

Neural Networks and Neurocontrollers





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Outline

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- ✓ Introduction to Neural Network
 - ✓ Biological Neuron
 - ✓ Artificial Neural Network
- ✓ Supervised Learning
 - ✓ Perceptron
 - ✓ Multilayer Perceptron
 - ✓ Back Propagation
 - ✓ Recurrent Neural Network
- ✓ Unsupervised Learning
- ✓ Competitive Learning
 - ✓ Kohonen Networks
- ✓ Reinforcement Learning
- ✓ Neurocontrollers





- Artificial Neural Networks (ANNs) are an abstract simulation of the nervous system, which contains a set of neurons exchanging information through connections (*axons*)
- The ANN model try to mimic axons and dendrites of the nervous system.
- The first neural model was proposed by McCulloch and Pitts (1943). The model was presented as a computational model of the nervous activity. After this, other models were proposed John von Neumann, Marvin Minsky, Frank Rosenblatt, etc.

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- Biological model. It has the objective of replicating biological neural systems, i.e. visual and auditive functionalities. These models are used to validate and verify hypothesis about biological systems.
- The second type is focused on the applications. The models are strongly influenced by application needs. They are called connectionist architectures.

We will focus on the second one!





Biological neuron



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- Human brain contain 100 million neurons. Neuroscientific evidences show each neuron can have 10000 sinapses in input or output
- Switching time of a neuron is few milliseconds. It is <u>slower</u> <u>than a logic gate</u>, but it has a greater connectivity
- A neuron receives from synapses information which are summed
- If the excitatory signal is leading, the neuroni s activated and it generates information through the synapse

Neural Network structure



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A neural network is composed by:

- A set of nodes (neurons), which is the basic unit
- A set of weights linked to connections.
- A set of thresholds or activation levels

The network **design** requires:

- 1. Number of basic unit.
- 2. Morphological structure.
- 3. Learning example encoding (input and output of the net)
- 4. Initialization and training of the weights linked to the connections, through a learning example set.



Main features:

- The objective function can have discrete/continuos values
- Learning data can be noisy
- Learning time is NOT real-time
- Fast evaluation of the learning rate of the neural network
- It is not crucial to get the semantics of the learned function

Robotics, Image Understanding, Biological Systems



Supervised Learning

-MLP and recurrent NN

- Unsupervised Learning - Clustering
- Competitive Learning
 - Kohonen networks
- Reinforcement Learning





- The perceptron is the neural network basic unit
- It was defined by Rosenblatt (1962)
- Try to replicate the single neuron function





- Output values are boolean: 0 1
- Inputs xi and weights wi are real (positive or negative)
- Learning consists in selecting value for weights and threshold



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$$in_i = \sum_j w_{ij} x_j = w_i x_i$$

b) Activation function, non linear (THRESHOLD)







If the threshold function is *sign()* and x1..xn are the input values:

$$o(x) = 1 \text{ if } w_0 + w_1 x_1 + w_2 x_2 + \dots + w_n x_n > 0$$

$$o(x) = -1 \text{ else}$$

• Vector notation:

 $o(\vec{x}) = sign(\vec{w} \cdot \vec{x})$

The Perceptron (classification generalization)



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- Learning problem:
 - Set of points in a *n-dimensional* space
 - 1 classify into two groups (positives and negatives)
 - 2 Then, given a new point P, associate P with one group
 - 1 Classification problem
 - 2 generalizzation problem (learning concepts)

Perceptron – Training algorithm

- Initialize weights randomly
- Gives an example from the dataset<x,c(x)>
- Compute o(x)
- IF o(x)≠c(x) then update:
- η is the *learning rate*
- x_i is the ith feature value of x
- The perceptron error (E) is equal to (c-o)

 $w_i \leftarrow w_i + \Delta w_i$ $\Delta w_i = \eta (c(x) - o(x)) x_i$



- Suppose that o(x)=-1 (if the threshold function is sign(x)) and c(x)=1
- It is needed to modify weights
- Example:

$$x_i = 0, 8, \ \eta = 0, 1, \ c = 1, \ o = -1$$
$$\Delta w_i = \eta (c - o) x_i = 0, 1(1 - (-1))0, 8 = 0, 16$$

- The wi. value increases in order to reduce the error
- IF c(x)=-1 e o(x)=1

$$\Delta w_i = \eta(c-o)x_i = 0, 1(-1-(+1))0, 8 = -0, 16$$



- Convergence theorem of percepton (Rosemblatt, 1962)
 - The perceptron is a <u>linear classifier</u>, therefore it will never get to the state with all the input vectors classified correctly if the training set is not <u>linearly separable</u>
 - In other words.. No local minima!
- The way to solve nonlinear problems is using multiple layers
- Solution: Feed forward neural network and recurrent neural network



• There are no connections between neurons in the same layer and between non adjacent layers





- Objectives:
 - Perceptron weights initialized randomly
 - Fast learning
 - Generalization capability

Backpropagation (2)





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li Input units
Hj Hidden units
Ok Output units

Threshold function: sigmoid

$$o(x) = \sigma(w \cdot x) = \frac{1}{1 + e^{w \cdot x}}$$

Error function is as follow:

$$E(\overset{\mathbf{r}}{w}) = \frac{1}{2} \sum_{x \in D} (t(x) - o(x))^2 =$$
$$\frac{1}{2} \sum_{x \in D} \sum_{k \in N_{out}} (t_k(x) - o_k(x))^2$$



$$\delta_j = o_j (1 - o_j)(t_j - o_j)$$

NNs produce **m** output values

Backpropagation (4)

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- While unreached termination condition, <u>execute</u>:
- For each value $v \in D$: (x, t(x)) (x=(x₁,x₂...x_n), t(x)=(t₁,t₂,...t_m):
 - I set of the input nodes (1,2,,n), O set of output nodes(1,2..m), N set of the net nodes
 - Compute the output of the net generated by the input v and the output of each node of the net $n_u \in N$ (x_i input of the input node $i_i \in I$, o_j output yielded by the node $n_i \in N$)
 - Compute error of the output node $o_k \in O$ as follows: $\delta_k = o_k (1 - o_k)(t_k - o_k)$
 - Compute the error for the hidden units $h_h \in H= (N-O \cup I)$ connected the the output nodes, as follows: $\delta_h = o_h (1 o_h) \sum w_{kh} \delta_k$
 - Compute the error for the other nodes
 - Updates the net weights as follow:

 $w_{ji} \leftarrow w_{ji} + \Delta w_{ji}$ $\Delta w_{ji} = \eta \delta_j x_{ji}$

 $n_k \in O$



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- The process continues until all the examples (<x,t(x)>) have been processed
- When does the process stop? Minimize errors on the train set is not the best solution (*overfitting*)
- Minimize errors on a test set (T), this means to split D in D'∪T, training using D' and using T to verify the termination condition

Error in the training set





Does the algorithm converge? DI BIOROBOTICA



Gradient algorithm issues:

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- Can stop on local minima
- A local minimum can give solutions which are far from the global minimum
- Sometimes there are a lot of local minima...
- A possible solution: training with changing initial weight values

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• Considering the n-th valueof di D, updating rule becomes:

 $\Delta wij(n) \leftarrow \eta \delta_j x_{ij} \leftarrow \alpha \Delta w_{ij}(n-1)$ Momentum

- Pros:
 - Overcoming local minima
 - Keeping stable value for the weights in the «flat zones»
 - Increase velocity when gradient does not change
- Cons:
 - If momentum value is too high can stop on local MAXIMA
 - One more tuning value



- Initializing weight values is basic to reach convergence
- BP depends on the learning rate $\eta.$ This can make the net diverging.
- It can be useful to use different values of η for the network layers



They are networks that learn to associate an input pattern with a sequence of output patterns



A recurrent neural network (RNN) is a class of neural networks where connections between units form a directed cycle. This creates an internal state of the network which allows it to exhibit dynamic temporal behavior.

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Learning strategies

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- Supervised Learning
 -MLP and Recurrent NN
 Unsupervised Learning
 - Clustering
- Competitive Learning
 - Kohonen networks
- Reinforcement Learning





- Split non labeled input values in subsets(cluster)
 - Similar input values are in the same subset
 - Different input values are in different subsets
- Find new in an subsets in an unsupervised way (no labels provided)

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Learning strategies

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- Supervised Learning
 -MLP e reti neurali ricorrenti
 RBF
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Competitive Learning

- In some cases, the network output can be ambiguous
- Thanks to the **lateral inhibition**, neurons start competing to respond to a stimulus.
- The neuron having the greatest output wins the competition and specializes itself to recognize that stimulus.
- Thanks to the excitatory connections, neurons near the winner are also sensitive to similar inputs

An isomorphism is created between input and output space

Competitive Learning - Implementation

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- The winning neuron is selected using a global strategy just by comparing the outputs of the other neurons.
- Two techniques can be used:

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- 1. Select the neuron with the maximum output;
- 2. Select the neuron whose weight vector is more similar to the

$$y_j = \sum_{i=1}^n w_{ji} x_i = W_j \bullet X = |W_j| |X| \cos \theta$$



METHOD 1 - The winner on an input X is the one with the greatest output



METHOD 2 -The winner neuron on input X is that having its weight vector more similar to X

 j_0 : DIS(W_{i0} , X) < DIS(W_i , X)

 $\forall j \neq j_0$

Kohonen networks



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The Kohonen network (or "selforganizing map", or SOM, for short) has been developed by Teuvo Kohonen. The basic idea behind the Kohonen network is to setup a structure of interconnected processing units (neurons) which *compete* for the signal.



The SOM defines a **mapping** from the input data space spanned by $x_1..x_n$ onto a one- or two-dimensional array of nodes. The mapping is performed in a way that the topological relationships in the n-dimensional input space are maintained when mapped to the SOM. In addition, the local density of data is also reflected by the map: areas of the input data space which are represented by more data are mapped to a larger area of the SOM.
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Learning strategies

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 \mathbf{X}_k

Several actions are executed. Successful actions are stored (by weight variations).

Punishments and rewards

An agent operates in the environment and modify actions based on the produced consequences.





Robot control and neurocontrollers







How do you make the network learning?





✓ Creating a dataset of <joint_posistions, end effector_positions> using the direct kinematics



- It is not always possible to compute the inverse kinematic solution using the joint positions
- For soft continuum robots actuated by cables it is possible to exploit the relation between the cable tendion and the end effector position, in order to control the tip

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Learning the Inverse Static Solution (II)



- Control of the soft arm through the learning of the inverse model that allows to control the end effector position through the cable tension
- The inverse problem can be learned collecting points and exploiting the approximation capability of the NN as for the rigid robots





Learning the Inverse Static Solution: an adaptive approach (I)

A Neural Network can be used to solve the inverse solution generating an adaptive approach :

The direct model relation is : x = f(q) (1) This particular representation is not invertible when m<n (redundant)

where, $x \in \Re^m$ is the position and orientation vector of the end effector; $q \in \Re^n$ is the joint vector f is a surjective function



We can develop local representations by linearizing the function at a point (q^o) thereby obtaining :

 $\delta x = J(q^o) \delta q$ (I)

Here $J(q^o)$ is the Jacobian matrix at the point q^o ; δx and δq are infinitesimally small changes in $x \in q$. The differential IK method involves generating of $(\delta x, \delta q, q)$ and learning the mapping $(\delta x, q^o)$ $\rightarrow \delta q$

The learning is feasible since the differential IK solutions form a convex set and therefore averaging multiple solutions still results in a valid solution



The method we have proposed involves expanding Eq. I and expressing it in terms of absolute positions, as shown below:

 $J(q_i)q_{i+1} = x_{i+1} - f(q_i) + J(q_i)q_i$ (II) q_{i+1} is the next actuator configuration for reaching a point x_{i+1} from the present configuration q_i . Note that Eq. II is only valid when the configurations are infinitesimally close. However, for practical purposes this can be a good approximation for larger regions.



Learning the Inverse Static Solution: an adaptive approach (IV)

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The analytical solution for the equation II can be written as:

 $q_{i+1} = G(x_{i+1} - f(q_i) + Jq_i) + (I_n - GJ)z$

where G is a generalized inverse of $J(q_i)$ and I_n is the identity matrix, and z is ar arbitrary n-dimensional vector. The first component represents the particular solution to the non-homogenous problem prescribed in Eq. II and the second component represents the infinite homogenous solutions. It can be proved that the solution space still forms a convex set. Therefore, a **universal function approximator (i.e. NN)** can be used for learning the mapping

 $(q_i, x_{i+1}) \rightarrow (q_{i+1})$ The samples (q_i, q_{i+1}, x_{i+1}) genereted are such that $|q_{i+1} - q_i| < \epsilon$

An appropriate value of ϵ is between 10%-5% of the maximum actuator range



Learning the Inverse Static Solution: an adaptive approach (V)

- We use a feed-forward NN to learn the relation:
 - $(\boldsymbol{q}_i, \boldsymbol{x}_i, \boldsymbol{x}_{i+1}) \rightarrow (\boldsymbol{q}_{i+1})$
 - The values of x_i, x_{i+1} are genereted using the direct model as showed for the learning of the IK of a rigid manipulator TRAINING PHASE
 TEST PHASE

$$(\boldsymbol{q}_{i-1}, \boldsymbol{x}_{i}, \boldsymbol{x}_{i-1}) \longrightarrow \text{INPUT} \quad (\boldsymbol{q}_{i}, \boldsymbol{x}_{i}, \boldsymbol{x}_{i+1}) \longrightarrow \text{INPUT}$$
$$(\boldsymbol{q}_{i}) \longrightarrow \begin{array}{c} \text{Desired} \\ \text{OUTPUT} \end{array} \quad (\boldsymbol{q}_{i+1}) \longrightarrow \begin{array}{c} \text{Network} \\ \text{OUTPUT} \end{array}$$



Learning the Inverse Static Solution: an adaptive approach (V): real robot implementation

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Six DoF Hybrid System (Pneumatic and Tendon)

EARNING

- 2000 sample points divided in the ratio 70:30 for training and testing respectively
- 2 hours for data collection, training and setting-up



ESTS

I-Support Prototype

Twenty Five random points selected from workspace

	Mean Error	Standard Deviation
Position (mm)	5.58	3.08
X- axis rotation (degrees)	2.76	5.42
Y- axis rotation (degrees)	1.84	1.83
Z- axis rotation (degrees)	3.85	7.02

External Disturbance (Only Position)



Unlike the case of rigid robots external disturbances modify the kinematics of the soft manipulator

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This is the first experimental implementation of soft robots tracking under external disturbances



- Using a NN to foresee the signal in the future
- Training the NN using past values





• Learning phase

y(t)



• The NN learns the relation between y(t) and $y(t-\Delta)$





• The NN yields an estimation of the y(t+ Δ)



Learning and prediction Inconsistent training set









- Generate a prediction with any a-priori knowledge of the signal
- Fast convergence with less input seen
- Simple model based on a single neuron (PERCEPTRON) receiving as input current and past values (x(t)-x (t-d))



Es. Ten steps ahead prediction d=10

training set (<input, desired
output>) :
<X1-X10, X20>,<X2-X11, X21>, ...
<Xn-Xn+10, Xn+20>

Network output: y10,y11... yn+10

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$$M(\boldsymbol{q})\ddot{\boldsymbol{q}} + V(\boldsymbol{q})\dot{\boldsymbol{q}} + P(\boldsymbol{q}) = \tau$$

 $\overline{M}(\boldsymbol{x})\ddot{\boldsymbol{x}} + \overline{V}(\boldsymbol{x})\dot{\boldsymbol{x}} + \overline{P}(\boldsymbol{x}) = \tau.$

 $\ddot{\boldsymbol{x}} \to F(\boldsymbol{\tau}, \boldsymbol{x}, \dot{\boldsymbol{x}})$

Complete system states

Chamber Pressures Tendon Forces

$$\boldsymbol{\tau} = F^{-1}(\ddot{\boldsymbol{x}}_d \boldsymbol{x}, \dot{\boldsymbol{x}})$$



•General Dynamic Model :

 Mapping from the manipulator states and inputs to the next states

$$(\tau, x, \dot{x}) \rightarrow \ddot{x}$$

Discretized Dynamic Model :

Allows for a recursive mapping by removing the first order and second order derivatives

$$(\tau, x^i, x^{i-1}) \rightarrow x^{i+1}$$



network



Recurrent Neural Networks are powerful for long-term time series predictions[1,2]

[1]Menezes J, Barreto G. Long-term time series prediction with the NARX network: An empirical evaluation. Neurocomputing 2008; 71: 3335-3343.

[2]Eugen Diaconescu. 2008. The use of NARX neural networks to predict chaotic time series. WSEAS Trans. Comp. Res. 3, 3 (March 2008), 182-191



Developing a controller

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With the new learned forward model, traditional trajectory optimization techniques can be used for developing open loop optimal control policies

We employ single shooting trajectory optimization with sequential quadratic programming method

 $\Pi(t)^* = \min_{\tau} \left\| \left\| x_{t_f}^{task} - x^{des} \right\|^2 + \sum_{k} \tau_k^T R \tau_k \right\|_{\tau_k}^{\tau_k}$ **Minimize Objective Function** Subject to Dynamic Model and Constraints $0 \leq \tau_k^m \leq \tau_{max}^m$ $x_{i+1} = f(x_i, x_{i-1}, \tau_i) \quad \forall i = 1 \dots \frac{t_f}{dt}$

Thuruthel, T. G., Falotico, E., Renda, F., & Laschi, C. (2017). Learning dynamic models for open loop predictive control of soft robotic manipulators. *Bioinspiration & biomimetics*, 12(6), 066003.



Thuruthel, T. G., Falotico, E., Renda, F., & Laschi, C. (2017). Learning dynamic models for open loop predictive control of soft robotic manipulators. *Bioinspiration & biomimetics*, *12*(6), 066003.





Thuruthel, T. G., Falotico, E., Renda, F., & Laschi, C. (2017). Learning dynamic models for open loop predictive control of soft robotic manipulators. *Bioinspiration & biomimetics*, *12*(6), 066003.



Dynamic Controllers: Closed-loop control



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What do we do if we want higher accuracy and if manipulator dynamics changes (For eg. By adding load) ?

- Closed loop control policies can be generated using model-based reinforcement learning.
- However they are task specific (reaching in this case)
- Learned policies are very robust to model changes



Thomas George Thuruthel, E. Falotico, F. Renda, and C. Laschi. Model-based reinforcement learning for closed-loop dynamic control of soft robotic manipulators. *IEEE Transactions on Robotics*, pages 1–11, 2018.

Head stabilization in biped locomotion



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The brain uses the information coming from vestibular system to generate a *unified inertial reference frame*, centred in the head, that allows whole-body coordinated movements and head-oriented locomotion.

Berthoz A., 2002, The sense of movement. Harvard University Press



- The controller is based on a feed feedback error learning (FEL) model. This model estimates the orientation of the head , which allow following a reference orientation.
- ✓ The output of this model is sent as input to a Neural Network which computes the joint positions relative to the estimated orientation



 Artificial Neural Network capable of solving the inverse kinematics problem without using the closed form solution.



 The network has one hidden layer of 20 units. It takes as input the head orientation (ν,φ,ψ) and as output the neck joints angles (θ1, θ2, θ3).



 ✓ The pitch and the roll rotation error are less than
 2 degrees, while the trunk peak-to-peak amplitude was almost 15 degrees for the pitch and 8 for the roll



E.Falotico, N. Cauli, K. Hashimoto, P. Kryczka, A. Takanishi, P. Dario, A.Berthoz, C.Laschi, (2012). "Head stabilization based on a feedback error learning in a humanoid robot". *Proceedings of the 21st IEEE International Symposium on Robot and Human Interactive Communication, Paris, France, September 9-13, 2012.*

Adaptive head stabilization model – results on the Sabian robot



WALKING TEST

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✓ This experiment started with the network weights and the regressor parameters of the FEL set to the values reached at the end of a training phase

✓The peak to peak amplitude of the head is less than 2 degrees during the whole pattern execution.

1. The scenario with the robot in the starting position and the red ball indicating the end of the path.

The scenario from the robot viewpoint.
 & 4. The robot performing the walking pattern with head stabilization.

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Anticipatory Visual perception as a bioinspired mechanism underlying robot

THE AVP SCHEME



Traditional Perception-Action cycle for the AVP architecture **DI BIOROBOTICA**



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• Visual Processing module takes as input current images from both robot cameras to reconstruct the environment producing the relevant feature position.

•The poses of relevant features are sent to a **Trajectory Planning** module to generate the walking path

•The **Controller** module then takes the first robot pose from the sequence of poses planned by the Trajectory Planning module and produces the corresponding motor commands

•This cycle continues until the robot reaches the target.



AVP based perception action cycle (I)

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• Internal Models of the environment and of the task to be performed are necessary to predict future visual perceptions.

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•Images of different features relevant to the locomotion task are captured and memorized.





AVP based perception action cycle (II)

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•At every step, the **Visual Comparator** module compares the current image of the environment with a synthetic image predicted by the **AVP Generator** module.

•To produce the synthetic image, AVP Generator computes the current robot pose taking into account the initial pose of the robot and the motor commands executed at the immediately previous time step.

•The current robot pose is sent to the **Learning** sub-module of Internal Models, where the neural network predicts the corresponding poses of the relevant features.

•The AVP Generator then takes as input the memorized images of the relevant features, and **creates a synthetic image** by pasting them on the environmental background at poses predicted by the neural net.
Implementation of internal models for EP generation



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- OF GENERATOR: generates the Optical Flow from the camera image with Lucas-Kanade algorithm.
- OF SIMPLIFIER: generates a Simple OF dividing in zones the Optical Flow and calculating the mean flow vector for each zone.
- **OF PREDICTOR**: predicts the next step OF using an ESN. Learning is performed off-line.
- SYNTHETIC IMAGE CREATOR: generates a sinthetic image rapresenting the next step camera image.
- IMAGE COMPARATOR: generates the error between the synthetic image and the corresponding camera image.



Output signal composed by next step simple Optic Flow.





Background and floor were replaced by a white screen.
A matrix of coloured spheres was placed in front of the robot.

Implementation of internal models for EP generation Scuola Superiore EXPERIMENTS

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- A) Motor command: sinusoid of amplitude 15 and frequency 0.1. Dataset of 1500 elements (1200 training, 300 test).
- B) Motor command: sequence of sinusoids of amplitude 15 and frequencies between 0.1 and 0.5. Dataset of 1500 elements (1200 training, 300 test).
- C) Training set composed by the sum of 3 sinusoids with amplitude 5 and frequency 0.1 0.25 and 0.4 (1200 elements). Test set composed by a sinusoid with amplitude 15 and frequency 0.2 (300 elements).
- D) Motor command: same sinusoid as A. Environment: the matrix of spheres has columns more distants than before. Dataset of 1500 elements (1200 training, 300 test).



• Input dim: 5, Spectral radius: 0.7, Reservoir dim: 75

- Train Error (pixels MSE): 0.16558 0.15946 0.15664 0.16499
- Test Error (pixels MSE): 0.17732 0.18431 0.15675 0.16521



- Train Error (pixels MSE): 0.23676 0.23667 0.23405 0.25608
- Test Error (pixels MSE): 0.28956 0.31899 0.31899 0.32285



- Input dim: 5, Spectral radius: 0.7, Reservoir dim: 75
- Train Error (pixels MSE): 0.21956 0.2316 0.2188 0.23137
- Test Error (pixels MSE): 0.31305 0.28552 0.28264 0.30684

