Chapter 6
Trajectory planning

In previous chapters we studied mathematical models of robot mechanisms. First of all we were interested in robot kinematics and dynamics. Before applying this knowledge to robot control, we must become familiar with the planning of robot motion. The aim of trajectory planning is to generate the reference inputs to the robot control system, which will ensure that the robot end-effector will follow the desired trajectory.

Robot motion is usually defined in the rectangular world coordinate frame placed in the robot workspace most conveniently for the robot task. In the simplest task we only define the initial and the final point of the robot end-effector. The inverse kinematic model is then used to calculate the joint variables corresponding to the desired position of the robot end-effector.

6.1 Interpolation of the trajectory between two points

When moving between two points, the robot manipulator must be displaced from the initial to the final point in a given time interval t_f. In most cases we are not interested in the precise trajectory between the two points. Nevertheless, we must determine the time course of the motion for each joint variable and provide the calculated trajectory to the control input. The joint variable is either the angle ϑ for the rotational or the displacement d for the translational joint. When considering the interpolation of the trajectory we shall not distinguish between the rotational and translational joints, so that the joint variable will be more generally denoted as q. With industrial manipulators moving between two points we most often select the so called trapezoidal velocity profile. The robot movement starts at $t = 0$ with constant acceleration, followed by the phase of constant velocity and finished by the constant deceleration phase (Figure 6.1). The resulting trajectory of either the joint angle or displacement consists of the central linear interval, which is started and concluded with a parabolic segment. The initial and final velocities of the movement between the two points are zero. The duration of the constant acceleration phase is equal

T. Bajd et al., *Robotics*, Intelligent Systems, Control and Automation: Science and Engineering 43, DOI 10.1007/978-90-481-3776-3_6,

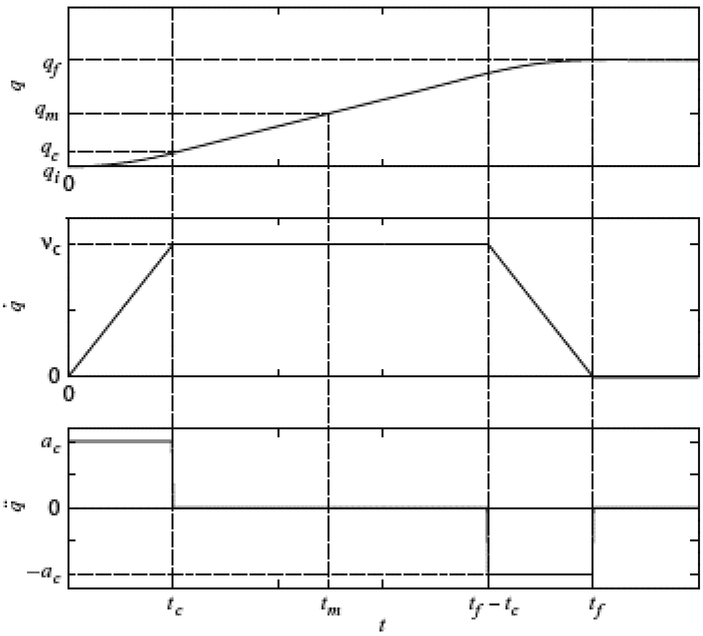

Fig. 6.1 The time dependence of the joint variables with trapezoidal velocity profile

to the interval with the constant deceleration. In both phases the magnitude of the acceleration is a_c. In this way we deal with a symmetric trajectory, where

$$q_m = \frac{q_f + q_i}{2} \quad \text{at the moment} \quad t_m = \frac{t_f}{2}. \tag{6.1}$$

The trajectory $q(t)$ must satisfy several constraints in order that the robot joint will move from the initial point q_i to the final point q_f in the required time interval t_f. The velocity at the end of the initial parabolic phase must be equal to the constant velocity in the linear phase. The velocity in the first phase is obtained from the equation describing the constant acceleration motion

$$v = a_c t. \tag{6.2}$$

At the end of the first phase we have

$$v_c = a_c t_c. \tag{6.3}$$

The velocity in the second phase can be determined by the help of Figure 6.1

$$v_c = \frac{q_m - q_c}{t_m - t_c}, \tag{6.4}$$

where q_c represents the value of the joint variable at the end of the initial parabolic phase, i.e. at the time t_c. Until that time the motion with constant acceleration a_c

takes place, so the velocity is determined by equation (6.2). The time dependence of the joint position is obtained by integrating equation (6.2)

$$q = \int v dt = a_c \int t dt = a_c \frac{t^2}{2} + q_i, \tag{6.5}$$

where the initial joint position q_i is taken as the integration constant. At the end of the first phase we have

$$q_c = q_i + \frac{1}{2} a_c t_c^2. \tag{6.6}$$

The velocity at the end of the first phase (6.3) is equal to the constant velocity in the second phase (6.4)

$$a_c t_c = \frac{q_m - q_c}{t_m - t_c}. \tag{6.7}$$

By inserting equation (6.6) into equation (6.7) and considering the expression (6.1), we obtain, after rearrangement, the following quadratic equation

$$a_c t_c^2 - a_c t_f t_c + q_f - q_i = 0. \tag{6.8}$$

The acceleration a_c is determined by the selected actuator and the dynamic properties of the robot mechanism. For chosen q_i, q_f and t_f the time interval t_c is

$$t_c = \frac{t_f}{2} - \frac{1}{2} \sqrt{\frac{t_f^2 a_c - 4(q_f - q_i)}{a_c}}. \tag{6.9}$$

To generate the movement between the initial q_i and the final position q_f the following polynomial must be generated in the first phase

$$q_1(t) = q_i + \frac{1}{2} a_c t^2 \qquad 0 \leq t \leq t_c. \tag{6.10}$$

In the second phase a linear trajectory must be generated starting in the point (t_c, q_c) with the slope v_c

$$(q - q_c) = v_c (t - t_c). \tag{6.11}$$

After rearrangement we obtain

$$q_2(t) = q_i + a_c t_c \left(t - \frac{t_c}{2}\right) \qquad t_c < t \leq (t_f - t_c). \tag{6.12}$$

In the last phase the parabolic trajectory must be generated similar to the first phase, only that now the extreme point is in (t_f, q_f) and the curve is turned upside down

$$q_3 = q_f - \frac{1}{2} a_c (t - t_f)^2 \qquad (t_f - t_c) < t \leq t_f. \tag{6.13}$$

In this way we obtained analytically the time dependence of the angle or displacement of the rotational or translational joint moving from point to point.