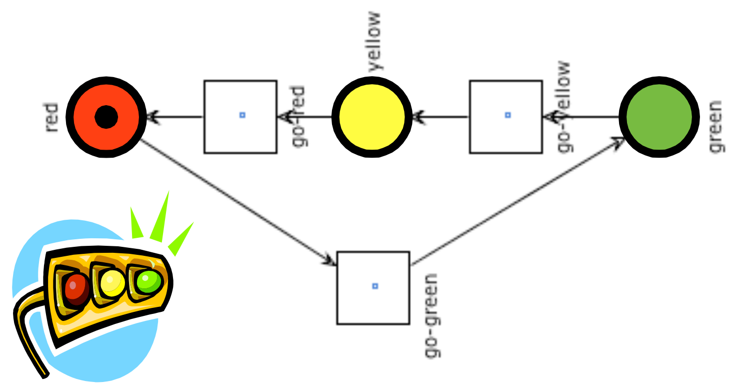

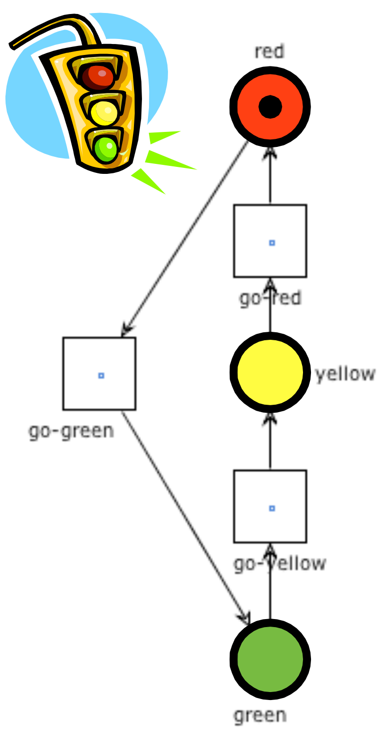

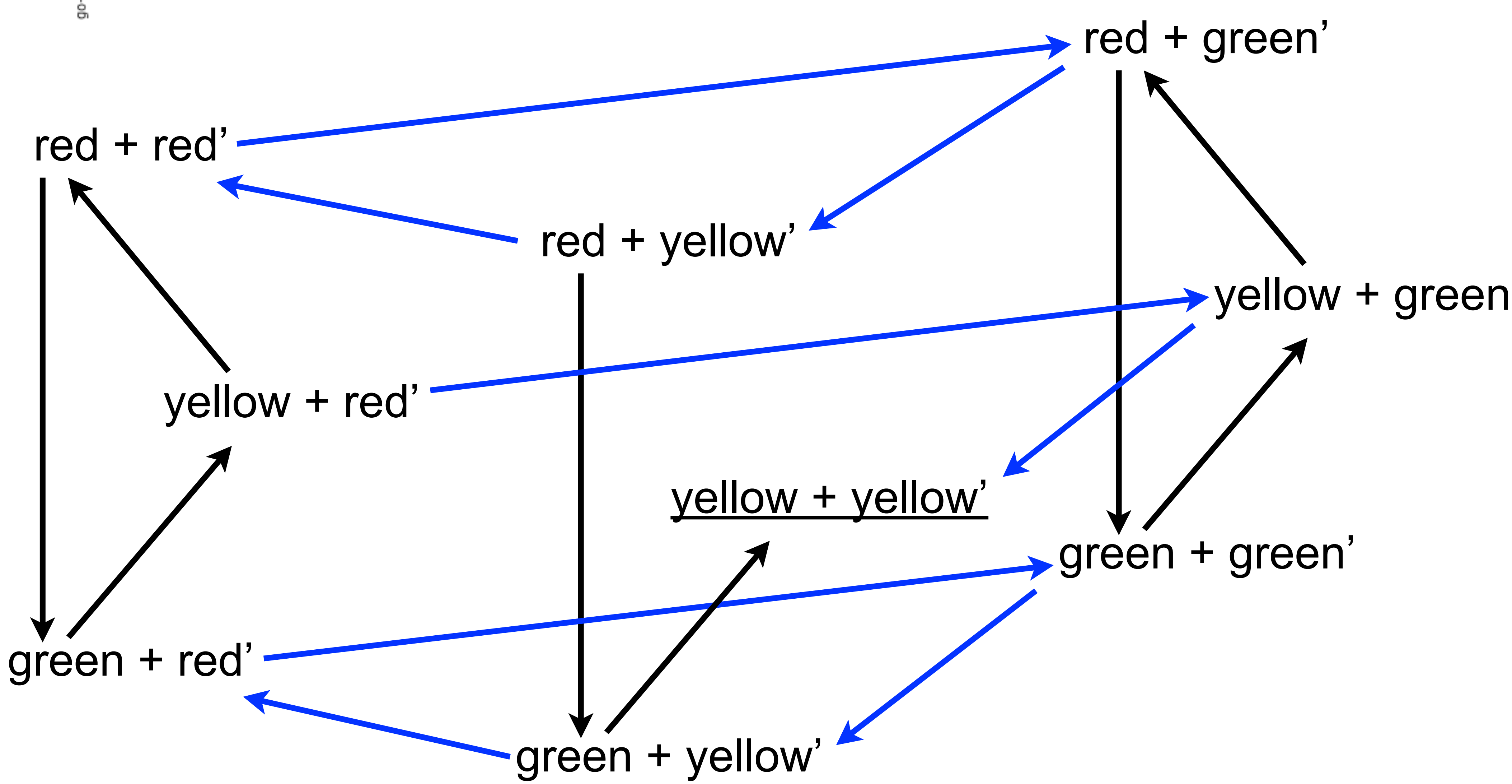

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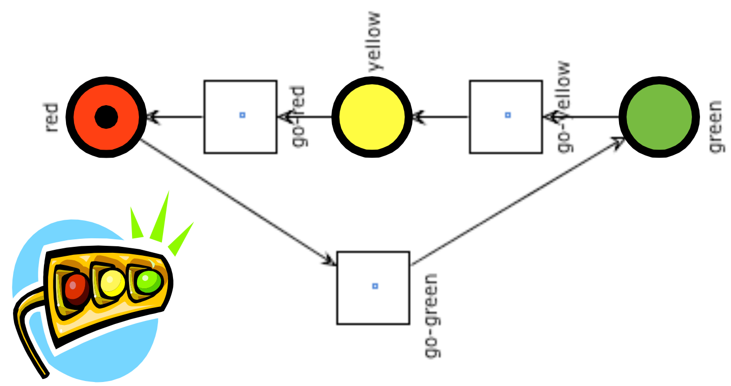

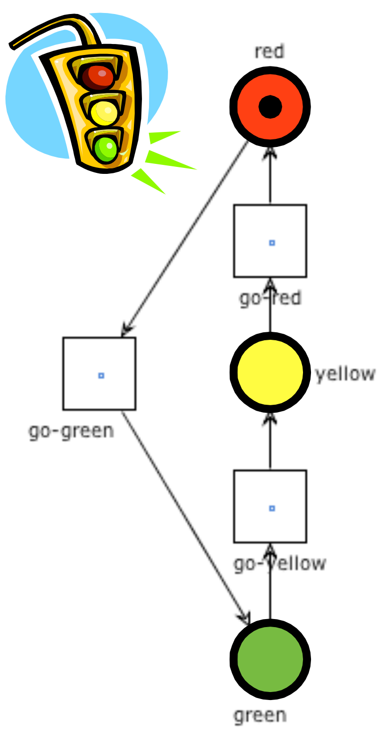

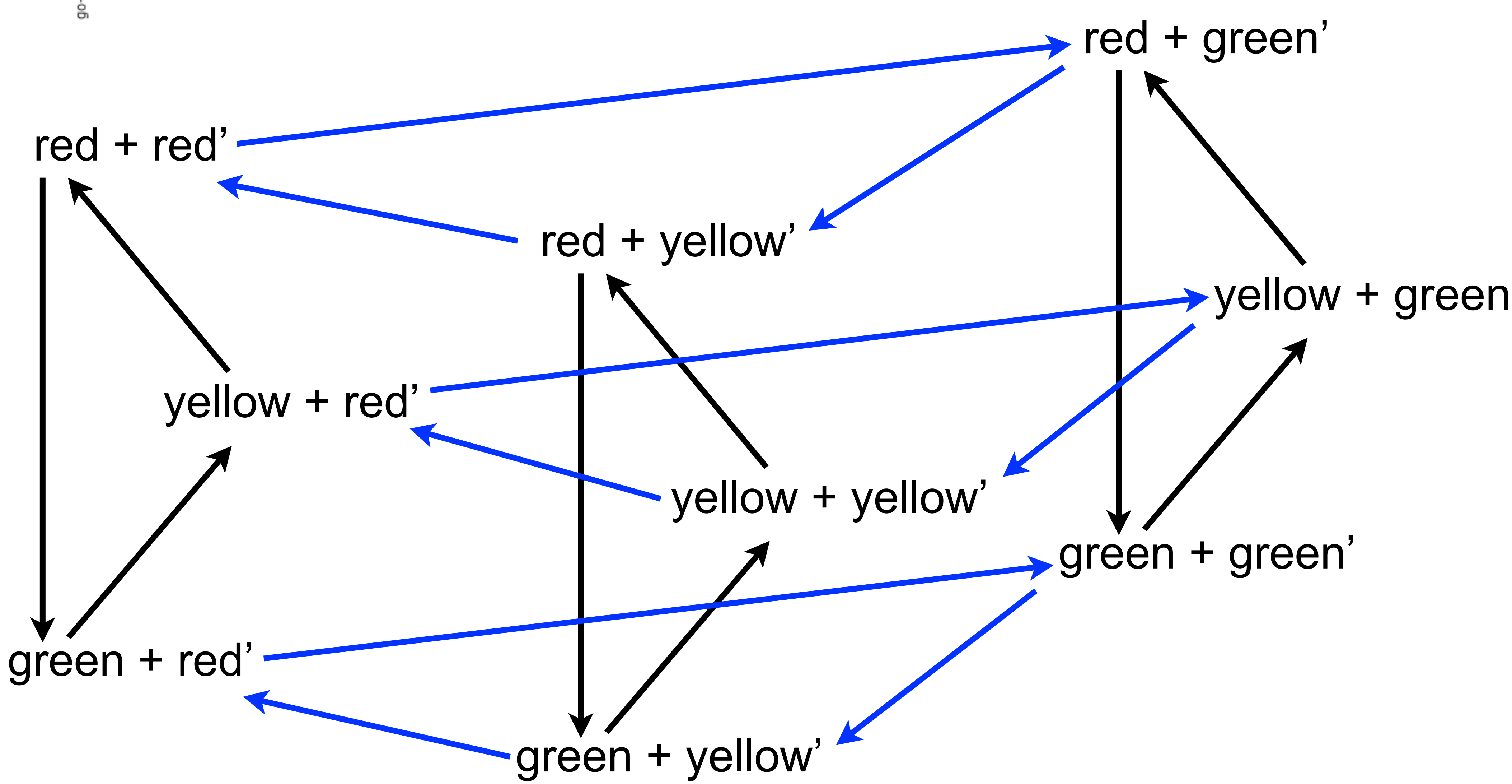

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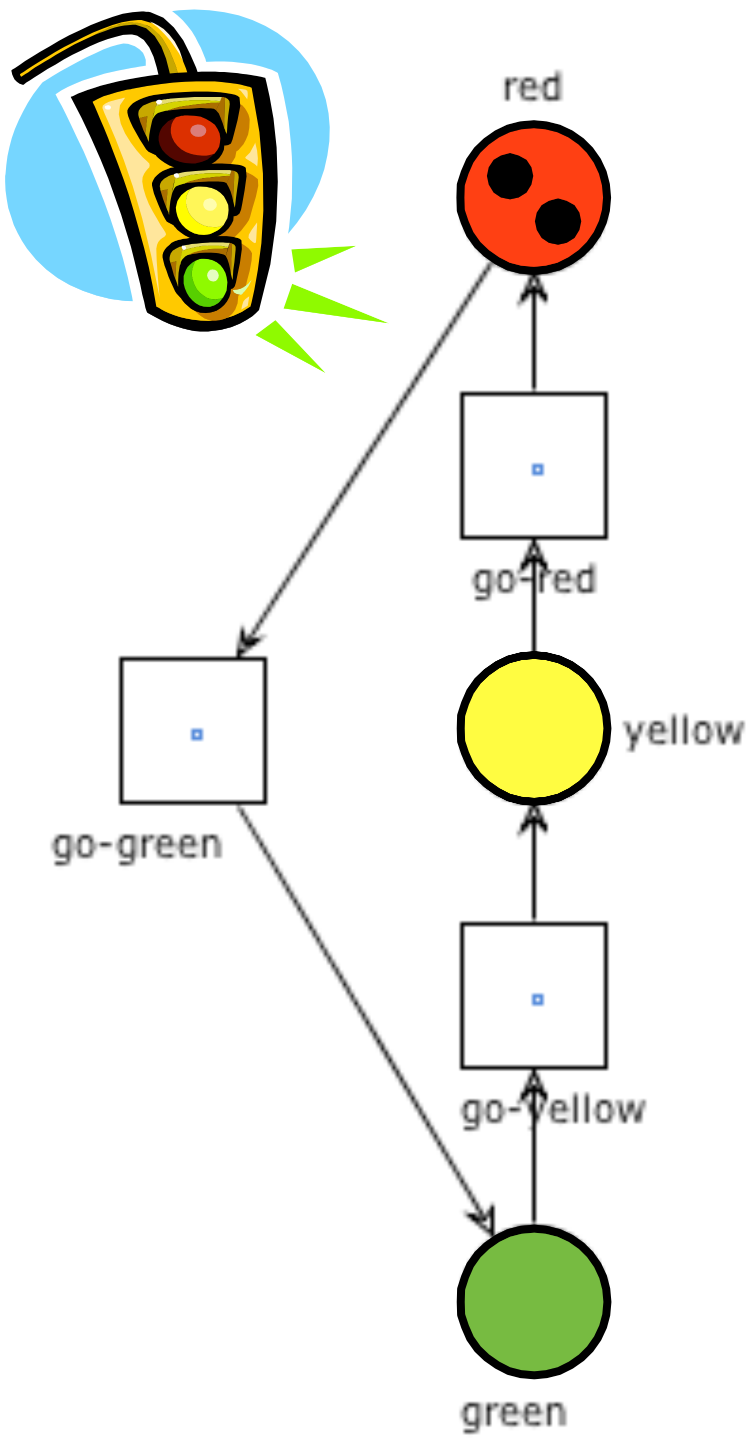
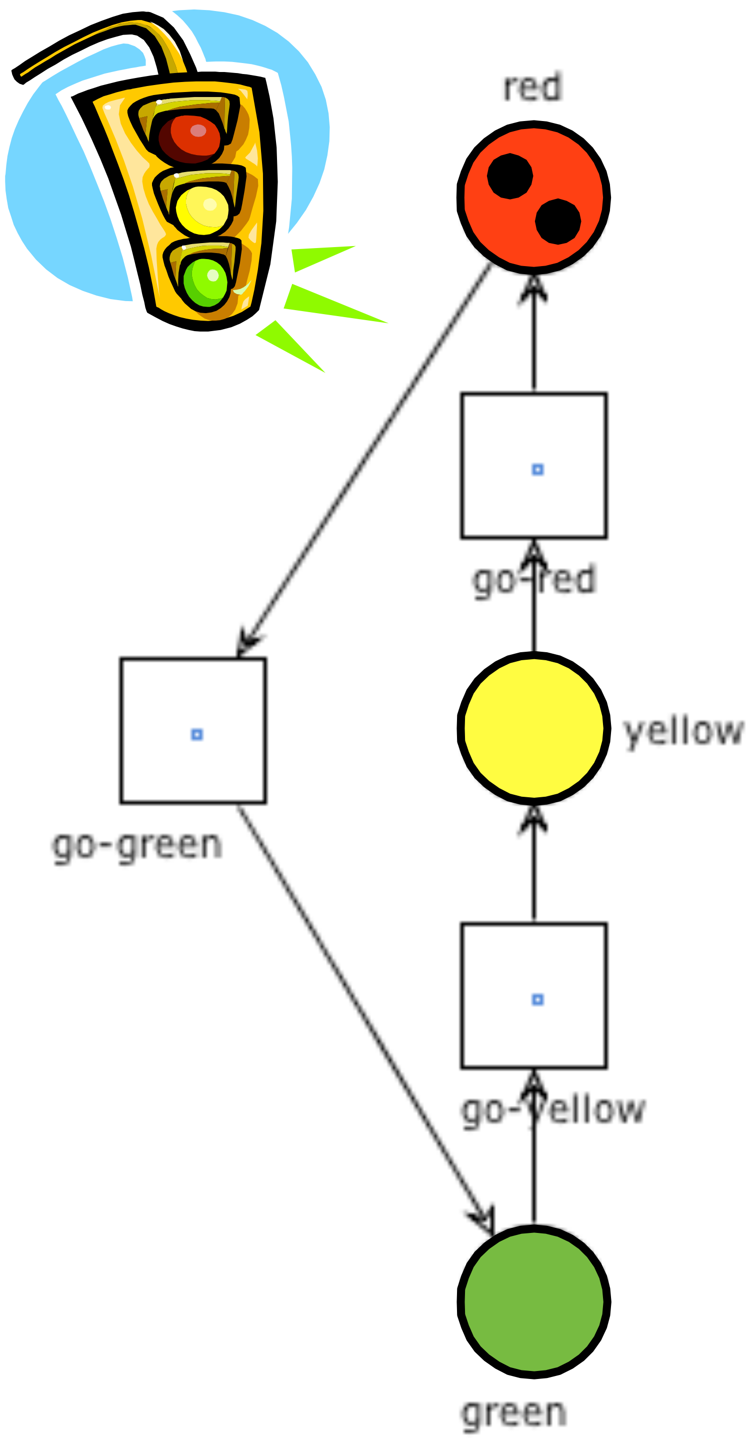


<u>2 red</u>

Example: two traffic lights

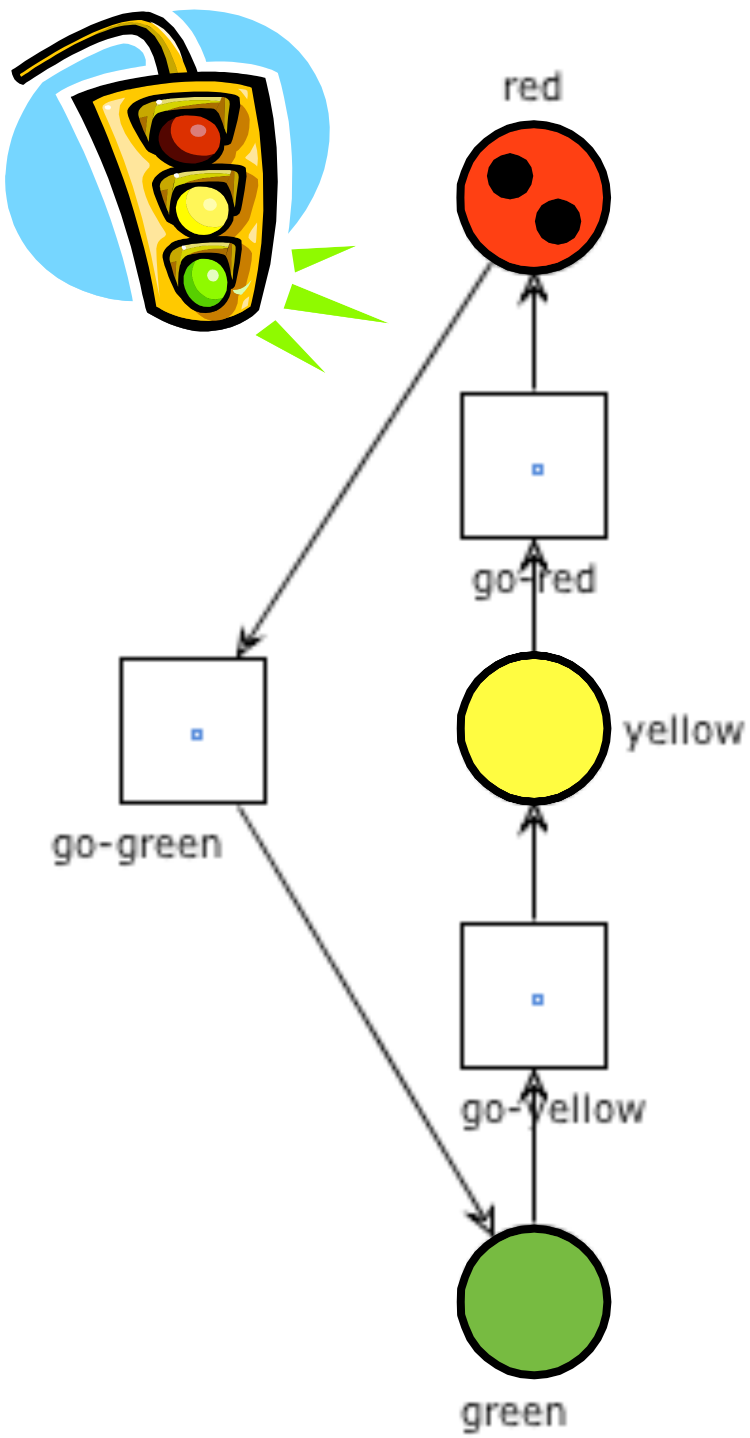


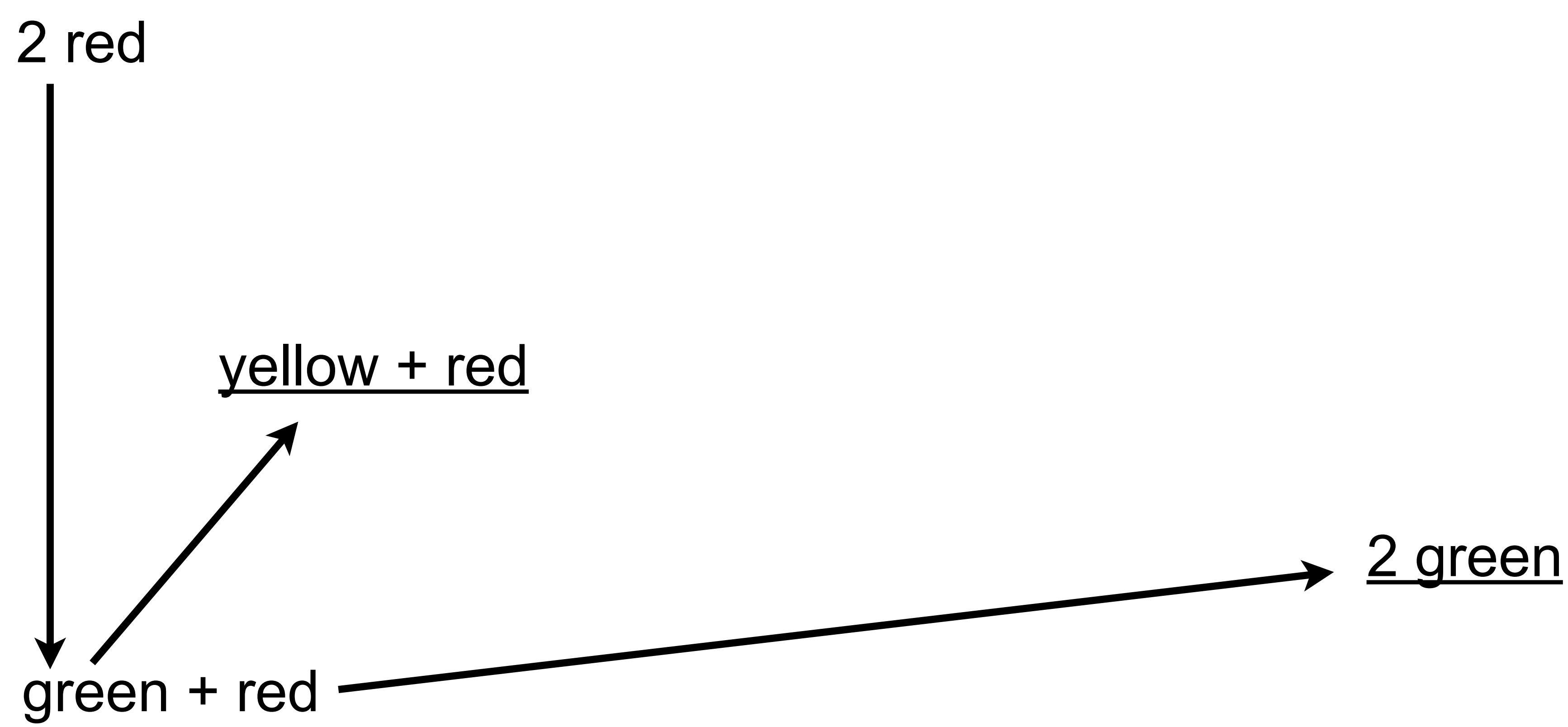
2 red

↓

green + red

Example: two traffic lights





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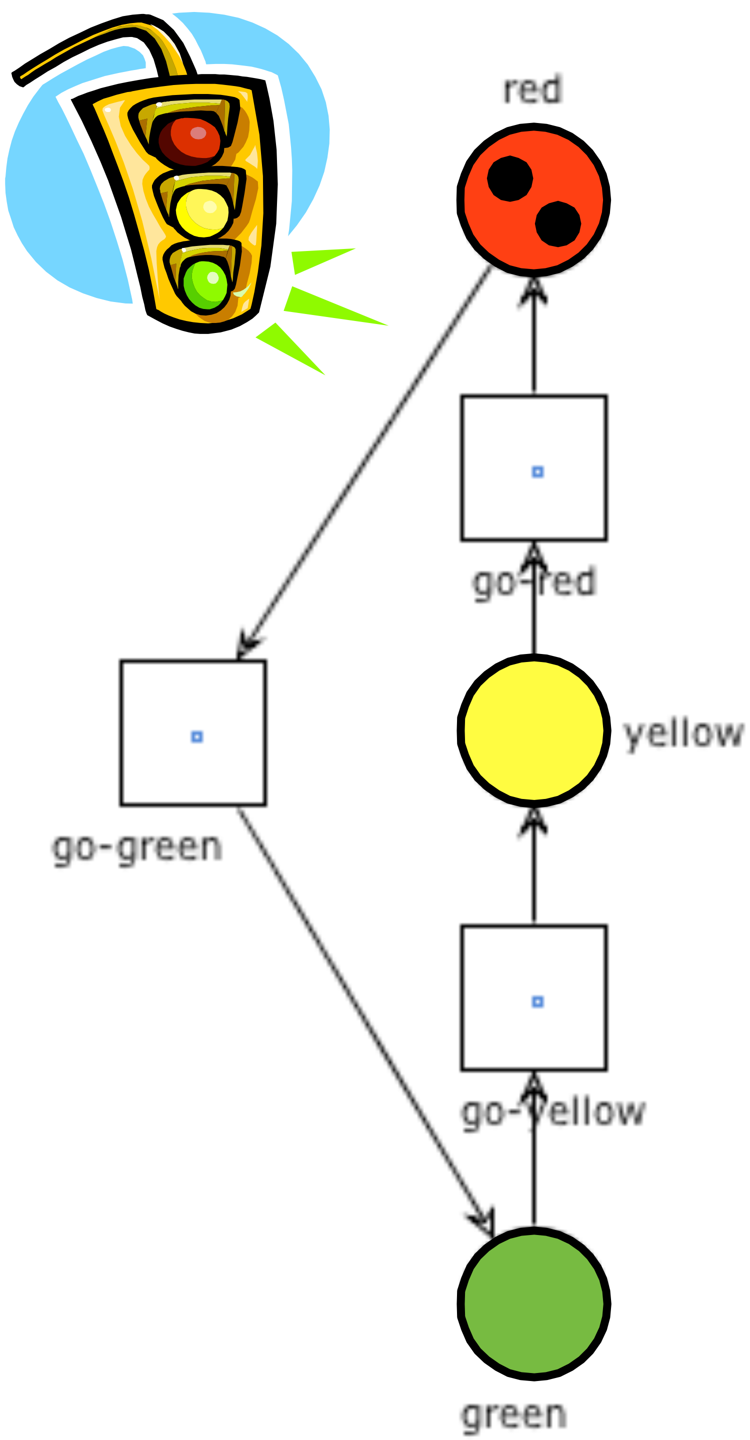

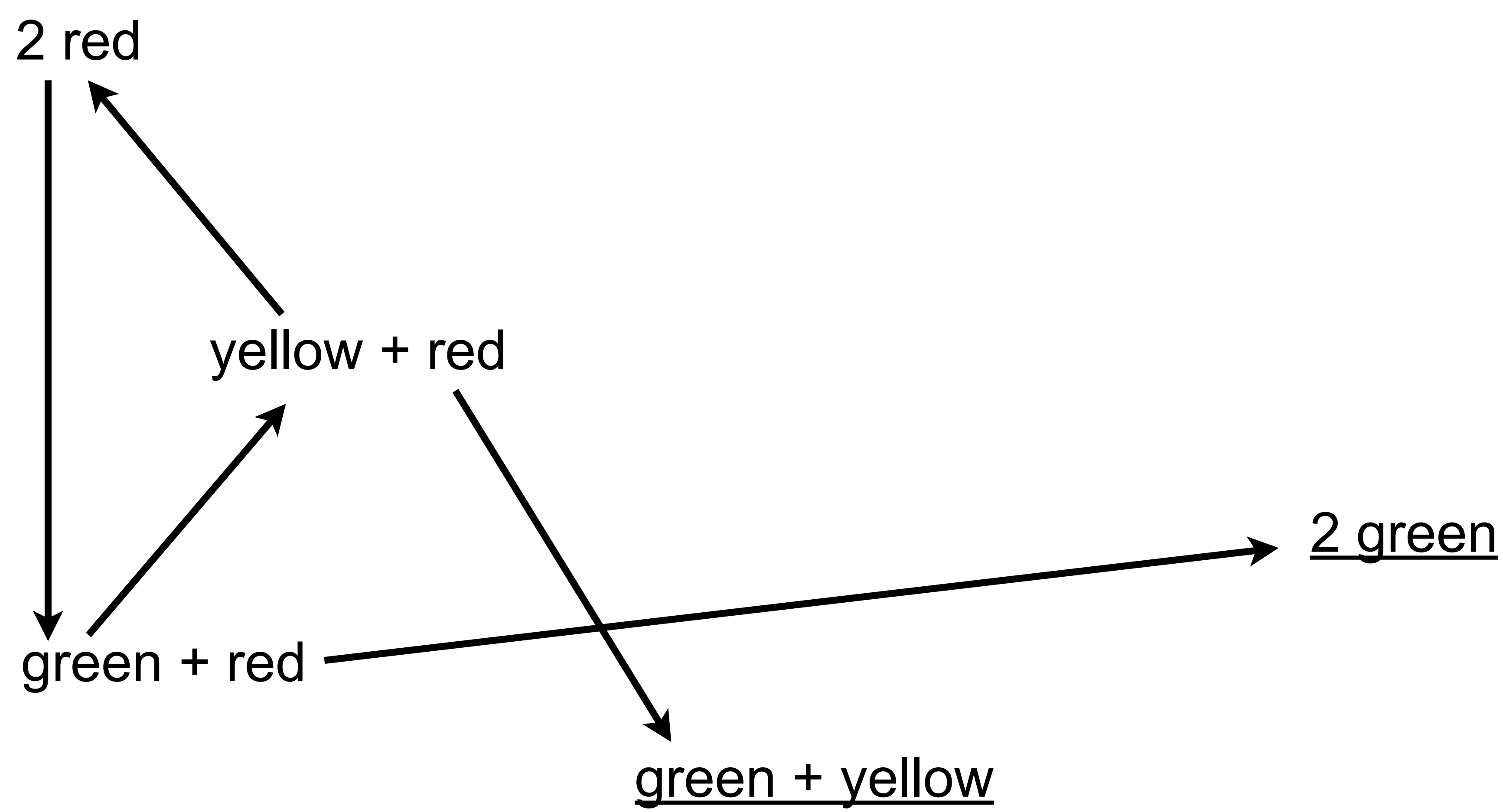

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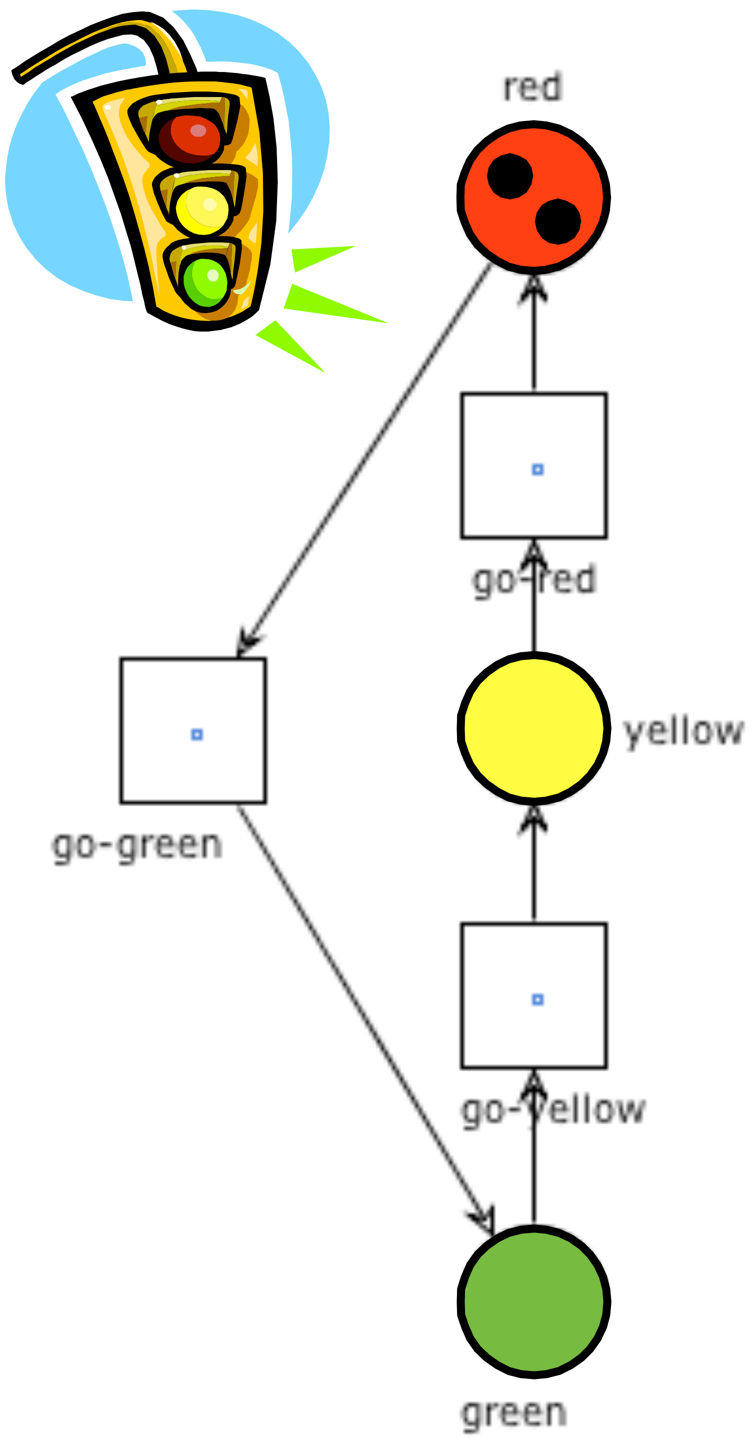

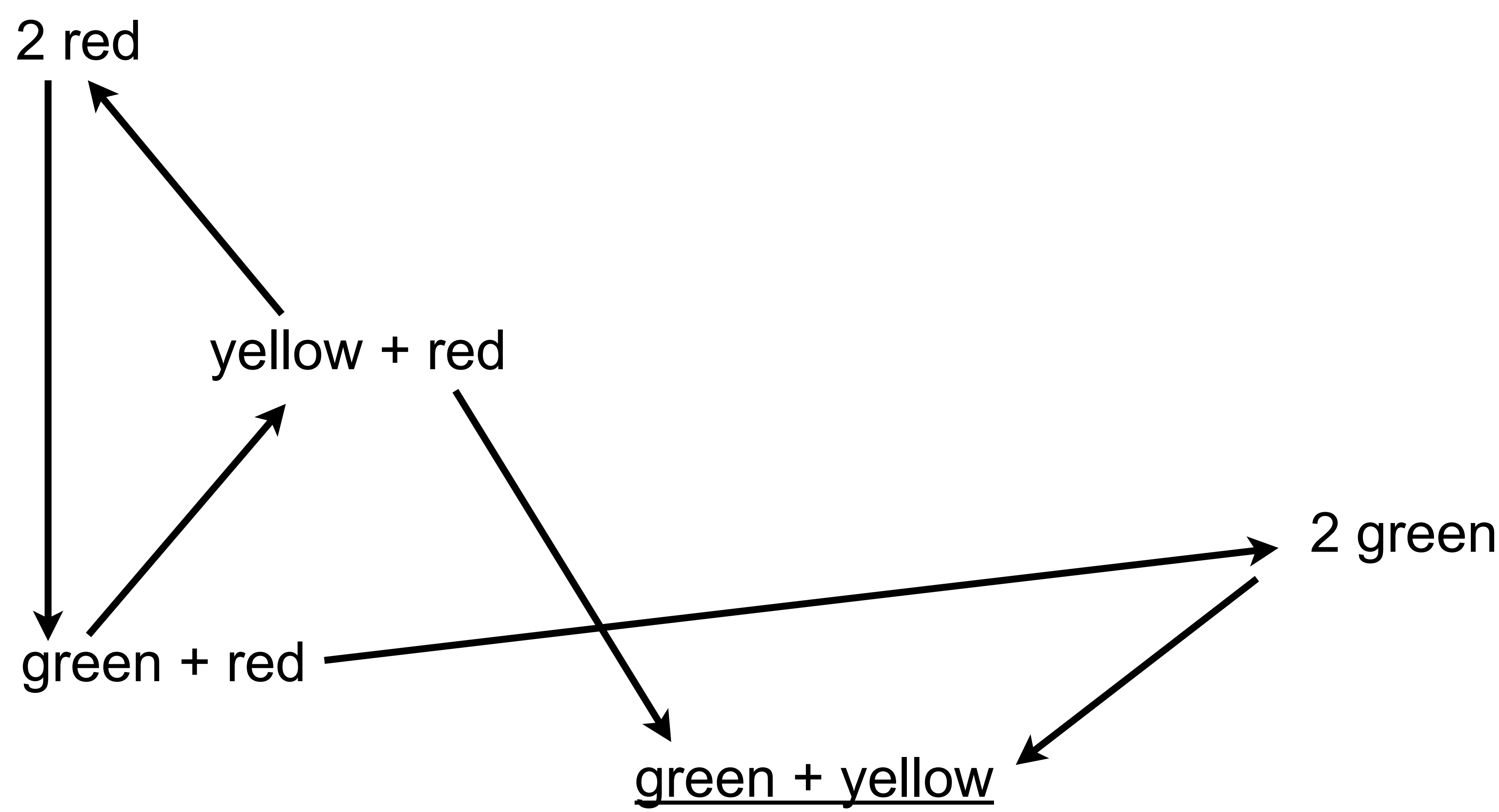

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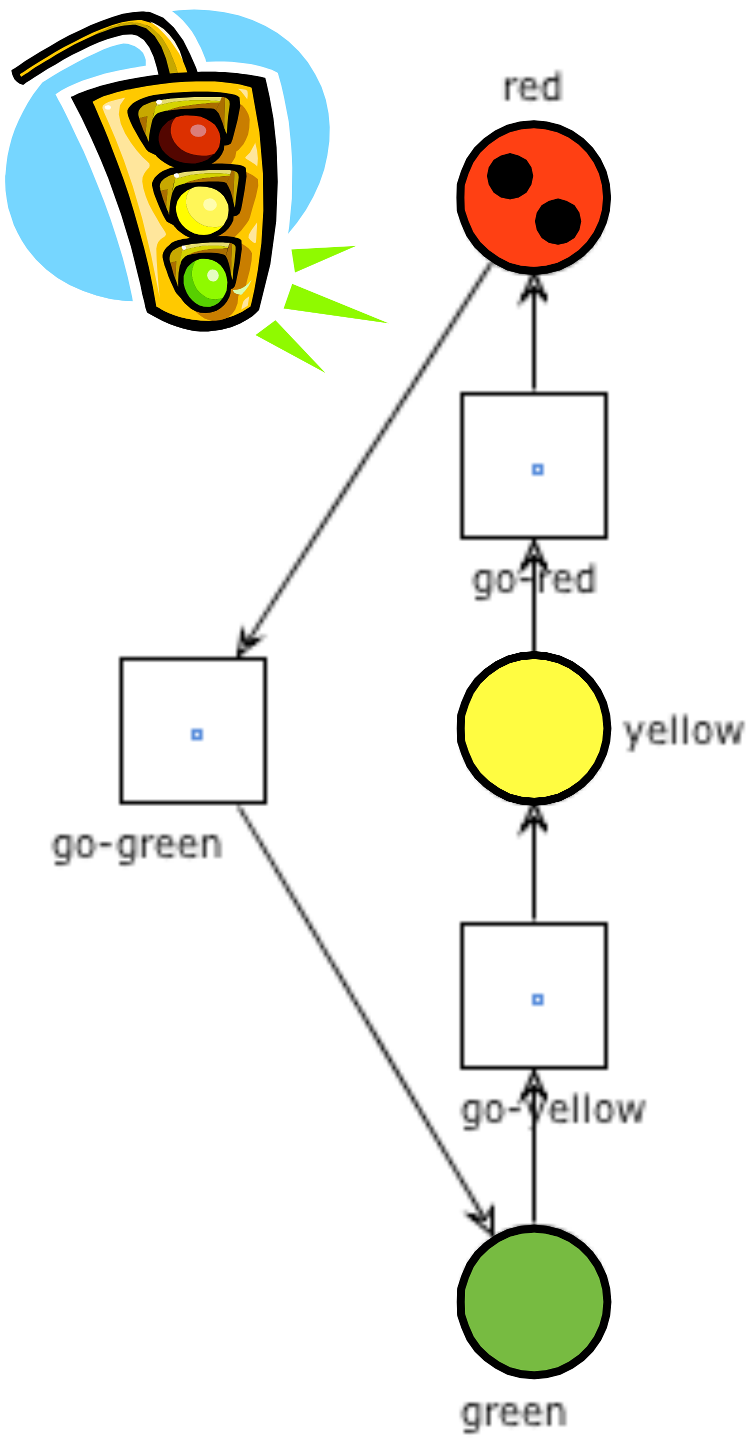

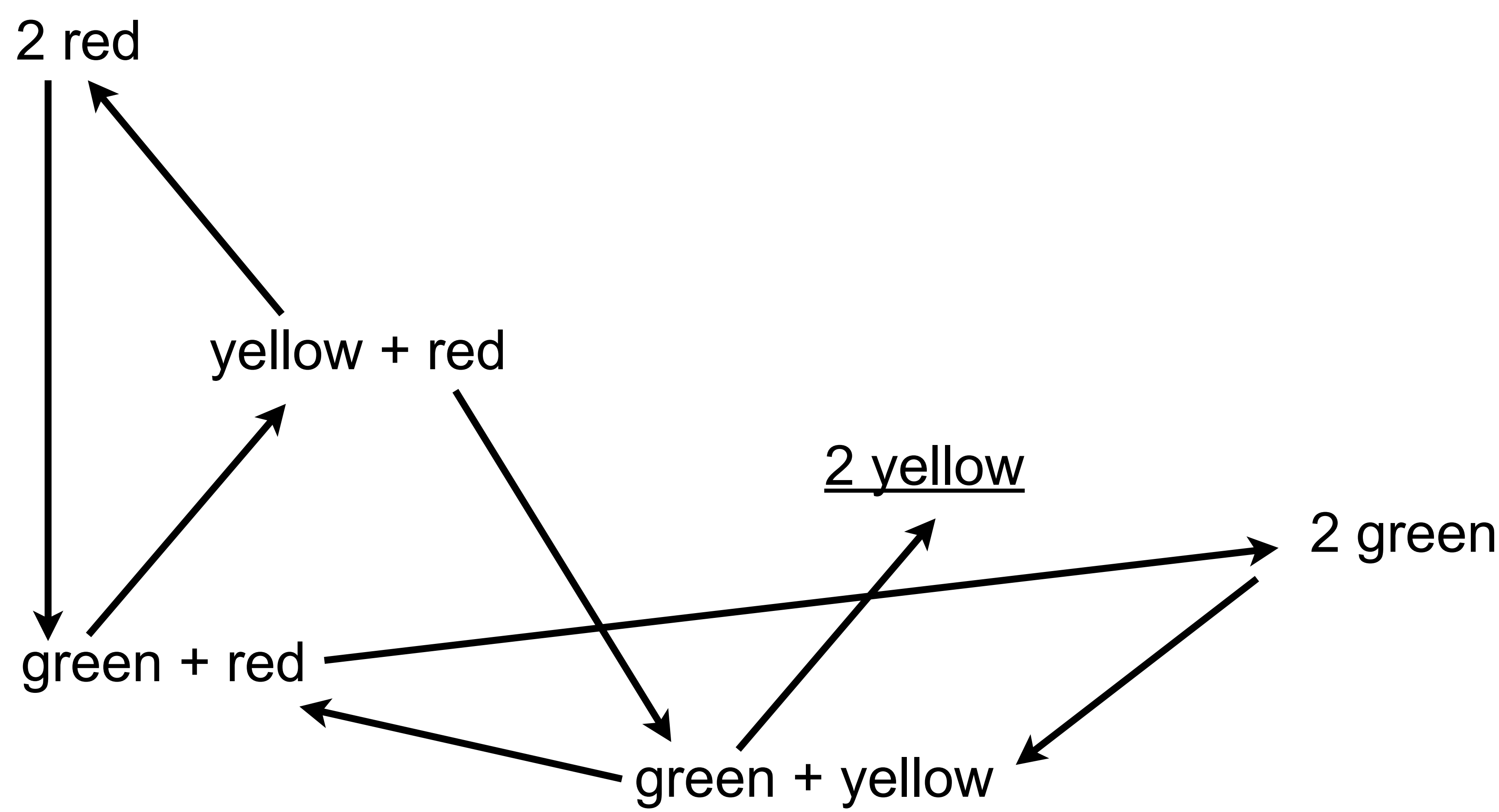

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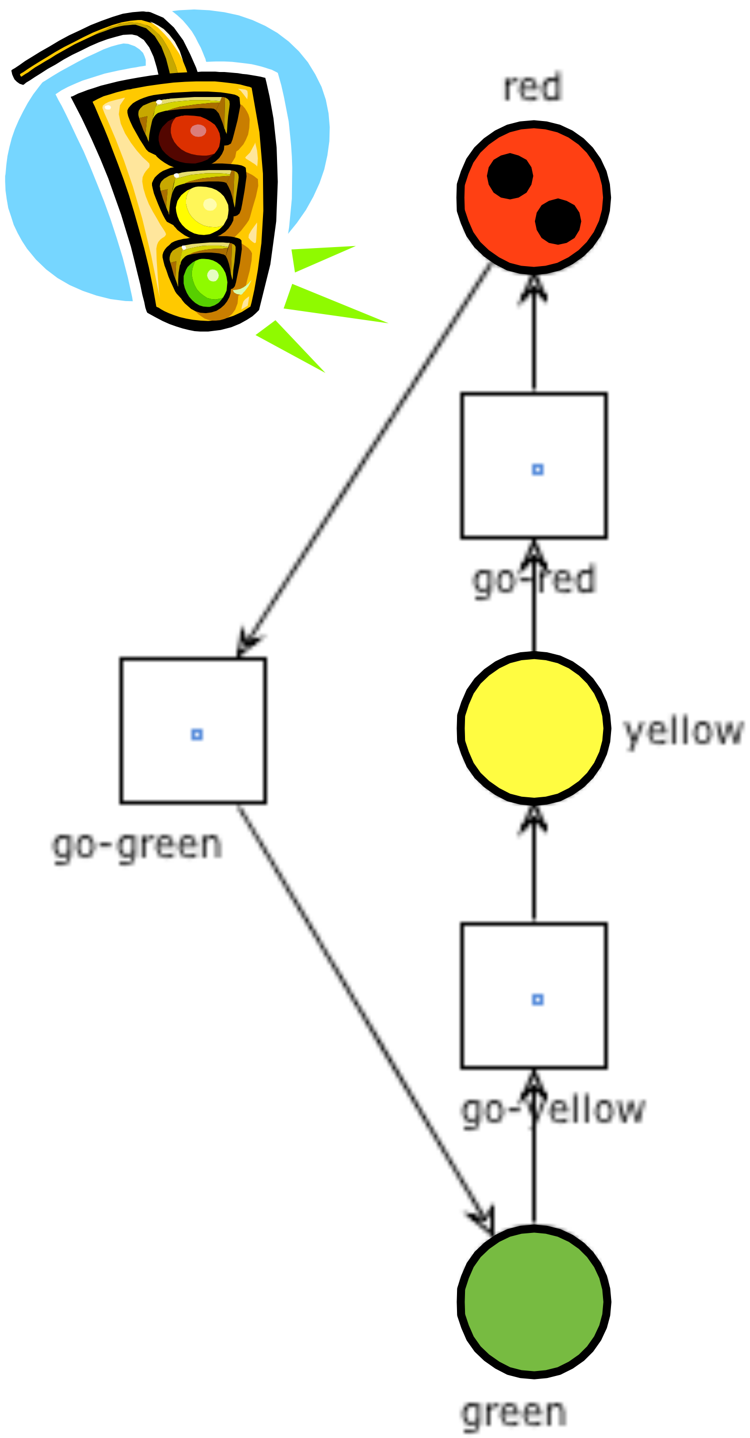

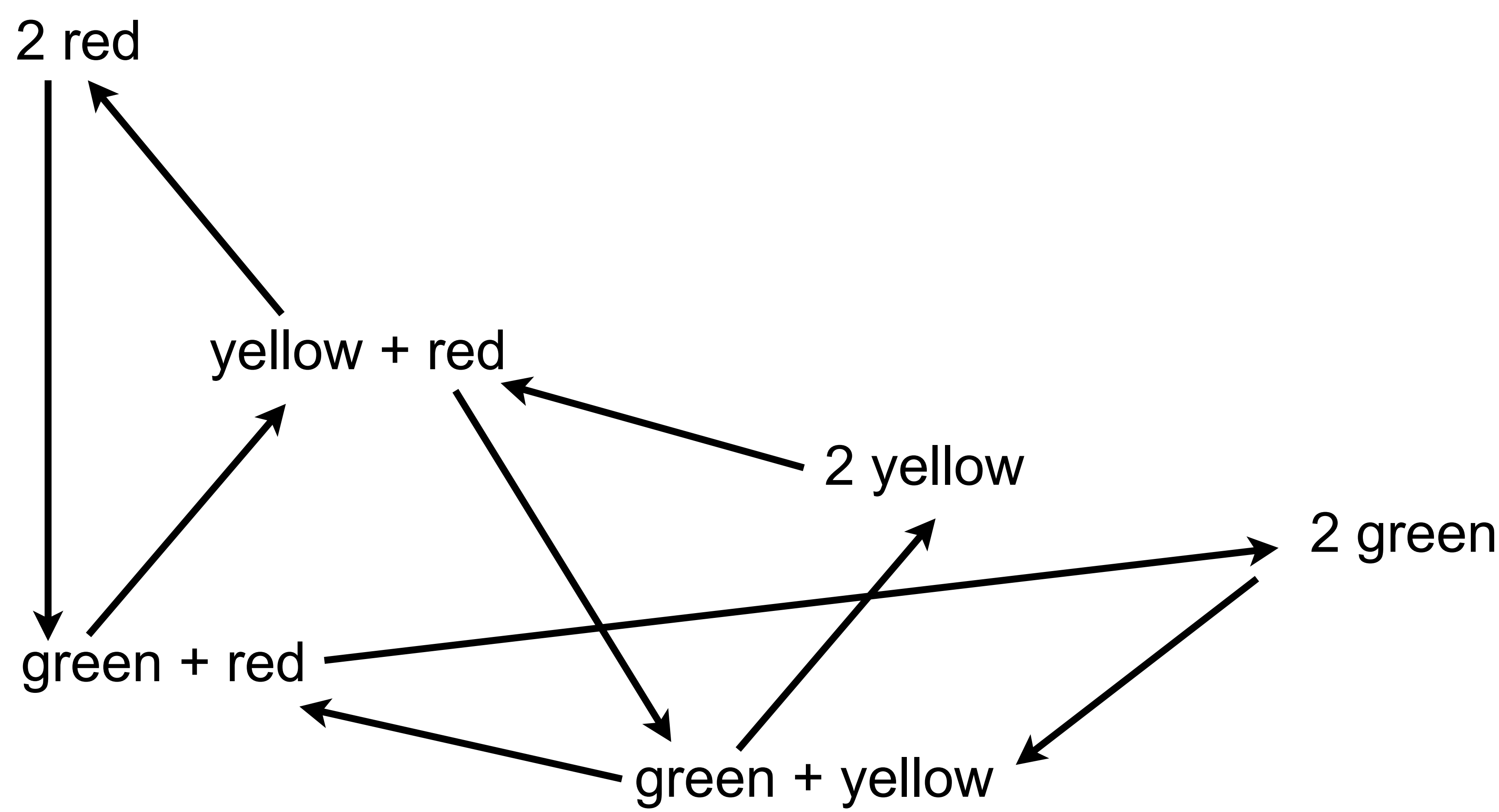

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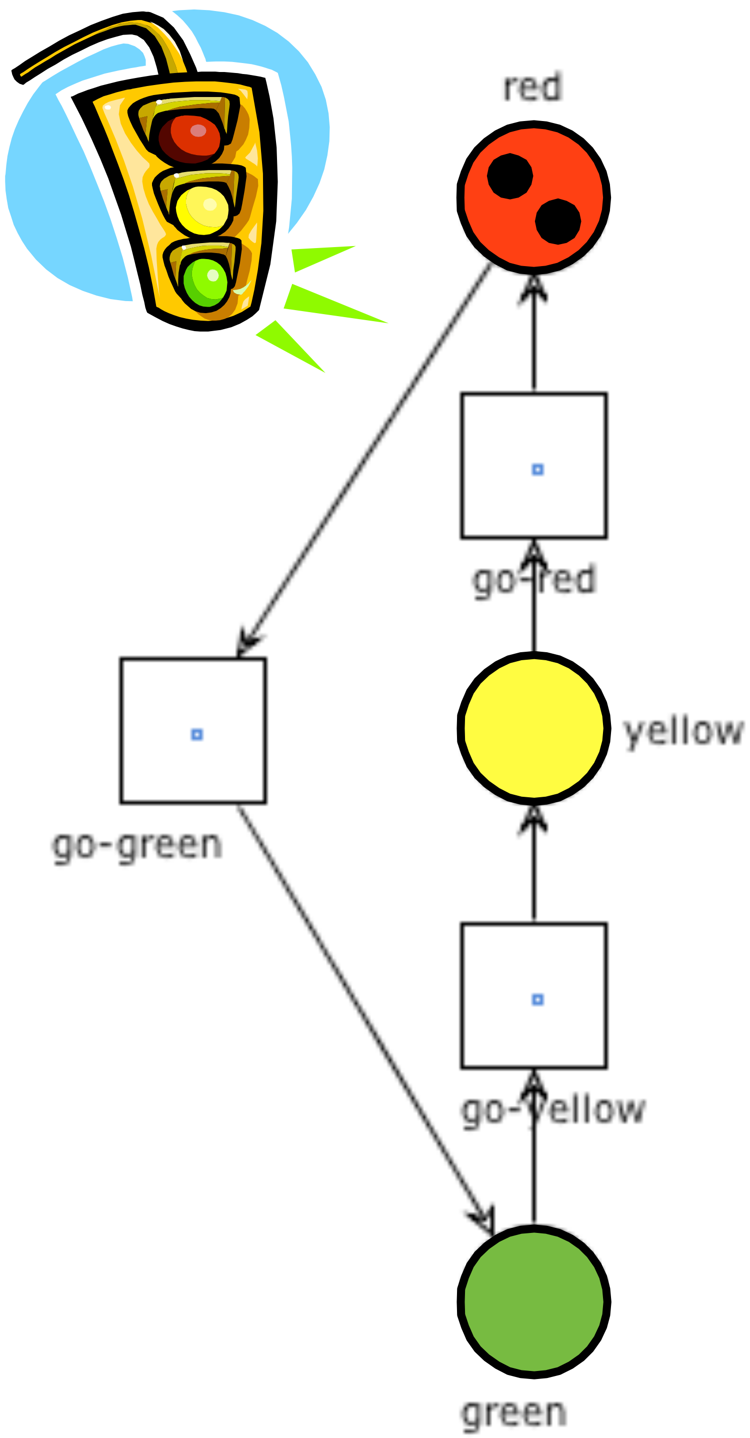
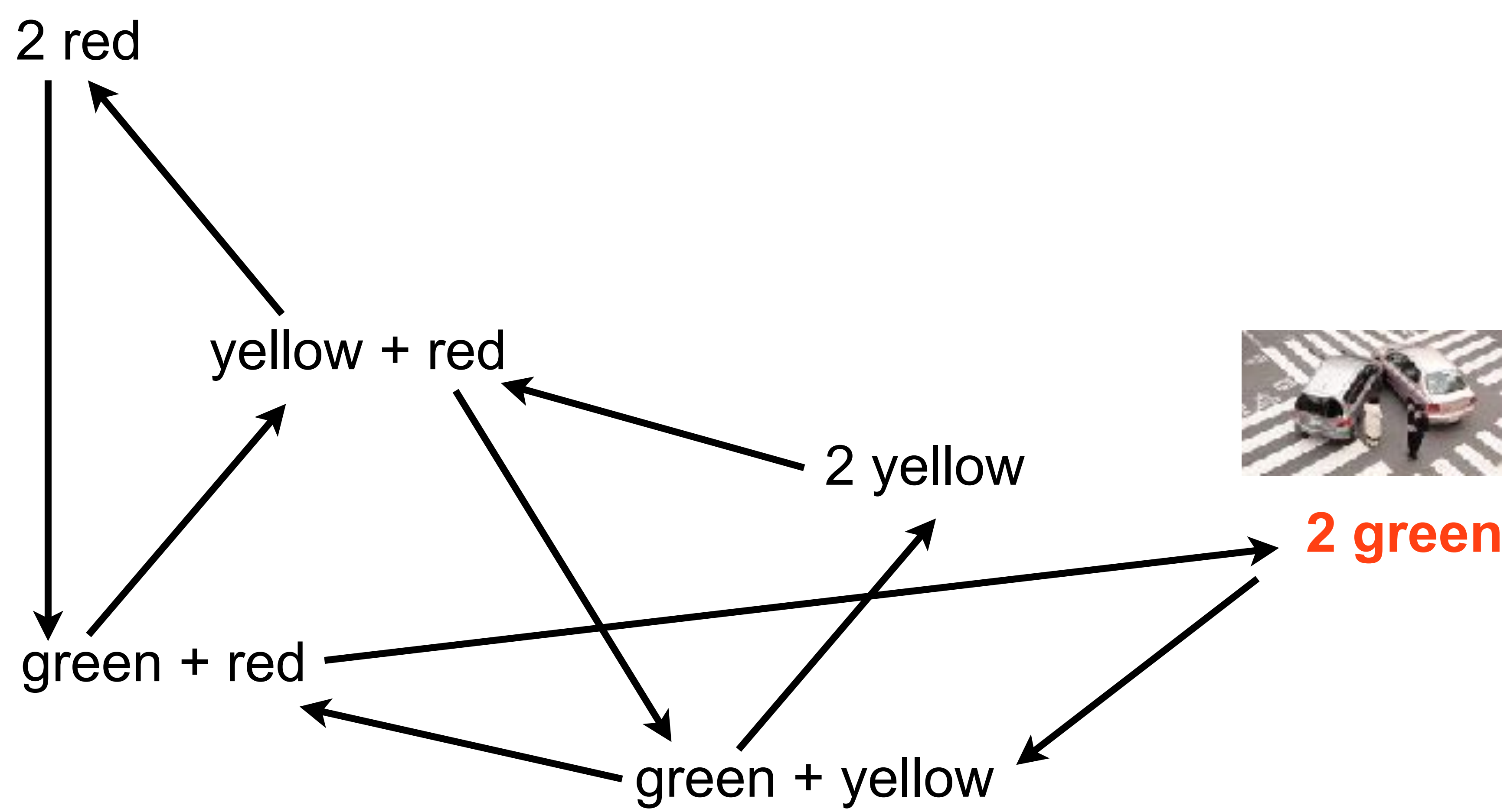
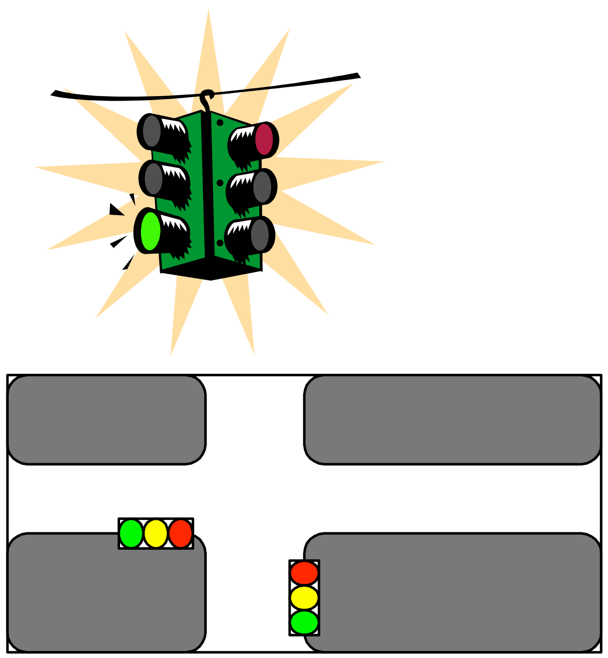
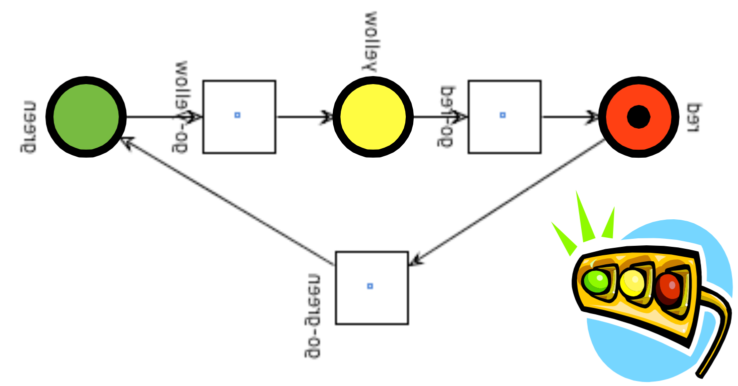


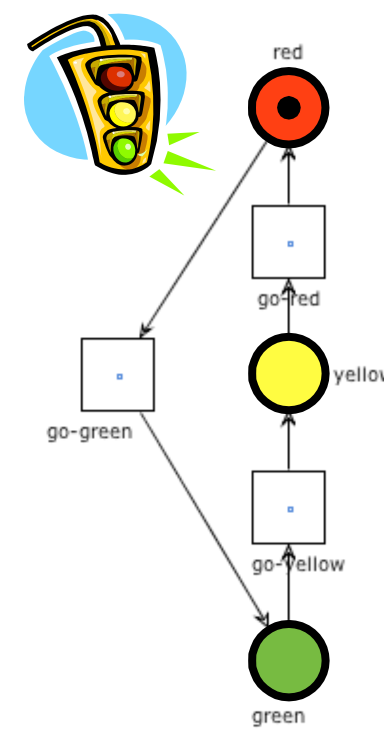


Question time



Complete the net in such a way that the two lights can never be green at the same time





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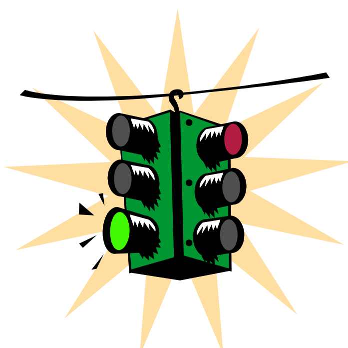

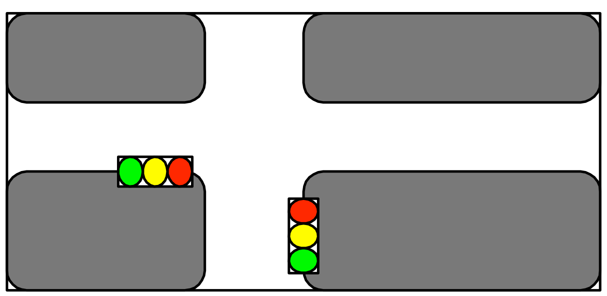

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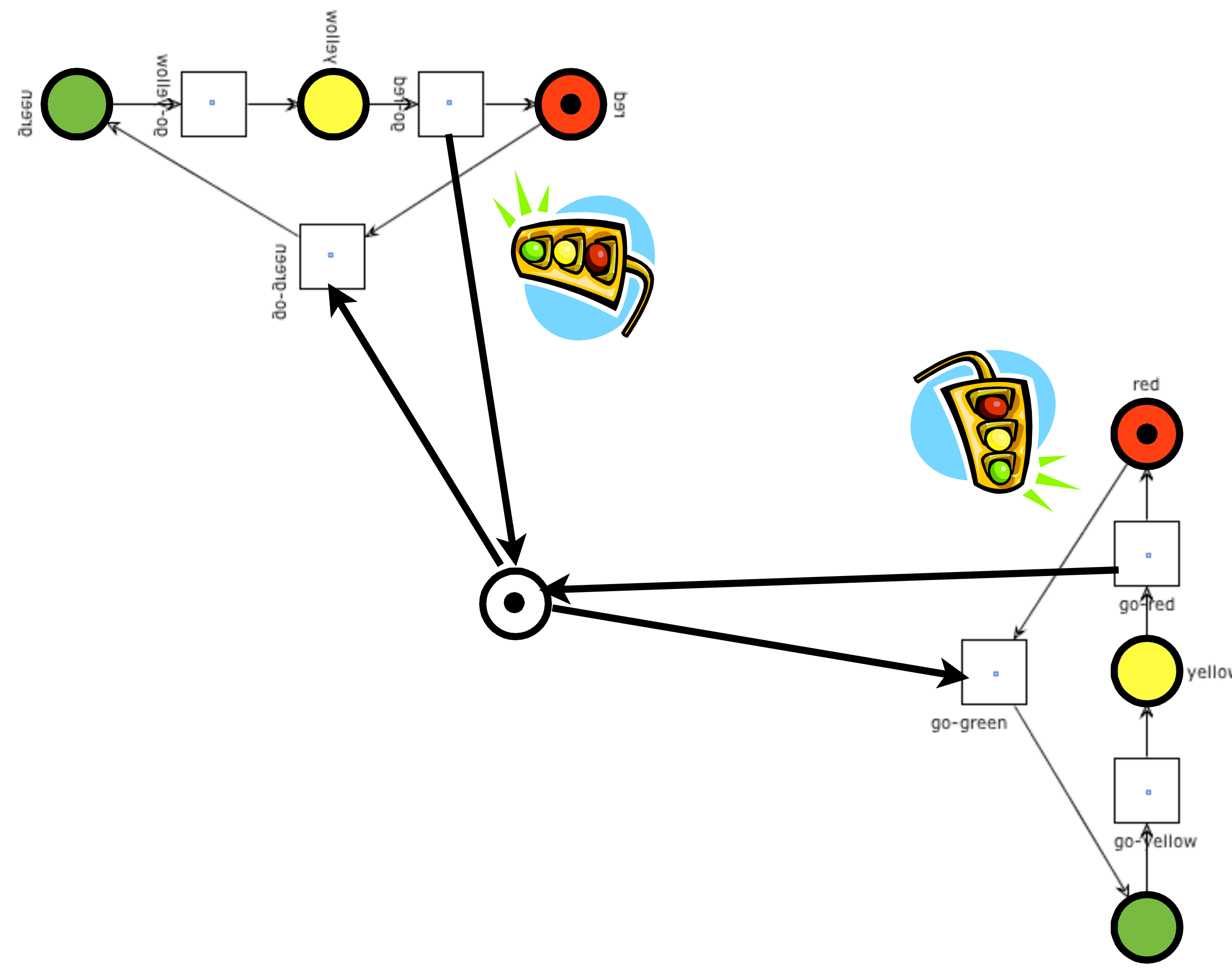
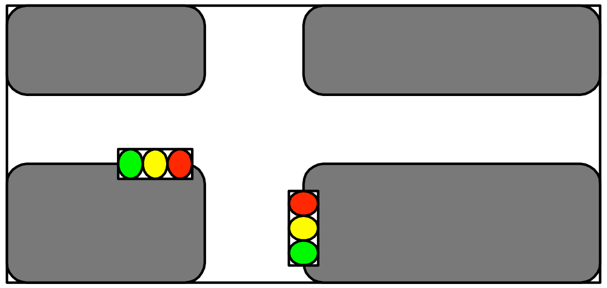


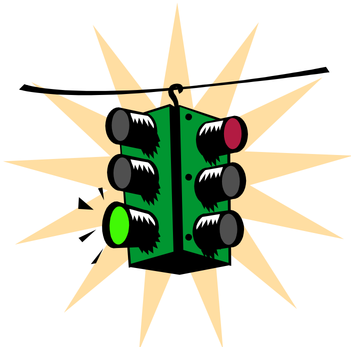


Exercises

Draw the reachability graph of the last net

Modify the net so to guarantee that green alternate on the two traffic lights and then draw the reachability graph

Play the "token games" on the above nets using Workflow Petri net Designer: http://www.woped.org

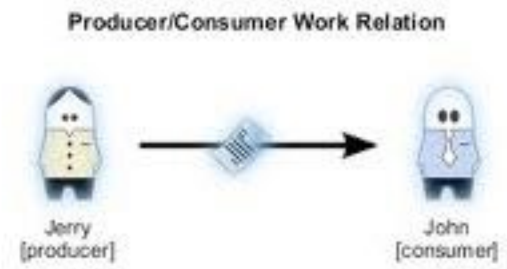


Exercise:
German traffic lights

German traffic lights have an extra phase:
traffic lights do not turn suddenly from red to green but
give a red light together with a yellow light before turning to green.

Identify the possible states and model the automaton that lists all possible states and state transitions.

Design a Petri net that behaves exactly like a German traffic light. There should be three places indicating the state of each light and make sure that the Petri net does not allow state transitions which should not be possible.

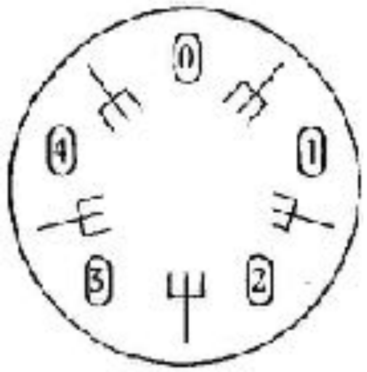


Exercise:

Producer and consumer

Model a process with one producer and one consumer:
Each one is either busy or free.
Each one alternates between these two states
After every production cycle the producer puts a product in a buffer and the consumer consumes one product from this buffer (when available) per cycle.

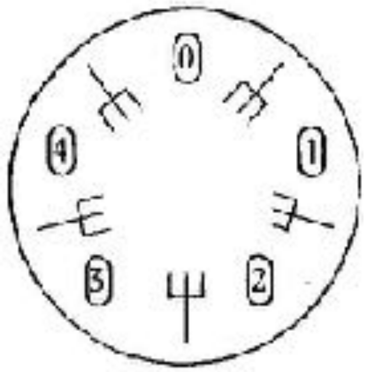
Draw the reachability graph
How to model 4 producers and 3 consumers connected through a single buffer?
How to limit the size of the buffer to 2 items?



Exercise: Dining philosophers

The problem is originally due to E.W. Dijkstra (and soon elaborated by T. Hoare) as an examination question on a synchronization problem where five computers competed for access to five shared tape drive peripherals.

It can be used to illustrate several important concepts in concurrency (mutual exclusion, deadlock, starvation)

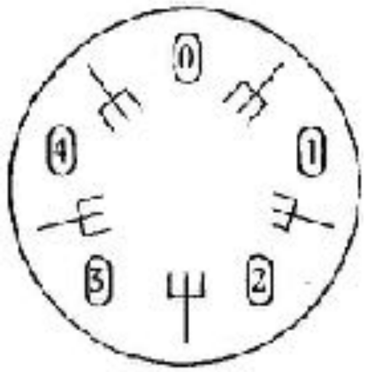


Exercise: Dining philosophers

The life of a philosopher consists of an alternation of thinking and eating

Five philosophers are living in a house where a table is laid for them, each philosopher having his own place at the table

Their only problem (besides those of philosophy) is that the dish served is a very difficult kind of spaghetti, that has to be eaten with two forks. There are two forks next to each plate, so that presents no difficulty: as a consequence, however, no two neighbours may be eating simultaneously.



Exercise: Dining philosophers

Design a net for representing the dining philosophers problem, then use WoPeD to compute the reachability graph

image taken from wikipedia
philosophers clockwise from top:
Plato, Konfuzius, Socrates,
Voltaire and Descartes

Exercise: Railway system

Use a Petri net to model a circular railway system with four stations (st_1, st_2, st_3, st_4) and one train

At each station passengers may "hop on" or "hop off" (this is impossible when the train is moving)

The train has a capacity of 50 persons (if the train is full no passenger can hop on, if the train is empty no passenger can hop off)

What is the number of reachable states?