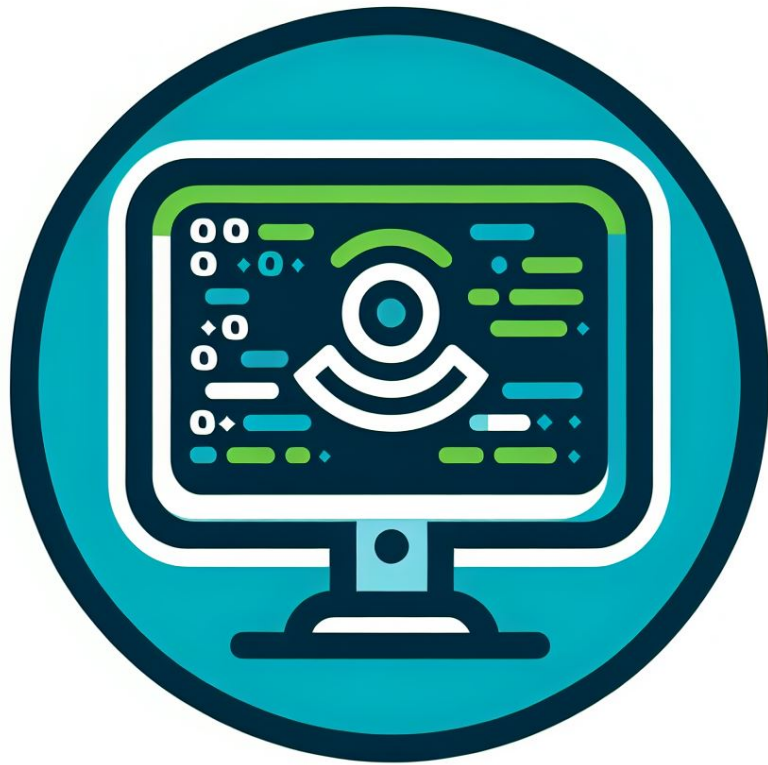




PSC 2024/25 (375AA, 9CFU)

Principles for Software Composition

Roberto Bruni

<http://www.di.unipi.it/~bruni/>

[http://didawiki.di.unipi.it/doku.php/
magistraleinformatica/psc/start](http://didawiki.di.unipi.it/doku.php/magistraleinformatica/psc/start)

27 - PEPA

PEPA

Performance Evaluation Process Algebra

Building models

Conceptualise your system as a Markov chain

Construct your Markov chain (infinitesimal generator matrix)

Solve your equations to derive quantitative information

Building models

Conceptualise your system as a Markov chain

Construct your Markov chain (infinitesimal generator matrix)

Solve your equations to derive quantitative information

Monolithic approach: not suitable for complex systems

PEPA project



the PEPA project started in Edinburgh in 1991

motivated by the performance analysis
of large computer and communication systems

exploit interplay between Process Algebras and CTMC

Process Algebras (PA):
compositional description of complex systems,
formal reasoning (for correctness)

CTMC:
numerical analysis

compositional construction of CTMC

PEPA meets CTMC

PA mutual influence CTMC

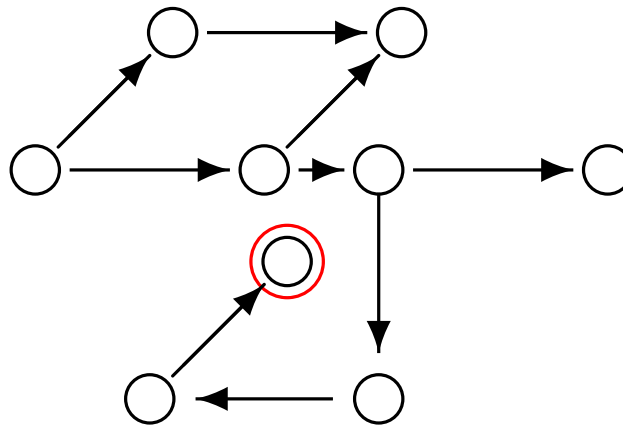
interaction designed around CTMC	ease of construction
actions have durations	design of independent components
add rates to labels	cooperation between components
probabilistic branching	explicit interaction
quantitative measures	reusable sub-models
probabilistic model checking	easy to understand models
quantitative logics	space reduction techniques
	functional verification

Formal models

qualitative

vs

quantitative



reachability:
will the system arrive to a
particular state?

how long will it take
the system to arrive to a
particular state?

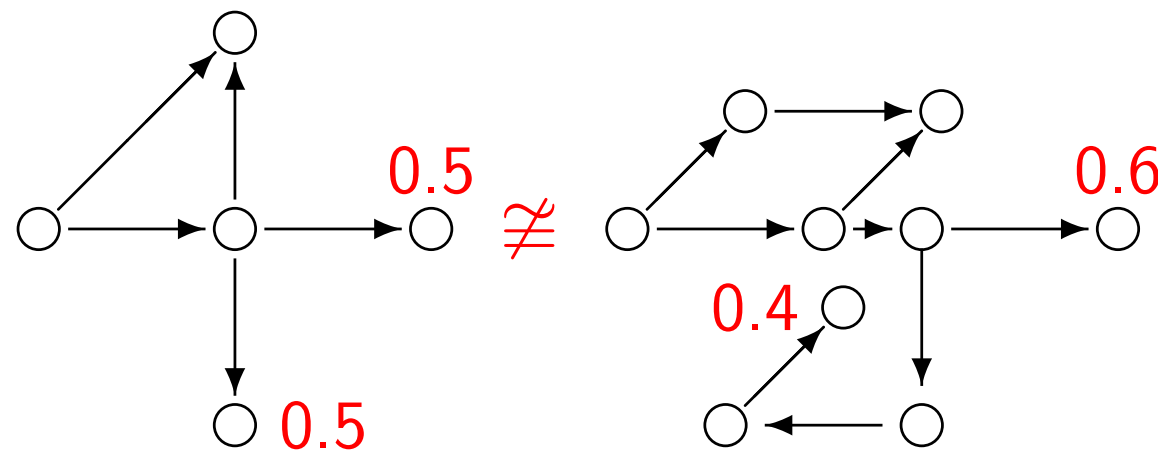
(taken from Jane Hillston's slides)

Formal models

qualitative

vs

quantitative



conformance:
does system behaviour
match its specification?

how likely is that
system behaviour will
match its specification?

does the frequency profile
of the system match
that of its specification?

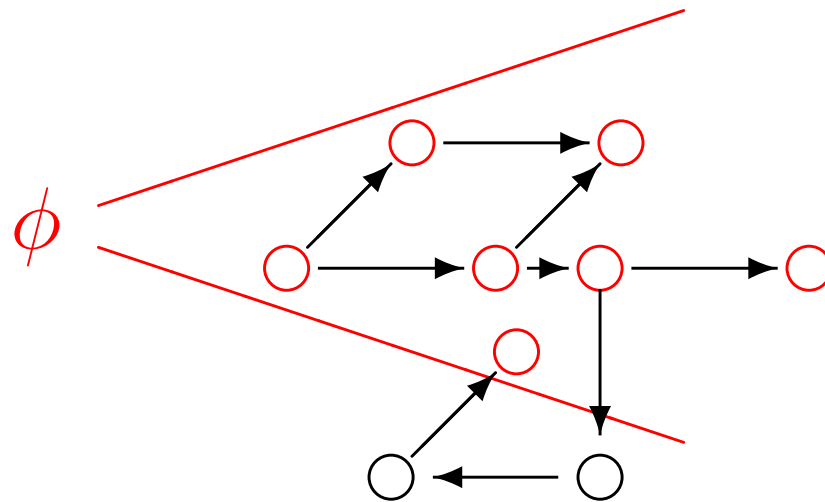
(taken from Jane Hillston's slides)

Formal models

qualitative

vs

quantitative



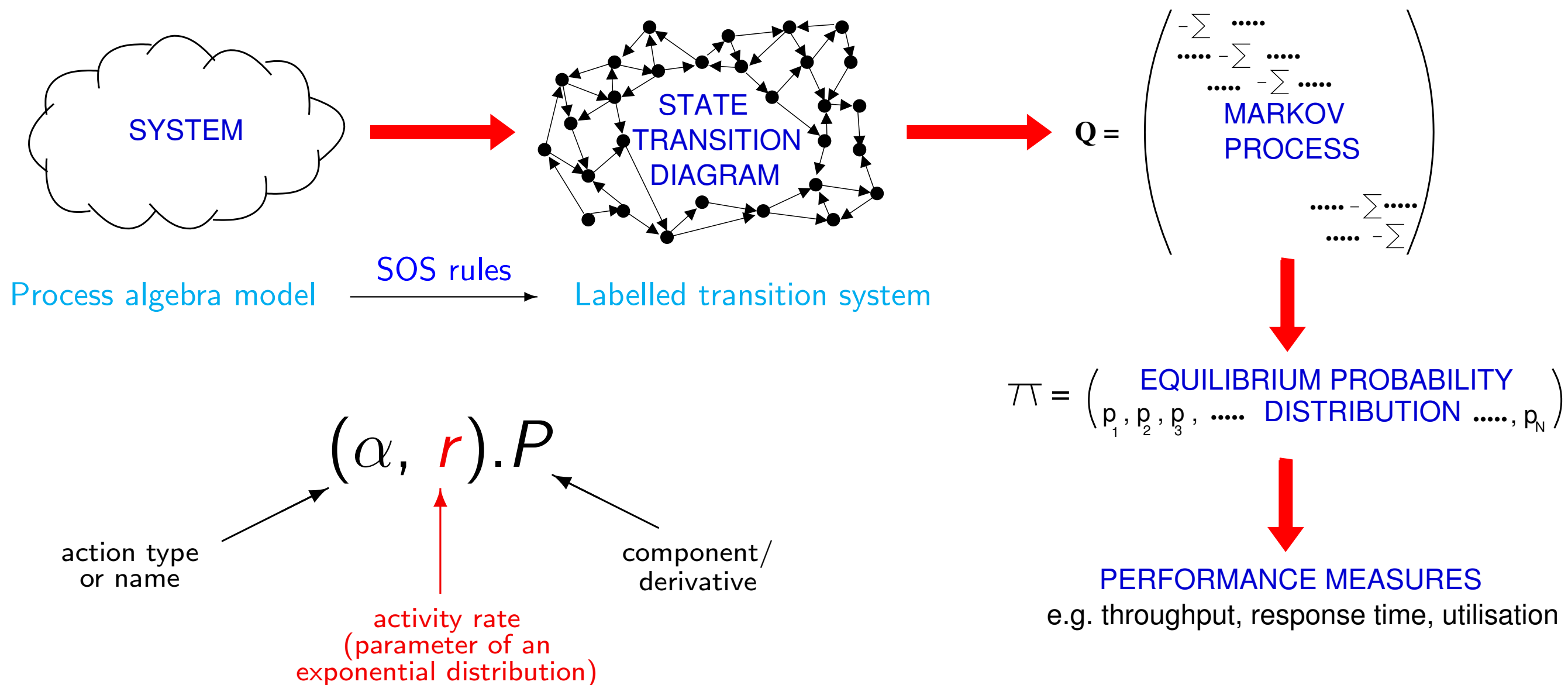
verification:
does a given property
hold within the system?

Does a given property
hold within the system
with a given probability?

How long is it until
a given probability hold?

(taken from Jane Hillston's slides)

PEPA workflow



(taken from Jane Hillston's slides)

Communication style

PEPA parallel composition is based on Tony Hoare's CSP

CCS-style

actions α and co-actions $\bar{\alpha}$

binary synchronisation

conjugate sync $\alpha \mid \bar{\alpha}$

result in a silent action τ

restriction $P \backslash \alpha$ (blocks α , not τ)

parallel composition $P \mid Q$

one operator

CSP-style

no i/o distinction α

multiple cooperation

shared name sync $\alpha \mid \alpha \mid \dots \mid \alpha$

result in the same name α

hiding P / α (renames α to τ)

cooperation combinator $P \bowtie_L Q$

parametric operator

CSP cooperation combinator

$$P \bowtie_L Q$$

cooperation set

cooperation

$$\frac{P_1 \xrightarrow{\alpha} Q_1 \quad P_2 \xrightarrow{\alpha} Q_2 \quad \boxed{\alpha \in L}}{P_1 \bowtie_L P_2 \xrightarrow{\alpha} Q_1 \bowtie_L Q_2}$$

interleaving

$$\frac{P_1 \xrightarrow{\alpha} Q_1 \quad \boxed{\alpha \notin L}}{P_1 \bowtie_L P_2 \xrightarrow{\alpha} Q_1 \bowtie_L P_2} \quad \frac{P_2 \xrightarrow{\alpha} Q_2 \quad \boxed{\alpha \notin L}}{P_1 \bowtie_L P_2 \xrightarrow{\alpha} P_1 \bowtie_L Q_2}$$

pure interleaving

$$P \parallel Q \triangleq P \bowtie_{\emptyset} Q$$

PEPA

syntax and semantics

PEPA syntax

P, Q	$::=$	nil	inactive process
		$(\alpha, r).P$	action prefix
		$P + Q$	choice
		$P \boxtimes_L Q$	cooperation combinator
		P/L	hiding
		C	process constant

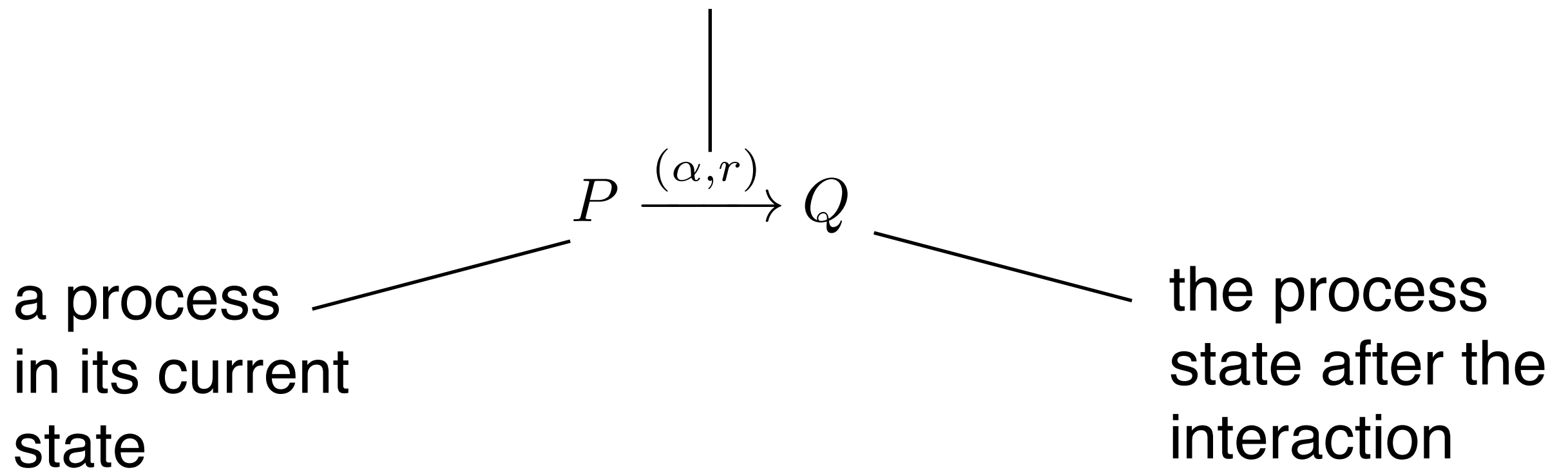
$\alpha \in \Lambda$ action

$L \subseteq \Lambda$ set of actions

$\Delta = \{C_i \triangleq P_i\}_{i \in I}$ set of process declarations

PEPA LTS

ongoing interaction
with the environment
(with other processes)
and its rate



small-step semantics

PEPA semantics (basics)

$$\frac{}{(\alpha, r).P \xrightarrow{(\alpha, r)} P}$$

$$\frac{P_1 \xrightarrow{(\alpha, r)} Q}{P_1 + P_2 \xrightarrow{(\alpha, r)} Q}$$

$$\frac{P_2 \xrightarrow{(\alpha, r)} Q}{P_1 + P_2 \xrightarrow{(\alpha, r)} Q}$$

$$\frac{C \triangleq P \in \Delta \quad P \xrightarrow{(\alpha, r)} Q}{C \xrightarrow{(\alpha, r)} Q}$$

SAME AS CCS!!!

Example

Server $\triangleq$ get . $\overline{\text{download}}$. rel . Server

Browser $\triangleq$ display . cache . Browser
+ display . $\overline{\text{get}}$. download . $\overline{\text{rel}}$. Browser

Example

Server $\triangleq (get, \top).(download, \mu).(rel, \top).Server$

extremely high rate
cannot influence the overall rate
of interacting components

Browser $\triangleq (display, \lambda_1).(cache, m).Browser$
 $+$ $(display, \lambda_2).(get, g).(download, \top).(rel, r).Browser$

a local choice

taken with probability $\frac{\lambda_i}{\lambda_1 + \lambda_2}$

Hiding and interleaving

$$\frac{P \xrightarrow{(\alpha, r)} Q \quad \boxed{\alpha \notin L}}{P/L \xrightarrow{(\alpha, r)} Q/L}$$

$$\frac{P \xrightarrow{(\alpha, r)} Q \quad \boxed{\alpha \in L}}{P/L \xrightarrow{(\tau, r)} Q/L}$$

$$\frac{P_1 \xrightarrow{(\alpha, r)} Q_1 \quad \boxed{\alpha \notin L}}{P_1 \boxtimes_L P_2 \xrightarrow{(\alpha, r)} Q_1 \boxtimes_L P_2}$$

$$\frac{P_2 \xrightarrow{(\alpha, r)} Q_2 \quad \boxed{\alpha \notin L}}{P_1 \boxtimes_L P_2 \xrightarrow{(\alpha, r)} P_1 \boxtimes_L Q_2}$$

Cooperation

$$\frac{P_1 \xrightarrow{(\alpha, r_1)} Q_1 \quad P_2 \xrightarrow{(\alpha, r_2)} Q_2 \quad \boxed{\alpha \in L}}{P_1 \boxtimes_L P_2 \xrightarrow{(\alpha, r)} Q_1 \boxtimes_L Q_2}$$

which rate should we put here?

Multiway synchronization

$$\begin{aligned}
 F &\stackrel{\text{def}}{=} (\text{fork}, r_f).(\text{join}, r_j).F' \\
 W_1 &\stackrel{\text{def}}{=} (\text{fork}, r_{f_1}).(\text{doWork}_1, r_1).W'_1 \\
 W_2 &\stackrel{\text{def}}{=} (\text{fork}, r_{f_2}).(\text{doWork}_2, r_2).W'_2 \\
 F' &\stackrel{\text{def}}{=} \dots, \quad W'_1 \stackrel{\text{def}}{=} \dots, \quad W'_2 \stackrel{\text{def}}{=} \dots \\
 \text{System} &\stackrel{\text{def}}{=} (F \boxtimes_{\{\text{fork}\}} W_1) \boxtimes_{\{\text{fork}\}} W_2
 \end{aligned}$$

$$\frac{F \xrightarrow{(\text{fork}, r_f)} (\text{join}, r_j)F' \quad W_1 \xrightarrow{(\text{fork}, r_{f_1})} (\text{doWork}_1, r_1).W'_1}{F \boxtimes_{\{\text{fork}\}} W_1 \xrightarrow{(\text{fork}, r')} (\text{join}, r_j).F' \boxtimes_{\{\text{fork}\}} (\text{doWork}_1, r_1).W'_1}$$



$$\frac{F \boxtimes_{\{\text{fork}\}} W_1 \boxtimes_{\{\text{fork}\}} W_2 \xrightarrow{(\text{fork}, r'')} (\text{join}, r_j).F' \boxtimes_{\{\text{fork}\}} (\text{doWork}_1, r_1)W'_1 \boxtimes_{\{\text{fork}\}} (\text{doWork}_2, r_2).W'_2}{F \boxtimes_{\{\text{fork}\}} W_1 \boxtimes_{\{\text{fork}\}} W_2 \xrightarrow{(\text{fork}, r'')} (\text{join}, r_j).F' \boxtimes_{\{\text{fork}\}} (\text{doWork}_1, r_1)W'_1 \boxtimes_{\{\text{fork}\}} (\text{doWork}_2, r_2).W'_2}$$

(taken from Mirco Tribastone's slides)

Exclusive cooperation

$$Premium \stackrel{def}{=} (dwn, r_p).Premium'$$

$$Basic \stackrel{def}{=} (dwn, r_b).Basic'$$

$$S \stackrel{def}{=} (dwn, r_s).S'$$

...

$$System \stackrel{def}{=} (Premium \parallel Basic) \bowtie_L S,$$

$$L = \{dwn\}$$

$$Premium \xrightarrow{(dwn, r_p)} Premium'$$

$$\frac{}{Premium \parallel Basic \xrightarrow{(dwn, r_p)} Premium' \parallel Basic} \quad \frac{}{S \xrightarrow{(dwn, r_s)} S'}$$

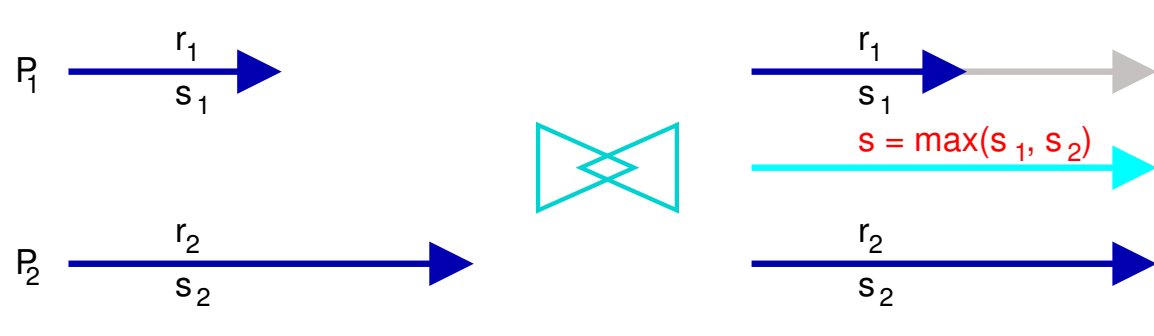
$$Premium \parallel Basic \bowtie_L S \xrightarrow{(dwn, r_{ps})} Premium' \parallel Basic \bowtie_L S'$$

$$System \xrightarrow{(dwn, r_{ps})} Premium' \parallel Basic \bowtie_L S'$$

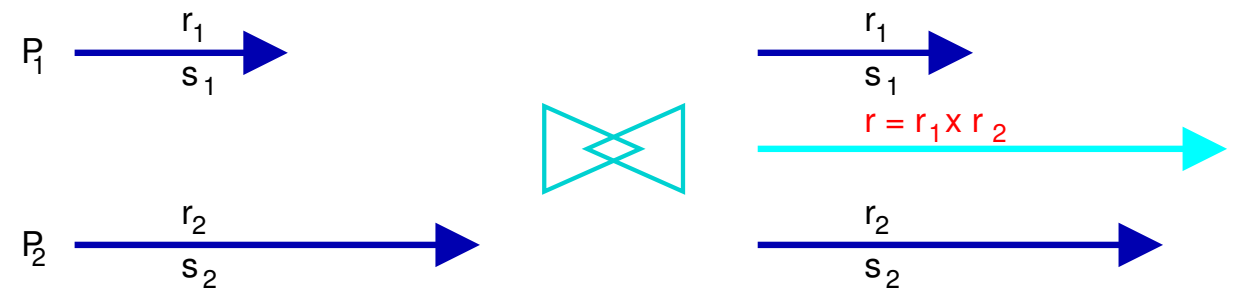
(taken from Mirco Tribastone's slides)

Which rate for sync?

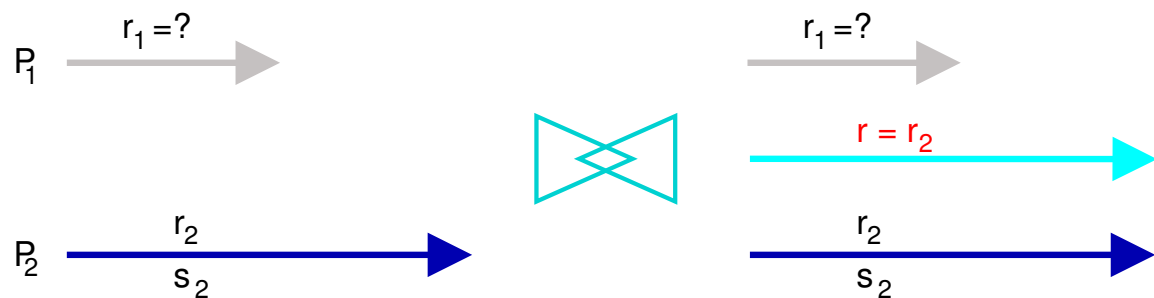
stochastic PA differ for the treatment of rates of synchronised actions



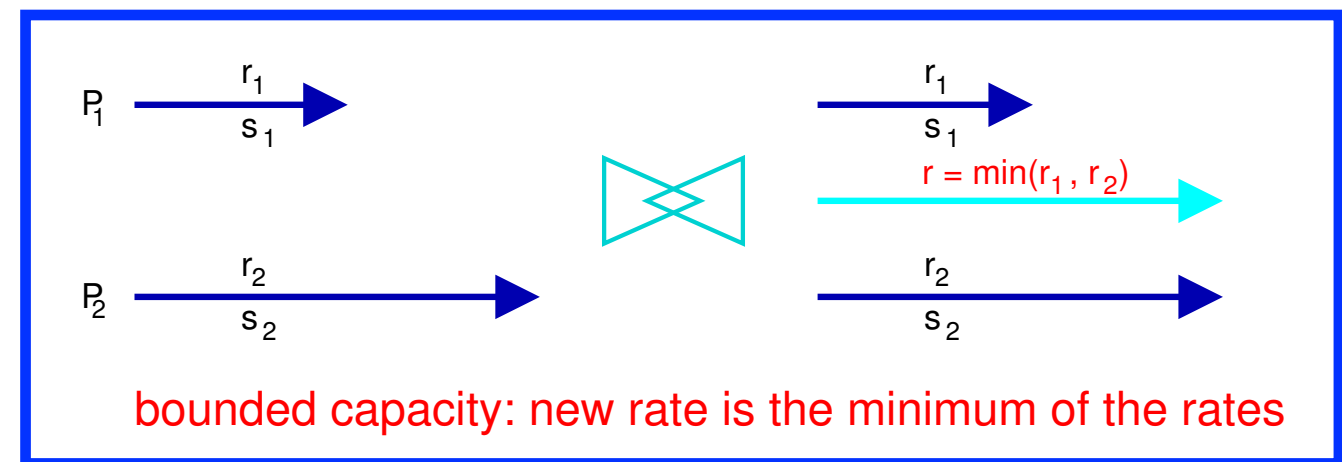
s is no longer exponentially distributed



TIPP: new rate is product of individual rates



EMPA: one participant is passive



bounded capacity: new rate is the minimum of the rates

PEPA's approach

(taken from Jane Hillston's slides)

PEPA: bounded capacity

Each component has a bounded capacity to carry out activities of some type, determined by the apparent rate for that type

cooperation cannot make a component exceed its bounded capacity

thus the apparent rate of a cooperation is the minimum of the apparent rates of the co-operands

PEPA: apparent rates

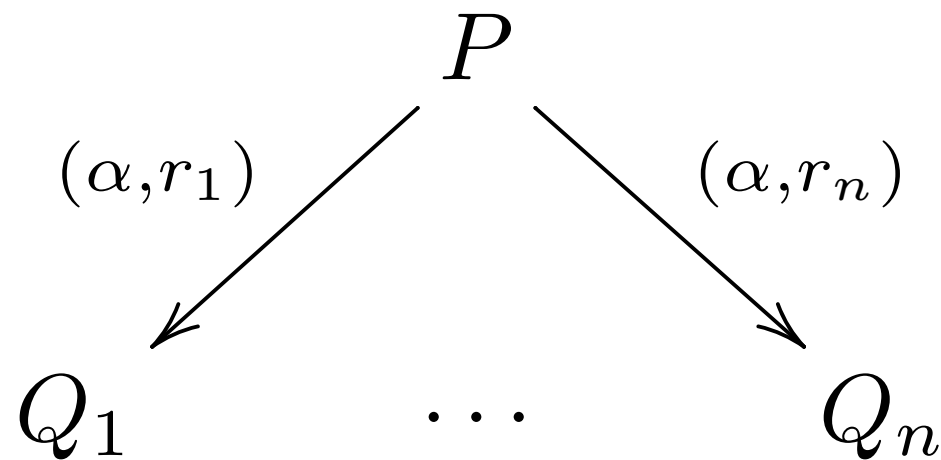
No component can be made to carry out an action in cooperation faster than its own defined rate for the actions

thus shared actions proceed at the minimum of the rates in the participating components

the apparent rates of independent actions is instead the sum of their rates within independent concurrent components

PEPA: apparent rate

$r_\alpha(P)$ is the observed rate of action α in P



$$r_\alpha(P) = r_1 + \dots + r_n$$

Properties: min

(X_1, λ_1) (X_2, λ_2) (independent, exponentially distributed)

$X(\omega) = \min\{X_1(\omega), X_2(\omega)\}$ is exponentially distributed

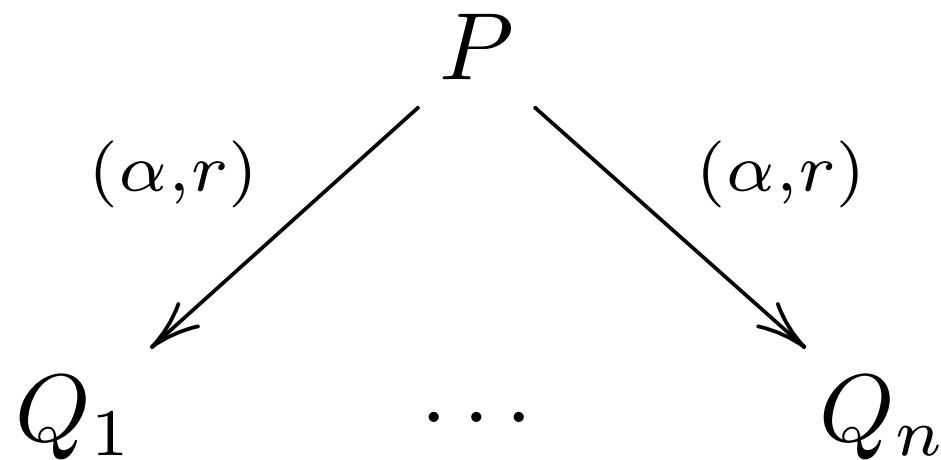
$$(X, \lambda_1 + \lambda_2)$$

$$P(X \leq x) \triangleq 1 - e^{-(\lambda_1 + \lambda_2)x}$$

(reminder, from CTMC slides)

PEPA: apparent rate

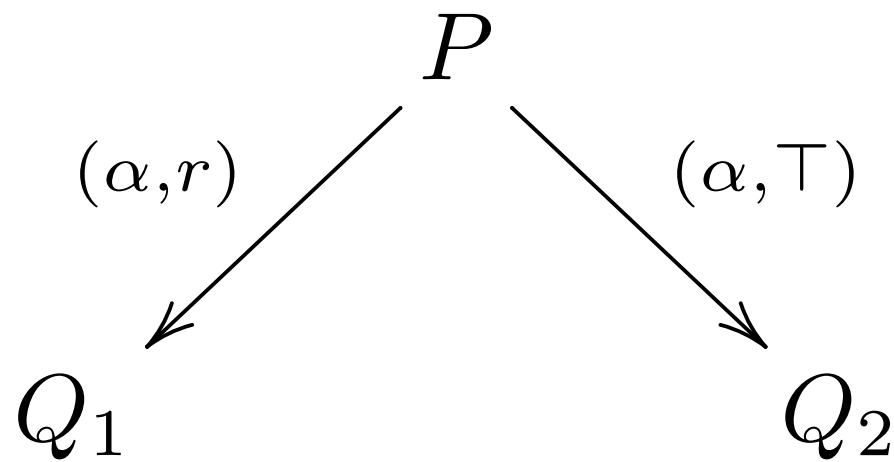
$r_\alpha(P)$ is the observed rate of action α in P



$$r_\alpha(P) = n \cdot r$$

PEPA: apparent rate

$r_\alpha(P)$ is the observed rate of action α in P



NOT ALLOWED!

PEPA: apparent rate

$r_\alpha(P)$ is the observed rate of action α in P

$$r_\alpha(\mathbf{nil}) \triangleq 0$$

$$r_\alpha((\beta, r).P) \triangleq \begin{cases} r & \text{if } \alpha = \beta \\ 0 & \text{if } \alpha \neq \beta \end{cases}$$

$$r_\alpha(P + Q) \triangleq r_\alpha(P) + r_\alpha(Q) \quad (+ \text{ is not idempotent!})$$

$$r_\alpha(P/L) \triangleq \begin{cases} r_\alpha(P) & \text{if } \alpha \notin L \\ 0 & \text{if } \alpha \in L \end{cases}$$

$$r_\alpha(P \boxtimes_L Q) \triangleq \begin{cases} r_\alpha(P) + r_\alpha(Q) & \text{if } \alpha \notin L \\ \min \{r_\alpha(P), r_\alpha(Q)\} & \text{if } \alpha \in L \end{cases}$$

$$r_\alpha(C) \triangleq r_\alpha(P) \quad \text{if } C \triangleq P \in \Delta$$

actions are
interleaved

the slowest must
be waited for

Cooperation

$$\frac{P_1 \xrightarrow{(\alpha, r_1)} Q_1 \quad P_2 \xrightarrow{(\alpha, r_2)} Q_2 \quad \boxed{\alpha \in L}}{P_1 \boxtimes_L P_2 \xrightarrow{(\alpha, r)} Q_1 \boxtimes_L Q_2}$$

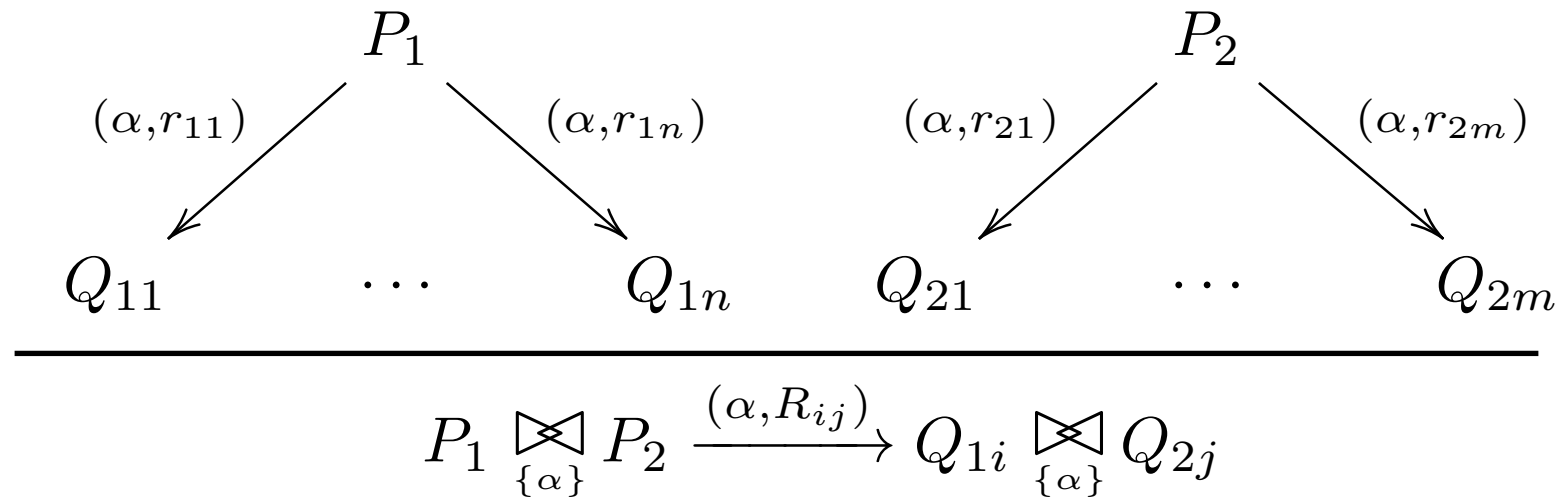
$$r = r_\alpha(P_1 \boxtimes_L P_2) \cdot \frac{r_1}{r_\alpha(P_1)} \cdot \frac{r_2}{r_\alpha(P_2)}$$

apparent rate

probability of specific action (α, r_i)
among the α -transitions of P_i

the sum of the rates of all the
 α -transitions that $P_1 \boxtimes_L P_2$ can do

Cooperation: example



$$R_{ij} = r_{\alpha}(P_1 \boxtimes_L P_2) \cdot \frac{r_{1i}}{r_{\alpha}(P_1)} \cdot \frac{r_{2j}}{r_{\alpha}(P_2)}$$

$$= \min\left\{\sum_k r_{1k}, \sum_k r_{2k}\right\} \cdot \frac{r_{1i}}{\sum_k r_{1k}} \cdot \frac{r_{2j}}{\sum_k r_{2k}}$$

Cooperation: example

For r_1, r_2 positive reals,

$$\frac{(\alpha, r_1).P_1 \xrightarrow{(\alpha, r_1)} P_1 \quad (\alpha, r_2).P_2 \xrightarrow{(\alpha, r_2)} P_2}{(\alpha, r_1).P_1 \boxtimes_{\{\alpha\}} (\alpha, r_2).P_2 \xrightarrow{(\alpha, R)} P_1 \boxtimes_{\{\alpha\}} P_2},$$

where

$$\begin{aligned} R &= \frac{r_1}{r_\alpha((\alpha, r_1).P_1)} \frac{r_2}{r_\alpha((\alpha, r_2).P_2)} \min \left(r_\alpha((\alpha, r_1).P_1), r_\alpha((\alpha, r_2).P_2) \right) \\ &= \frac{r_1}{r_1} \frac{r_2}{r_2} \min(r_1, r_2) = \min(r_1, r_2). \end{aligned}$$

We recover the intuitive definition of the minimum between the two rates.

(taken from Mirco Tribastone's slides)

Cooperation: example

For r a positive real,

$$\frac{(\alpha, r).P_1 \xrightarrow{(\alpha, r)} P_1 \quad (\alpha, \top).P_2 \xrightarrow{(\alpha, \top)} P_2}{(\alpha, r).P_1 \boxtimes_{\{\alpha\}} (\alpha, \top).P_2 \xrightarrow{(\alpha, R)} P_1 \boxtimes_{\{\alpha\}} P_2},$$

where

$$\begin{aligned} R &= \frac{r}{r_\alpha((\alpha, r).P_1)} \frac{\top}{r_\alpha((\alpha, \top).P_2)} \min \left(r_\alpha((\alpha, r).P_1), r_\alpha((\alpha, \top).P_2) \right) \\ &= \frac{r}{r} \frac{\top}{\top} \min(r, \top) = r. \end{aligned}$$

We recover the intuitive definition of infinite capacity — the rate of synchronisation is determined by the active component.

(taken from Mirco Tribastone's slides)

Apparent rates in active cooperation

$$Cli \stackrel{def}{=} (\alpha, r_d).Cli'$$

$$Ser \stackrel{def}{=} (\alpha, r_u).Ser'$$

$$Sys \stackrel{def}{=} (Cli \parallel Cli) \boxtimes_{\{\alpha\}} Ser$$

one server,
two clients

$$\frac{\frac{\frac{}{(\alpha, r_d).Cli' \xrightarrow{(\alpha, r_d)} Cli'}}{Cli \xrightarrow{(\alpha, r_d)} Cli'}}{Cli \parallel Cli \xrightarrow{(\alpha, r_d)} Cli' \parallel Cli} \quad \frac{\frac{}{(\alpha, r_u).Ser' \xrightarrow{(\alpha, r_u)} Ser'}}{Ser \xrightarrow{(\alpha, r_u)} Ser'}$$

$$Cli \parallel Cli \boxtimes_{\{\alpha\}} Ser \xrightarrow{(\alpha, R')} Cli' \parallel Cli \boxtimes_{\{\alpha\}} Ser',$$

$$R' = \frac{r_d}{r_d + r_d} \frac{r_u}{r_u} \min(r_d + r_d, r_u) = \frac{1}{2} \min(r_d + r_d, r_u)$$

(taken from Mirco Tribastone's slides)

Apparent rates in active cooperation

$$Cli \stackrel{def}{=} (\alpha, r_d).Cli'$$

$$Ser \stackrel{def}{=} (\alpha, r_u).Ser'$$

$$Sys \stackrel{def}{=} (Cli \parallel Cli) \boxtimes_{\{\alpha\}} Ser$$

one server,
two clients

$$\frac{\frac{\overline{(\alpha, r_d).Cli' \xrightarrow{(\alpha, r_d)} Cli'}}{Cli \xrightarrow{(\alpha, r_d)} Cli'} \quad \frac{(\alpha, r_u).Ser' \xrightarrow{(\alpha, r_u)} Ser'}{Ser \xrightarrow{(\alpha, r_u)} Ser'}}$$

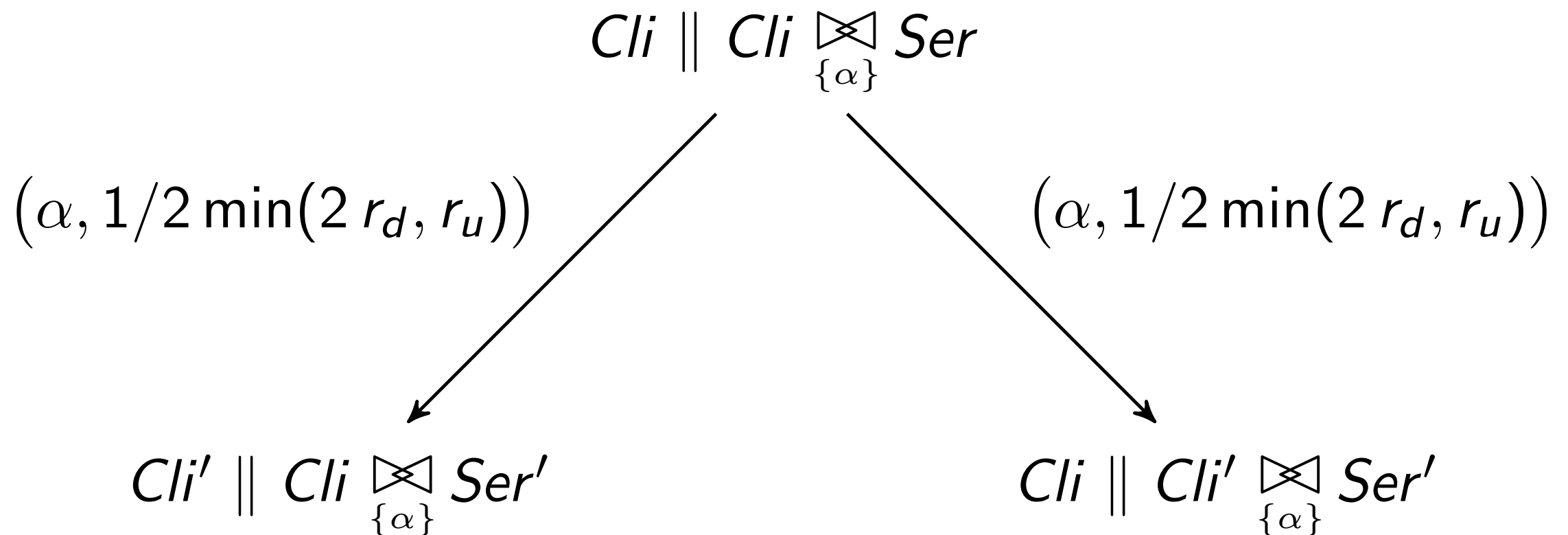
$$\frac{Cli \parallel Cli \xrightarrow{(\alpha, r_d)} Cli \parallel Cli' \quad Ser \xrightarrow{(\alpha, r_u)} Ser'}{Cli \parallel Cli \boxtimes_{\{\alpha\}} Ser \xrightarrow{(\alpha, R'')} Cli \parallel Cli' \boxtimes_{\{\alpha\}} Ser'},$$

$$R'' = \frac{r_d}{r_d + r_d} \frac{r_u}{r_u} \min(r_d + r_d, r_u) = \frac{1}{2} \min(r_d + r_d, r_u) = R'$$

(taken from Mirco Tribastone's slides)

Apparent rates in active cooperation

$$\begin{aligned}
 Cli &\stackrel{def}{=} (\alpha, r_d).Cli' \\
 Ser &\stackrel{def}{=} (\alpha, r_u).Ser' \\
 Sys &\stackrel{def}{=} (Cli \parallel Cli) \boxtimes_{\{\alpha\}} Ser
 \end{aligned}$$



(taken from Mirco Tribastone's slides)

Careful with that cooperation set

apparent rate of α : $\min \{ r , s , t \}$

- $\left((\alpha, r).P \underset{\{\alpha\}}{\bowtie} (\alpha, s).Q \right) \underset{\{\alpha\}}{\bowtie} (\alpha, t).R$

apparent rate of α : $\min \{ r+s , t \}$

- $\left((\alpha, r).P \parallel (\alpha, s).Q \right) \underset{\{\alpha\}}{\bowtie} (\alpha, t).R$

apparent rate of α : $\min \{ r , s \} + t$

- $\left((\alpha, r).P \underset{\{\alpha\}}{\bowtie} (\alpha, s).Q \right) \parallel (\alpha, t).R$

(taken from Jane Hillston's slides)

Example

Server $\triangleq (get, \top).(download, \mu).(rel, \top).Server$

S $\triangleq (get, \top).S1$

S1 $\triangleq (dnd, \mu).S2$

S2 $\triangleq (rel, \top).S$

Browser $\triangleq (display, \lambda_1).(cache, m).Browser$
 $+ (display, \lambda_2).(get, g).(download, \top).(rel, r).Browser$

B $\triangleq (dis, \lambda_1).B1 + (dis, \lambda_2).B2$

B1 $\triangleq (cac, m).B$

B2 $\triangleq (get, g).B3$

B3 $\triangleq (dnd, \top).B4$

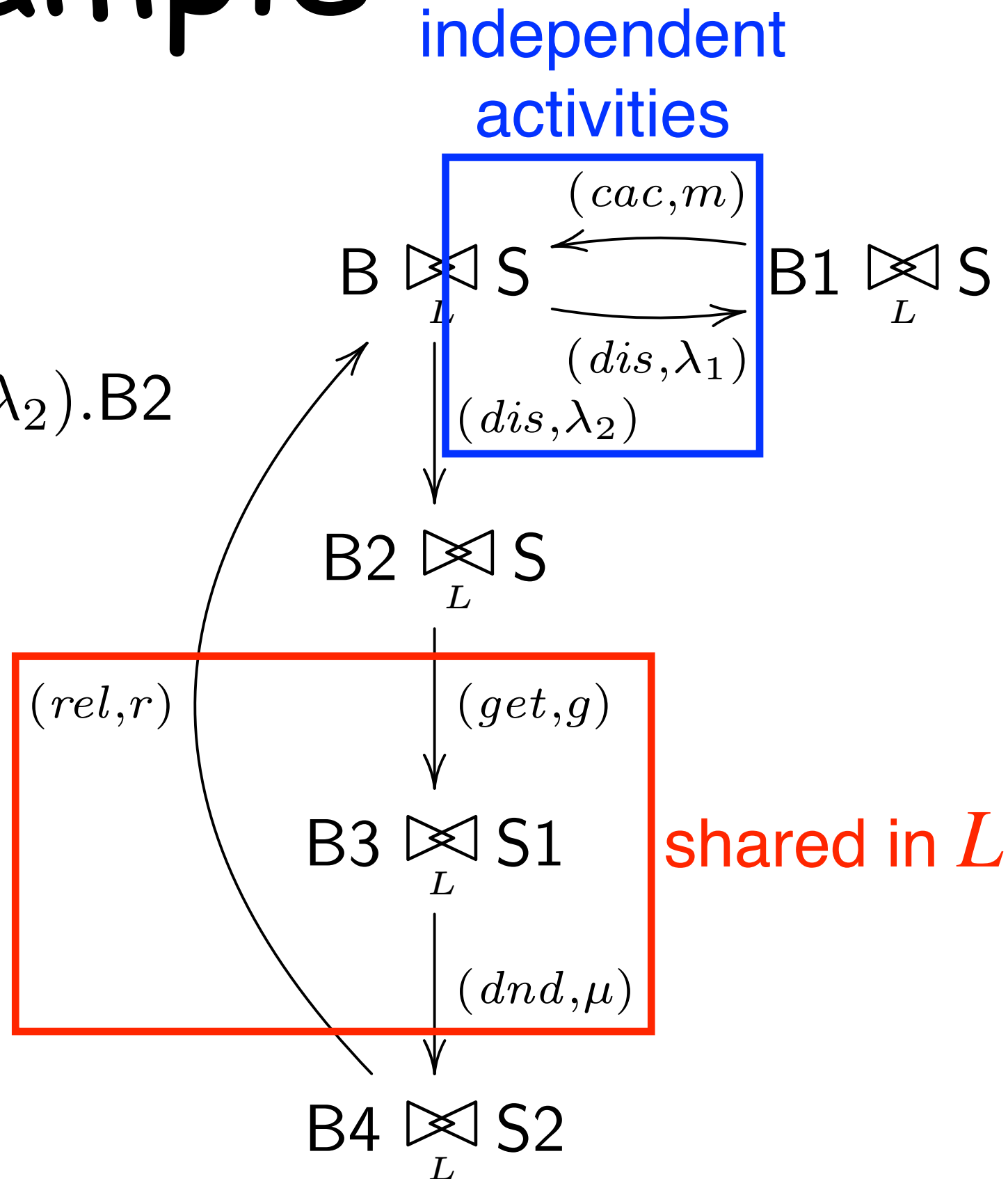
B4 $\triangleq (rel, r).B$

Example

$S \triangleq (get, \top).S1$
 $S1 \triangleq (dnd, \mu).S2$
 $S2 \triangleq (rel, \top).S$
 $B \triangleq (dis, \lambda_1).B1 + (dis, \lambda_2).B2$
 $B1 \triangleq (cac, m).B$
 $B2 \triangleq (get, g).B3$
 $B3 \triangleq (dnd, \top).B4$
 $B4 \triangleq (rel, r).B$

$L = \{get, dnd, rel\}$

$B \bowtie_L S$



Example

$$\begin{array}{ll}
 S & \triangleq (get, \top).S1 \\
 S1 & \triangleq (dnd, \mu).S2 \\
 S2 & \triangleq (rel, \top).S \\
 L & = \{get, dnd, rel\}
 \end{array}
 \qquad
 \begin{array}{ll}
 B & \triangleq (dis, \lambda_1).B1 + (dis, \lambda_2).B2 \\
 B1 & \triangleq (cac, m).B \\
 B2 & \triangleq (get, g).B3 \\
 B3 & \triangleq (dnd, \top).B4 \\
 B4 & \triangleq (rel, r).B
 \end{array}$$

$$(B \parallel B) \bowtie_L S$$

$$(B \parallel B) \bowtie_L S \xrightarrow{(dis, \lambda_2)} (B2 \parallel B) \bowtie_L S \xrightarrow{(dis, \lambda_2)} (B2 \parallel B2) \bowtie_L S$$

$$(B2 \parallel B2) \bowtie_L S \xrightarrow{(get, g)} (B3 \parallel B2) \bowtie_L S1$$

$$\begin{array}{c}
 (get, g) \\
 \downarrow
 \end{array}$$

$$(B2 \parallel B3) \bowtie_L S1$$

$$r_{get}(B2) = g$$

$$r_{get}(B2 \parallel B2) = 2g$$

$$r_{get}(S) = \top$$

$$r_{get}((B2 \parallel B2) \bowtie_L S) = 2g$$

Consumer/producer

Possible variants:

- A buffer with n places:

$$Cons_1 \stackrel{def}{=} (get, r_g).Cons_2$$

$$Cons_2 \stackrel{def}{=} (cons, r_c).Cons_1$$

$$Prod_1 \stackrel{def}{=} (make, r_m).Prod_2$$

$$Prod_2 \stackrel{def}{=} (put, r_p).Prod_1$$

$$Buf_2 \stackrel{def}{=} (get, \top).Buf_1$$

$$Buf_1 \stackrel{def}{=} (get, \top).Buf_0 \\ + (put, \top).Buf_2$$

$$Buf_0 \stackrel{def}{=} (put, \top).Buf_1$$

$$Sys \stackrel{def}{=} Cons_1 \begin{array}{c} \diagup \diagdown \\ \{get\} \end{array} Buf_2 \begin{array}{c} \diagdown \diagup \\ \{put\} \end{array} Prod_1$$

$$Buf_n \stackrel{def}{=} (get, \top).Buf_{n-1}$$

$$Buf_i \stackrel{def}{=} (get, \top).Buf_{i-1} \\ + (put, \top).Buf_{i+1}, \\ \text{for } 1 \leq i \leq n-1$$

$$Buf_0 \stackrel{def}{=} (put, \top).Buf_1$$

- and k consumers:

$$\overbrace{Cons_1 \parallel Cons_1 \parallel \dots \parallel Cons_1}^k \\ \begin{array}{c} \diagup \diagdown \\ \{get\} \end{array} Buf_n \begin{array}{c} \diagdown \diagup \\ \{put\} \end{array} Prod_1$$

(taken from Mirco Tribastone's slides)

Consumer/producer

$Cons_1 \stackrel{def}{=} (get, r_g).Cons_2$	$Prod_1 \stackrel{def}{=} (make, r_m).Prod_2$
$Cons_2 \stackrel{def}{=} (cons, r_c).Cons_1$	$Prod_2 \stackrel{def}{=} (put, r_p).Prod_1$
$Buf_2 \stackrel{def}{=} (get, \top).Buf_1$	$Buf_1 \stackrel{def}{=} (get, \top).Buf_0 + (put, \top).Buf_2$
$Buf_0 \stackrel{def}{=} (put, \top).Buf_1$	$Sys \stackrel{def}{=} Cons_1 \bowtie_{\{get\}} Buf_2 \bowtie_{\{put\}} Prod_1$

$$\begin{array}{c}
 \frac{Cons_1 \xrightarrow{(get, r_g)} Cons_2 \quad Buf_2 \xrightarrow{(get, \top)} Buf_1}{Cons_1 \bowtie_{\{get\}} Buf_2 \xrightarrow{(get, r_g)} Cons_2 \bowtie_{\{get\}} Buf_1} \\
 \hline
 Cons_1 \bowtie_{\{get\}} Buf_2 \bowtie_{\{put\}} Prod_1 \xrightarrow{(get, r_g)} Cons_2 \bowtie_{\{get\}} Buf_1 \bowtie_{\{put\}} Prod_1 \\
 \\
 Sys \xrightarrow{(get, r_g)} Cons_2 \bowtie_{\{get\}} Buf_1 \bowtie_{\{put\}} Prod_1
 \end{array}$$

(taken from Mirco Tribastone's slides)

Consumer/producer

$Cons_1 \stackrel{def}{=} (get, r_g).Cons_2$	$Prod_1 \stackrel{def}{=} (make, r_m).Prod_2$
$Cons_2 \stackrel{def}{=} (cons, r_c).Cons_1$	$Prod_2 \stackrel{def}{=} (put, r_p).Prod_1$
$Buf_2 \stackrel{def}{=} (get, \top).Buf_1$	$Buf_1 \stackrel{def}{=} (get, \top).Buf_0 + (put, \top).Buf_2$
$Buf_0 \stackrel{def}{=} (put, \top).Buf_1$	$Sys \stackrel{def}{=} Cons_1 \bowtie_{\{get\}} Buf_2 \bowtie_{\{put\}} Prod_1$

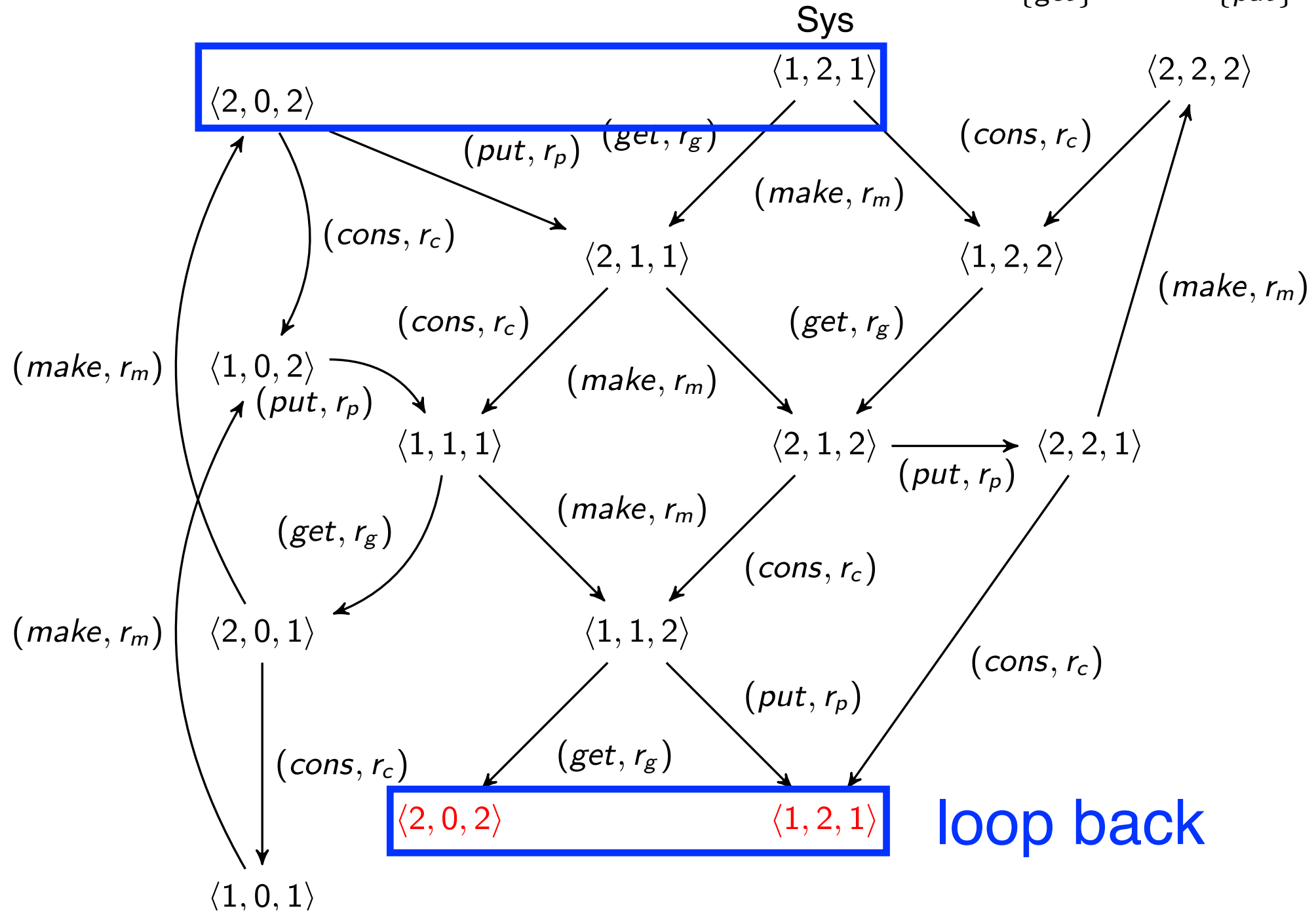
$$\frac{Prod_1 \xrightarrow{(make, r_m)} Prod_2}{Cons_1 \bowtie_{\{get\}} Buf_2 \bowtie_{\{get\}} Prod_1 \xrightarrow{(make, r_m)} Cons_1 \bowtie_{\{get\}} Buf_2 \bowtie_{\{put\}} Prod_2}$$

$$Sys \xrightarrow{(make, r_m)} Cons_1 \bowtie_{\{get\}} Buf_2 \bowtie_{\{put\}} Prod_2$$

(taken from Mirco Tribastone's slides)

Consumer/producer

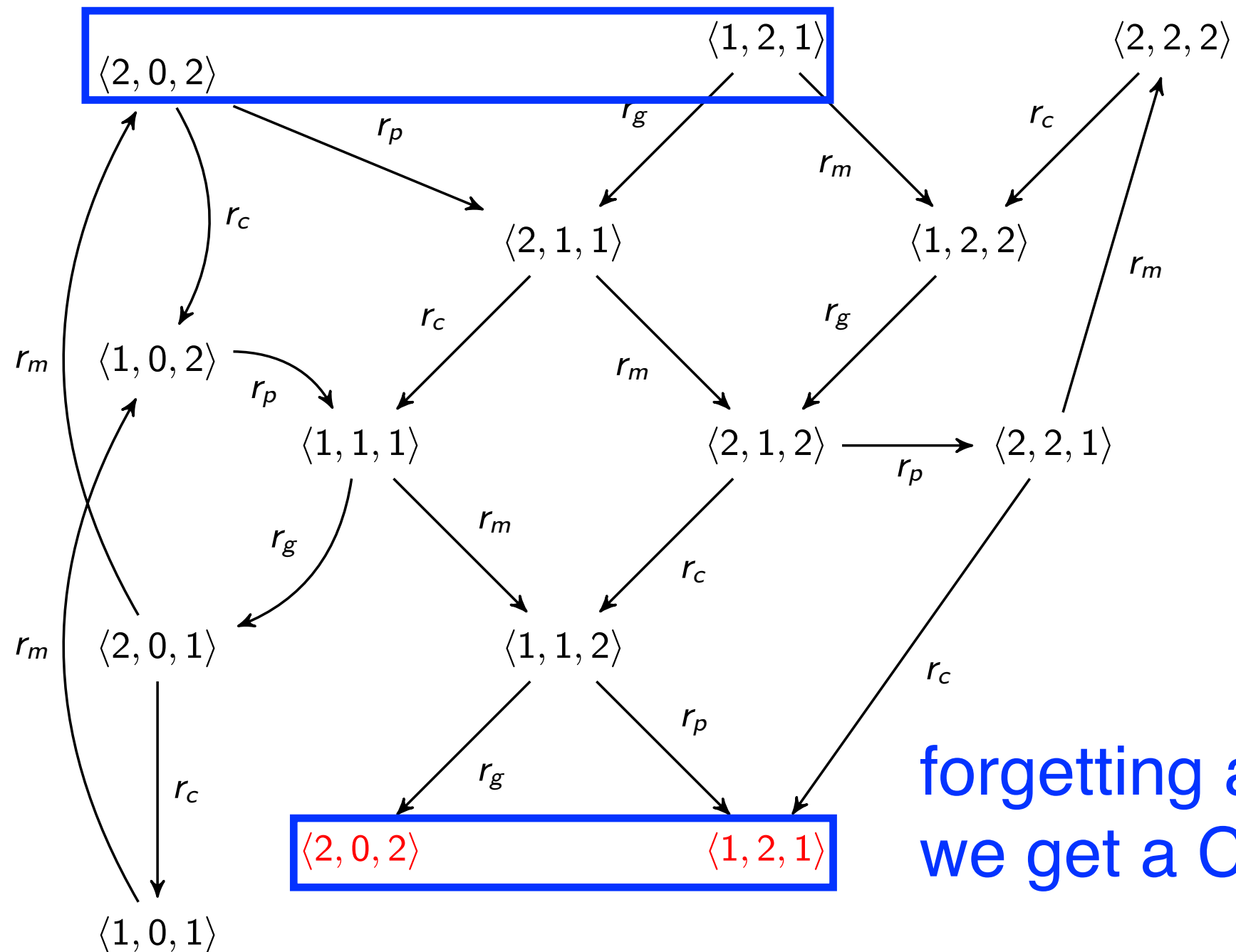
we may denote a state by $\langle i, j, k \rangle$ to indicate $Cons_i \bowtie_{\{get\}} Buf_j \bowtie_{\{put\}} Prod_k$



(taken from Mirco Tribastone's slides)

Consumer/producer

we may denote a state by $\langle i, j, k \rangle$ to indicate $Cons_i \boxtimes_{\{get\}} Buf_j \boxtimes_{\{put\}} Prod_k$



forgetting actions:
we get a CTMC!

(taken from Mirco Tribastone's slides)

Bus maps

A third party app receives requests from users for live bus positioning information. It sends requests to the the Google Map API and the TfE Bus Info API and then aggregates the results to present a map view of the bus data which is returned to the user.

Construct a PEPA model to represent this system.

(taken from Jane Hillston's slides)

Bus maps

A third party app receives requests from users for live bus positioning information. It sends requests to the the Google Map API and the TfE Bus Info API and then aggregates the results to present a map view of the bus data which is returned to the user.

$$\begin{aligned}
 \text{User} &\stackrel{\text{def}}{=} (\text{bus_pos_req}, r).(\text{bus_pos_resp}, \top).\text{User} \\
 \text{Map_finder} &\stackrel{\text{def}}{=} (\text{bus_pos_req}, r).(\text{google_req}, \lambda_1). \\
 &\quad (\text{google_resp}, \top).(\text{bus_pos_resp}, \top).\text{Map_finder} \\
 \text{Bus_finder} &\stackrel{\text{def}}{=} (\text{bus_pos_req}, r).(\text{tfe_req}, \lambda_2). \\
 &\quad (\text{tfe_resp}, \top).(\text{bus_pos_resp}, \top).\text{Bus_finder} \\
 \text{Google} &\stackrel{\text{def}}{=} (\text{google_req}, \top).(\text{google_resp}, \mu_1).\text{Google} \\
 \text{TfE} &\stackrel{\text{def}}{=} (\text{tfe_req}, \top).(\text{tfe_resp}, \mu_2).\text{TfE} \\
 \text{System} &\stackrel{\text{def}}{=} \text{User} \bowtie_L \left(\text{Bus_finder} \bowtie_L \text{Map_finder} \right) \bowtie_K (\text{Google} \parallel \text{TfE})
 \end{aligned}$$

where $L = \{\text{bus_pos_req}, \text{bus_pos_resp}\}$ and

$K = \{\text{google_req}, \text{google_resp}, (\text{tfe_req}, \top).(\text{tfe_resp}, \mu_2)\}$.

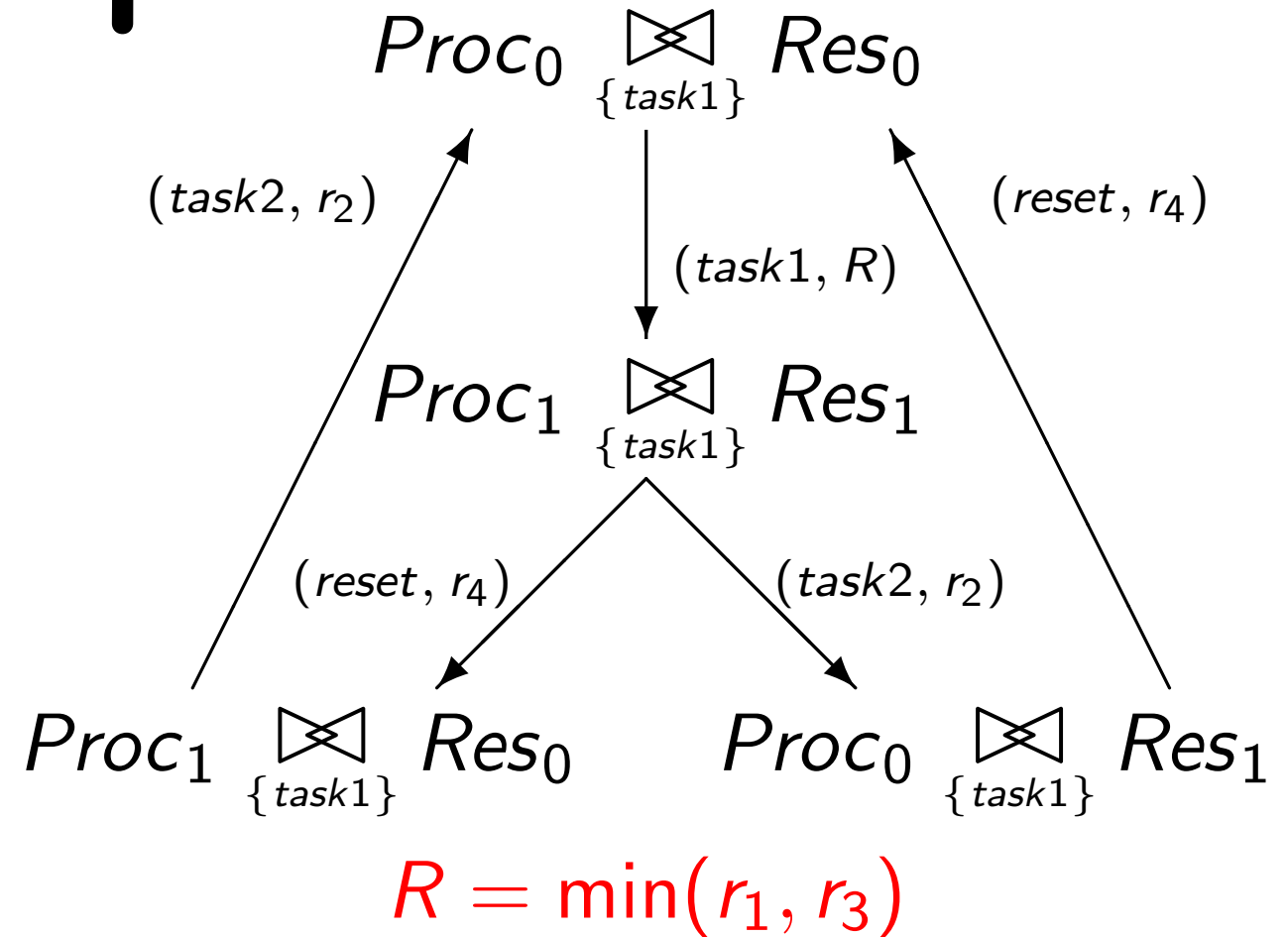
can you
spot the typos?

(taken from Jane Hillston's slides)

Example

$Proc_0 \stackrel{def}{=} (task1, r_1).Proc_1$
 $Proc_1 \stackrel{def}{=} (task2, r_2).Proc_0$
 $Res_0 \stackrel{def}{=} (task1, r_3).Res_1$
 $Res_1 \stackrel{def}{=} (reset, r_4).Res_0$

$Proc_0 \bowtie_{\{task1\}} Res_0$

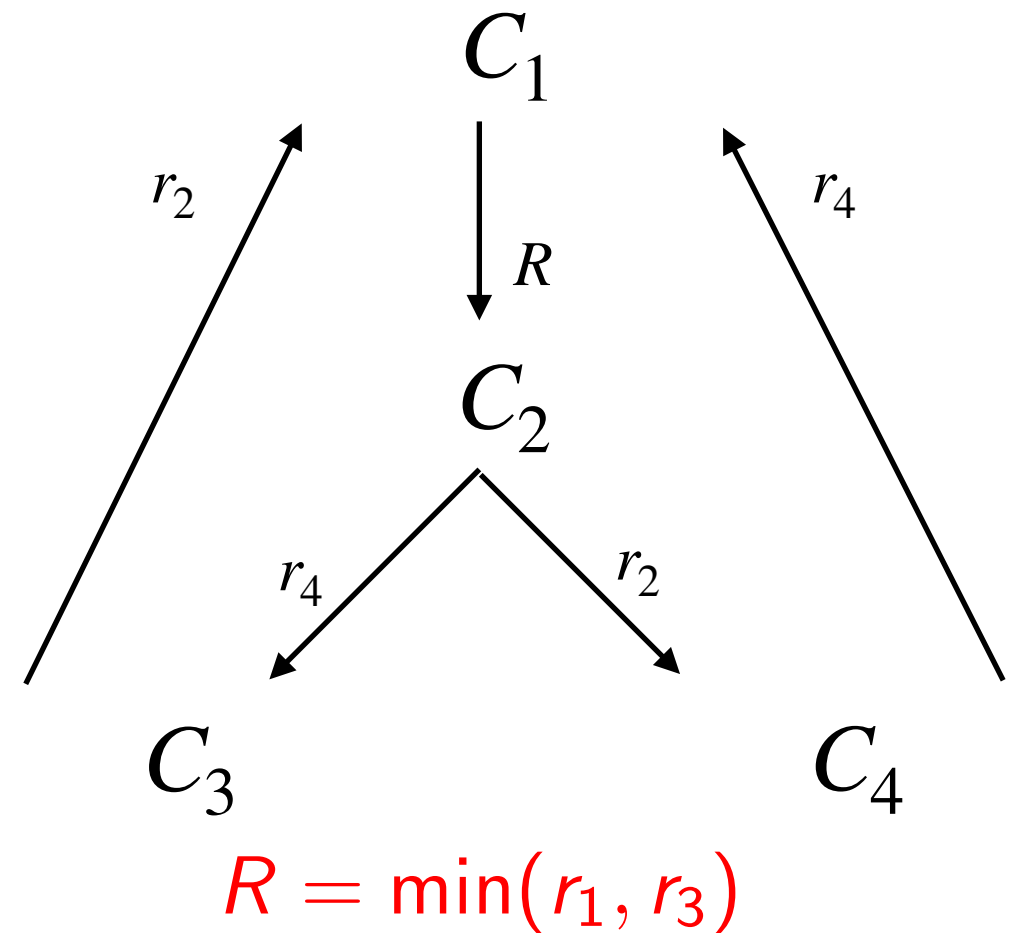


(taken from Jane Hillston's slides)

Example

$$\begin{aligned}
 Proc_0 &\stackrel{def}{=} (task1, r_1).Proc_1 \\
 Proc_1 &\stackrel{def}{=} (task2, r_2).Proc_0 \\
 Res_0 &\stackrel{def}{=} (task1, r_3).Res_1 \\
 Res_1 &\stackrel{def}{=} (reset, r_4).Res_0
 \end{aligned}$$

$$Proc_0 \boxtimes_{\{task1\}} Res_0$$



$$Q = \begin{pmatrix} -R & R & 0 & 0 \\ 0 & -(r_2 + r_4) & r_4 & r_2 \\ r_2 & 0 & -r_2 & 0 \\ r_4 & 0 & 0 & -r_4 \end{pmatrix} \quad \begin{cases} p \cdot Q = 0 \\ \sum_{i=1}^N p_i = 1 \end{cases}$$

(taken from Jane Hillston's slides)

Example

$$\mathbf{Q} = \begin{pmatrix} -R & R & 0 & 0 \\ 0 & -(r_2 + r_4) & r_4 & r_2 \\ r_2 & 0 & -r_2 & 0 \\ r_4 & 0 & 0 & -r_4 \end{pmatrix} \quad \begin{cases} p \cdot Q = 0 \\ \sum_{i=1}^N p_i = 1 \end{cases}$$

$$r_1 = 2 \quad r_2 = 2 \quad r_3 = 6 \quad r_4 = 8 \quad R = \min\{r_1, r_3\} = 2$$

$$p_1 = \frac{20}{41} \quad p_2 = \frac{4}{41} \quad p_3 = \frac{1}{41} \quad p_4 = \frac{16}{41}$$

Reward structure

$\mathcal{C}$ a set of PEPA components
 $\{C_1, \dots, C_n\}$

$\rho : \mathcal{C} \rightarrow \mathbb{R}$ a reward structure

p a steady state distribution

$$R_\rho \triangleq \sum_i p_i \cdot \rho(C_i)$$

sometimes rewards are defined in terms of activities

$$\rho : L \rightarrow \mathbb{R}$$

$$\rho(C) = \sum_{C \xrightarrow{(\alpha, r)} Q} \rho(\alpha)$$

Example: throughput

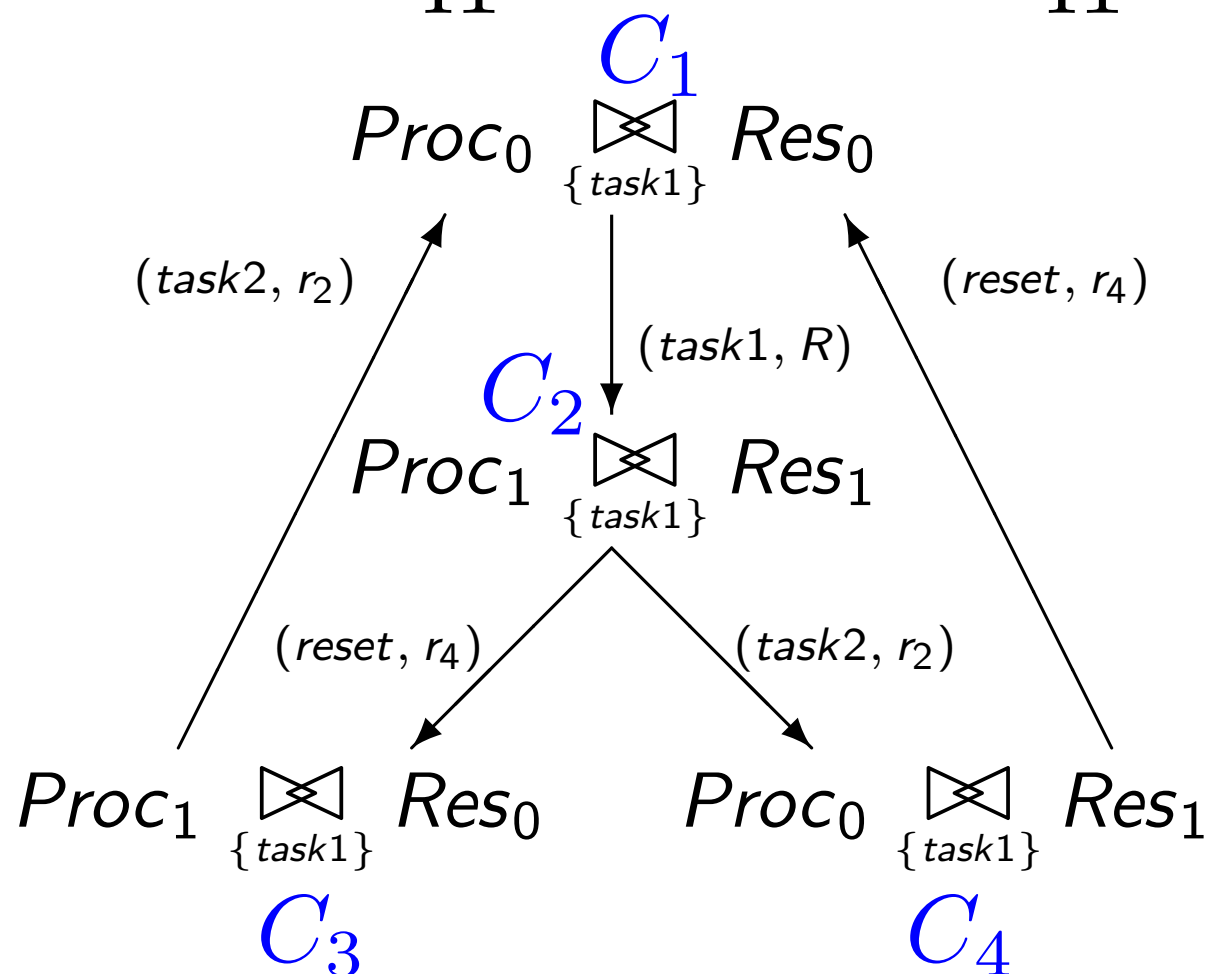
$$Q = \begin{pmatrix} -R & R & 0 & 0 \\ 0 & -(r_2 + r_4) & r_4 & r_2 \\ r_2 & 0 & -r_2 & 0 \\ r_4 & 0 & 0 & -r_4 \end{pmatrix} \quad \begin{cases} p \cdot Q = 0 \\ \sum_{i=1}^N p_i = 1 \end{cases}$$

$$p_1 = \frac{20}{41}$$

$$p_2 = \frac{4}{41}$$

$$p_3 = \frac{1}{41}$$

$$p_4 = \frac{16}{41}$$



$$\rho(task_i) = 1 \quad \rho(reset) = 0$$

$$\rho(C_1) = \rho(C_2) = \rho(C_3) = 1$$

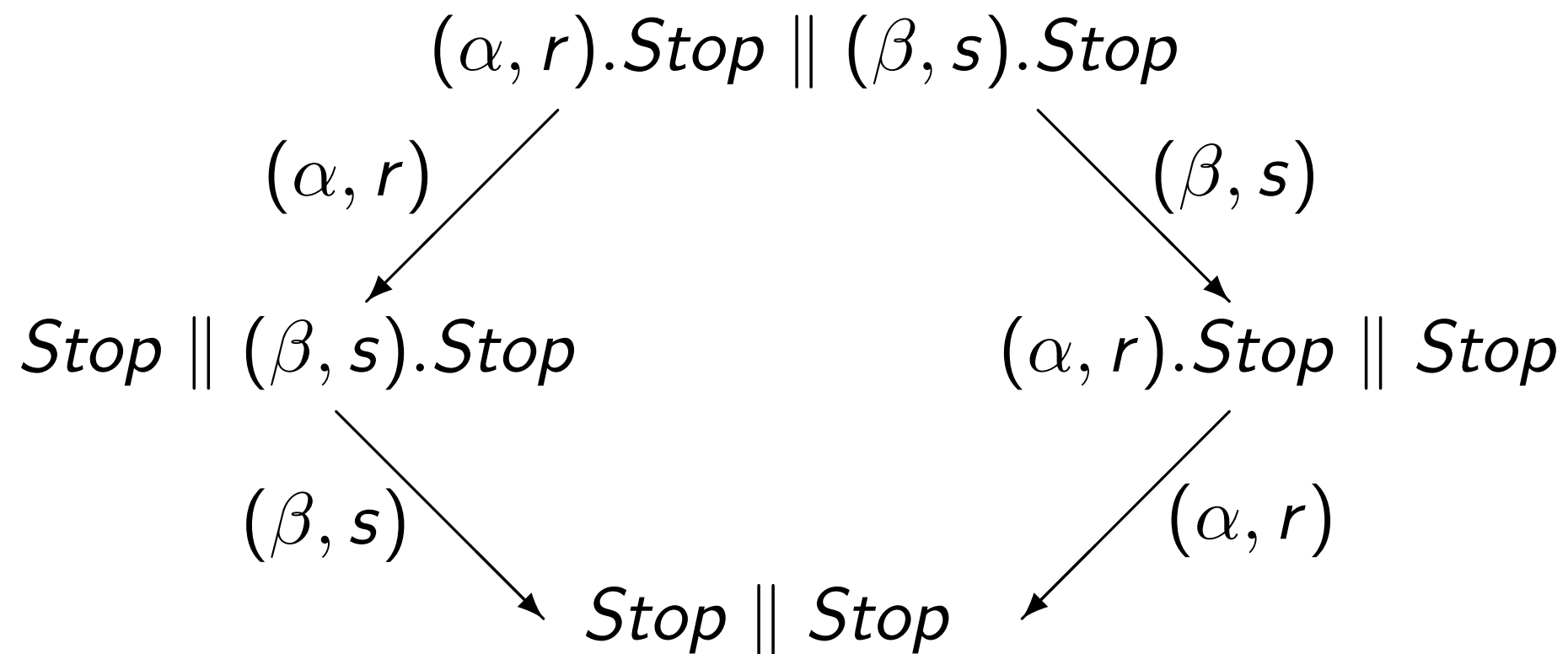
$$\rho(C_4) = 0$$

$$R = \frac{20 + 4 + 1}{41} = \frac{25}{41} = 61\%$$

PEPA

further considerations

The importance of being Exp



We retain the **expansion law** of classical process algebra:

$$\begin{aligned}
 (\alpha, r).Stop \parallel (\beta, s).Stop = \\
 (\alpha, r).(\beta, s).(Stop \parallel Stop) + (\beta, s).(\alpha, r).(Stop \parallel Stop)
 \end{aligned}$$

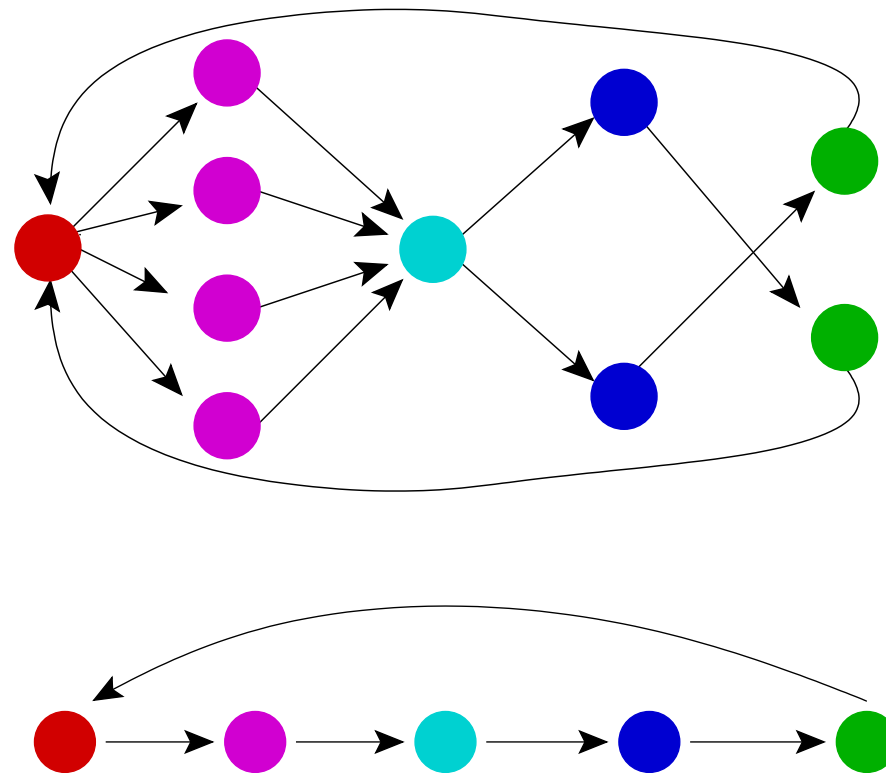
only if the **negative exponential distribution** is assumed.

(taken from Jane Hillston's slides)

Model aggregation

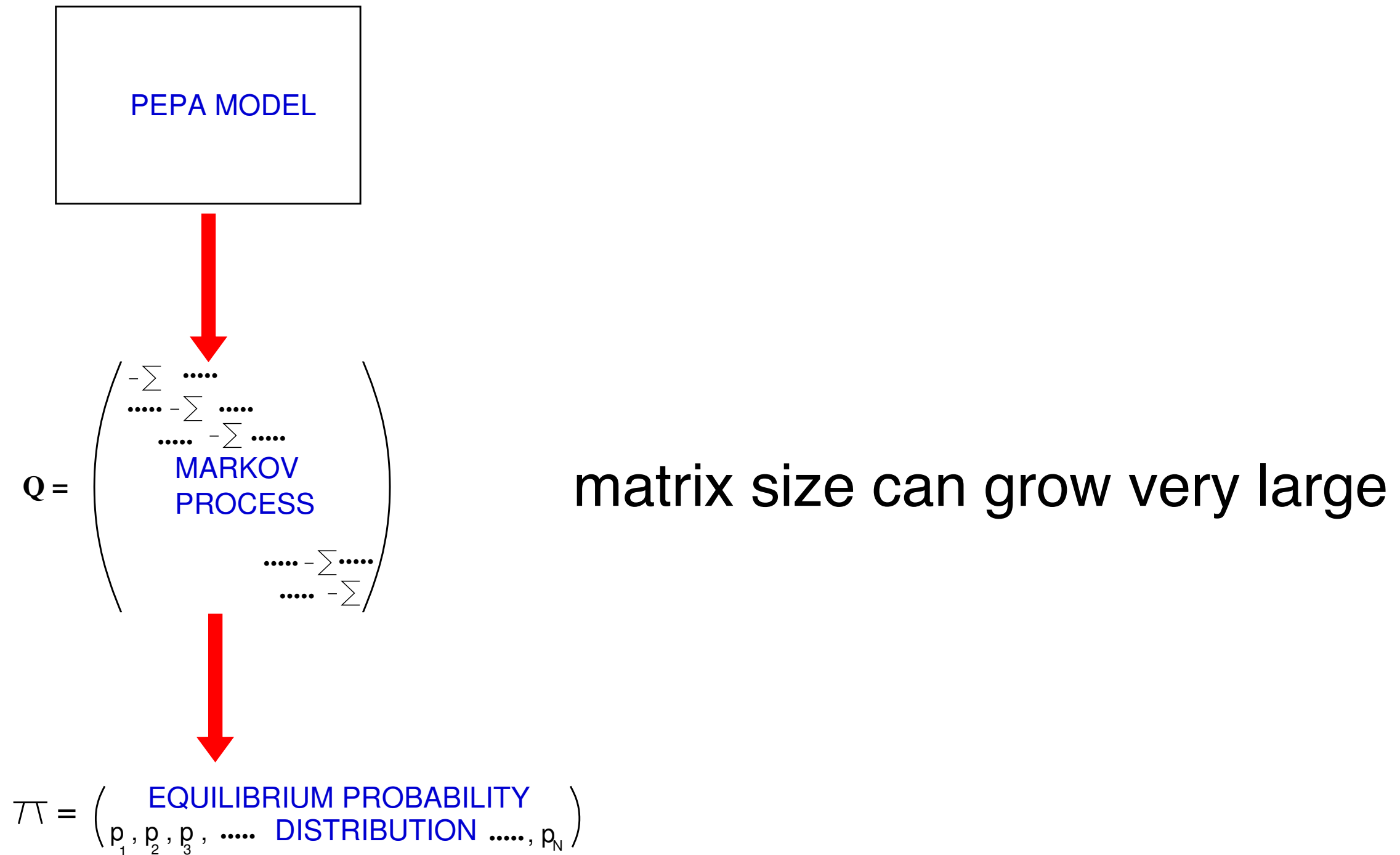
we can exploit CTMC bisimulation to reduce the state space
(notion of lumpable partition)

it is the only equivalence that preserves the Markov property



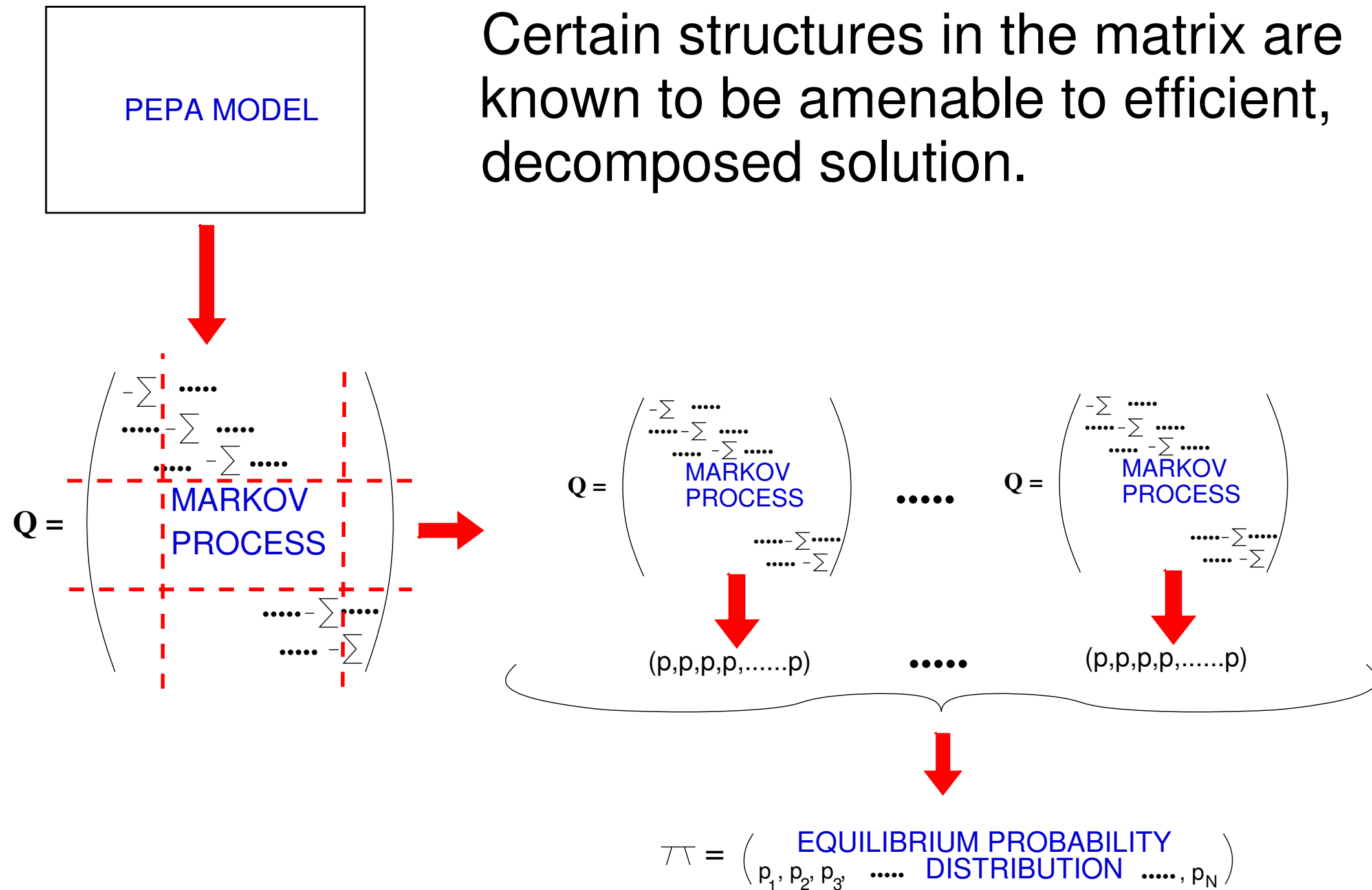
(taken from Jane Hillston's slides)

Compositionality



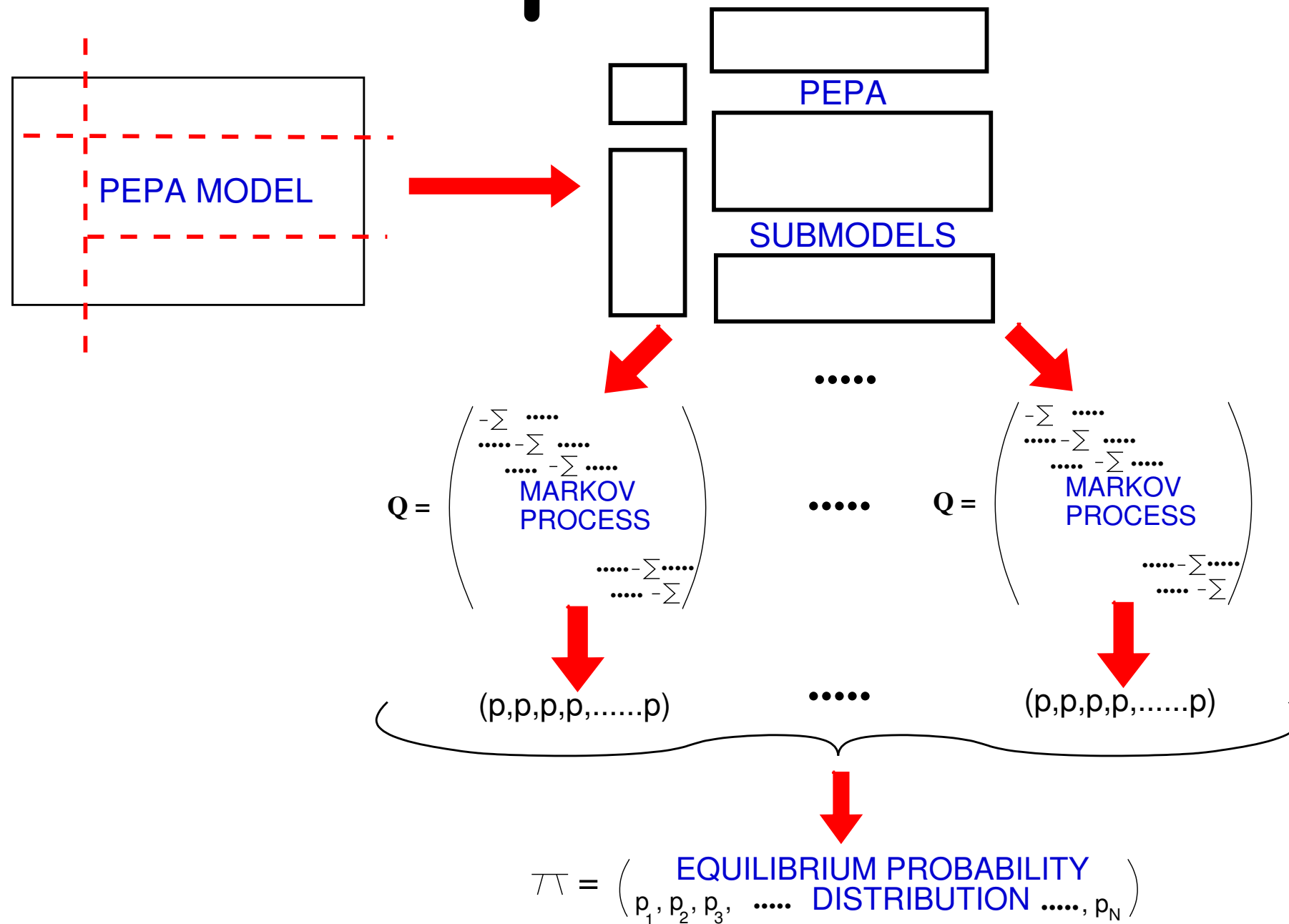
(taken from Jane Hillston's slides)

Compositionality



(taken from Jane Hillston's slides)

Compositionality



lift independent structures to the PEPA model!

(taken from Jane Hillston's slides)

Last badge



The final exam of a course consists of a list of 30 questions and a list of 30 answers. Each student has to draw a bijective correspondence between the two lists, linking each question to its answer.

The teacher will assign 1 point to each correct link and 0 to each wrong link.

Unfortunately, many students had no time to prepare for the exam, because they had a tight deadline to deliver a project and they will answer completely random.

1. What is the average score for such students?
2. Would the average score be improved by adding more questions and answers?