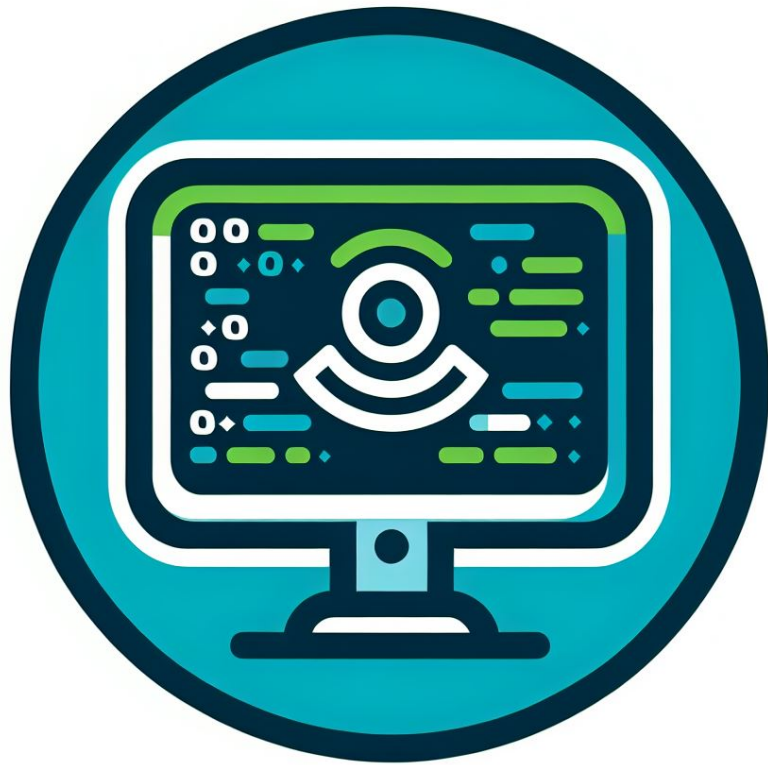




PSC 2024/25 (375AA, 9CFU)

Principles for Software Composition

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[http://didawiki.di.unipi.it/doku.php/
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20 - Weak semantics

CCS syntax

p, q	$::=$	nil	inactive process
		x	process variable (for recursion)
		$\mu.p$	action prefix
		$p \backslash \alpha$	restricted channel
		$p[\phi]$	channel relabelling
		$p + q$	nondeterministic choice (sum)
		$p q$	parallel composition
		rec $x. p$	recursion

(operators are listed in order of precedence)

CCS op. semantics

$$\begin{array}{c}
 \text{Act)} \frac{}{\mu.p \xrightarrow{\mu} p} \qquad \text{Res)} \frac{p \xrightarrow{\mu} q \quad \mu \notin \{\alpha, \bar{\alpha}\}}{p \setminus \alpha \xrightarrow{\mu} q \setminus \alpha} \qquad \text{Rel)} \frac{p \xrightarrow{\mu} q}{p[\phi] \xrightarrow{\phi(\mu)} q[\phi]} \\
 \\
 \text{SumL)} \frac{p_1 \xrightarrow{\mu} q}{p_1 + p_2 \xrightarrow{\mu} q} \qquad \text{SumR)} \frac{p_2 \xrightarrow{\mu} q}{p_1 + p_2 \xrightarrow{\mu} q} \\
 \\
 \text{ParL)} \frac{p_1 \xrightarrow{\mu} q_1}{p_1 | p_2 \xrightarrow{\mu} q_1 | p_2} \qquad \text{Com)} \frac{p_1 \xrightarrow{\lambda} q_1 \quad p_2 \xrightarrow{\bar{\lambda}} q_2}{p_1 | p_2 \xrightarrow{\tau} q_1 | q_2} \qquad \text{ParR)} \frac{p_2 \xrightarrow{\mu} q_2}{p_1 | p_2 \xrightarrow{\mu} p_1 | q_2} \\
 \\
 \text{Rec)} \frac{p[\mathbf{rec} \ x. \ p / x] \xrightarrow{\mu} q}{\mathbf{rec} \ x. \ p \xrightarrow{\mu} q}
 \end{array}$$

CCS

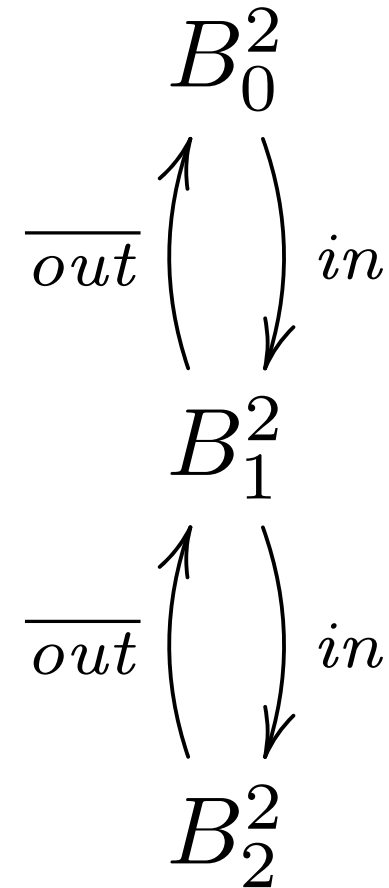
Weak transitions

Sequential buffer

$$B_0^2 \triangleq in.B_1^2$$

$$B_1^2 \triangleq in.B_2^2 + \overline{out}.B_0^2$$

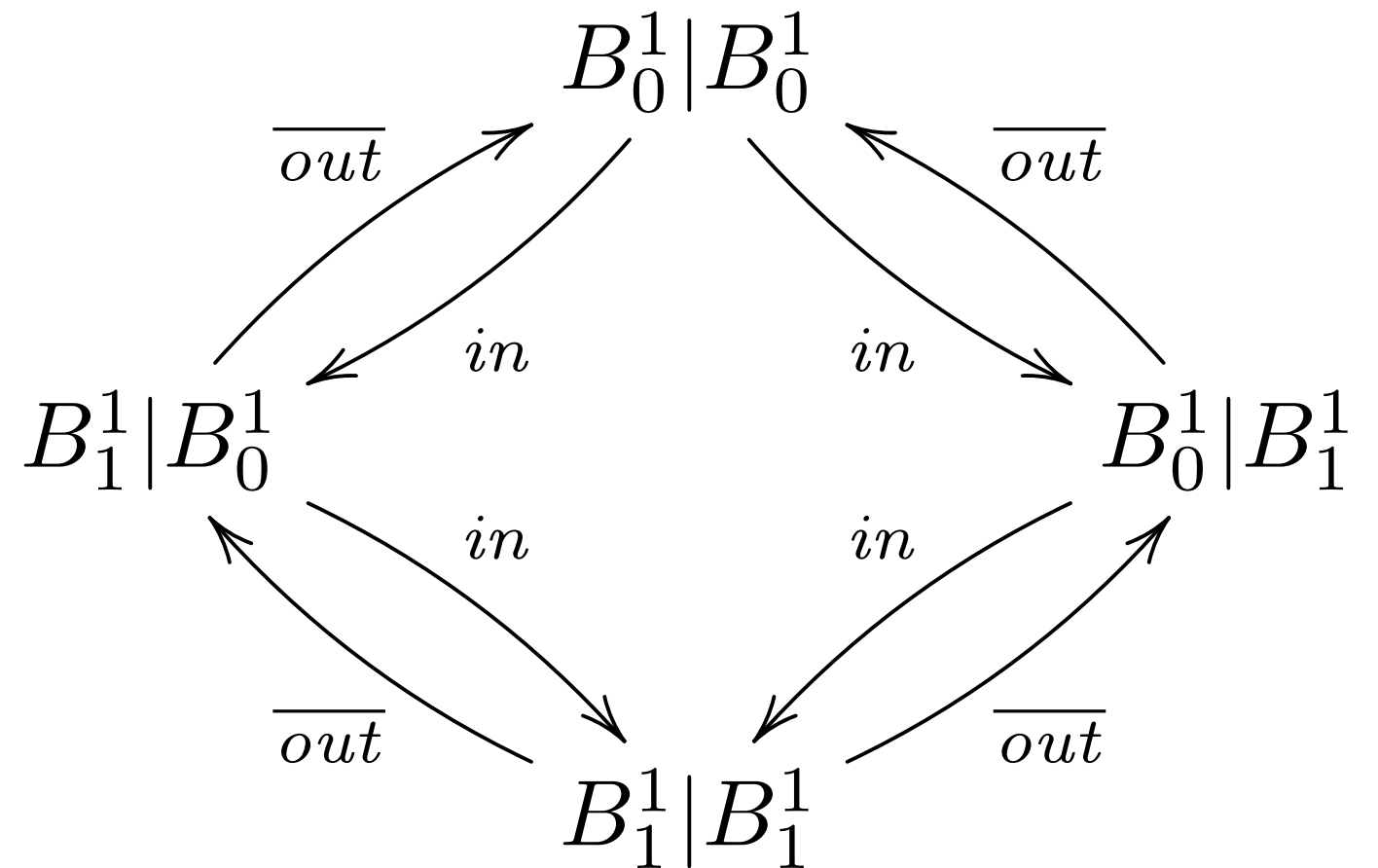
$$B_2^2 \triangleq \overline{out}.B_1^2$$



Parallel buffer

$$B_0^1 \triangleq in.B_1^1$$

$$B_1^1 \triangleq \overline{out}.B_0^1$$

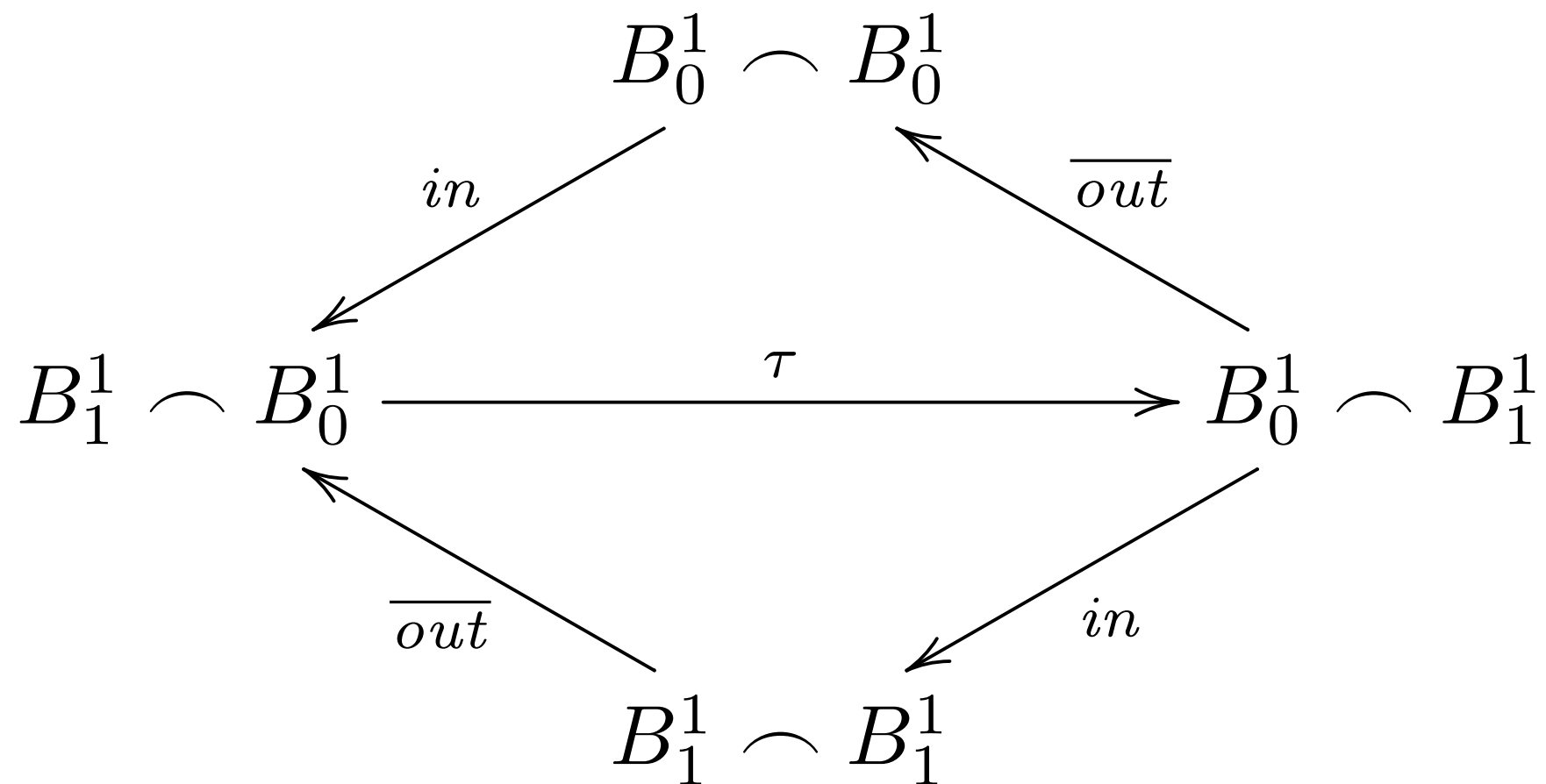


Linked buffer

$$B_0^1 \triangleq in.B_1^1 \quad \eta(out) = c$$

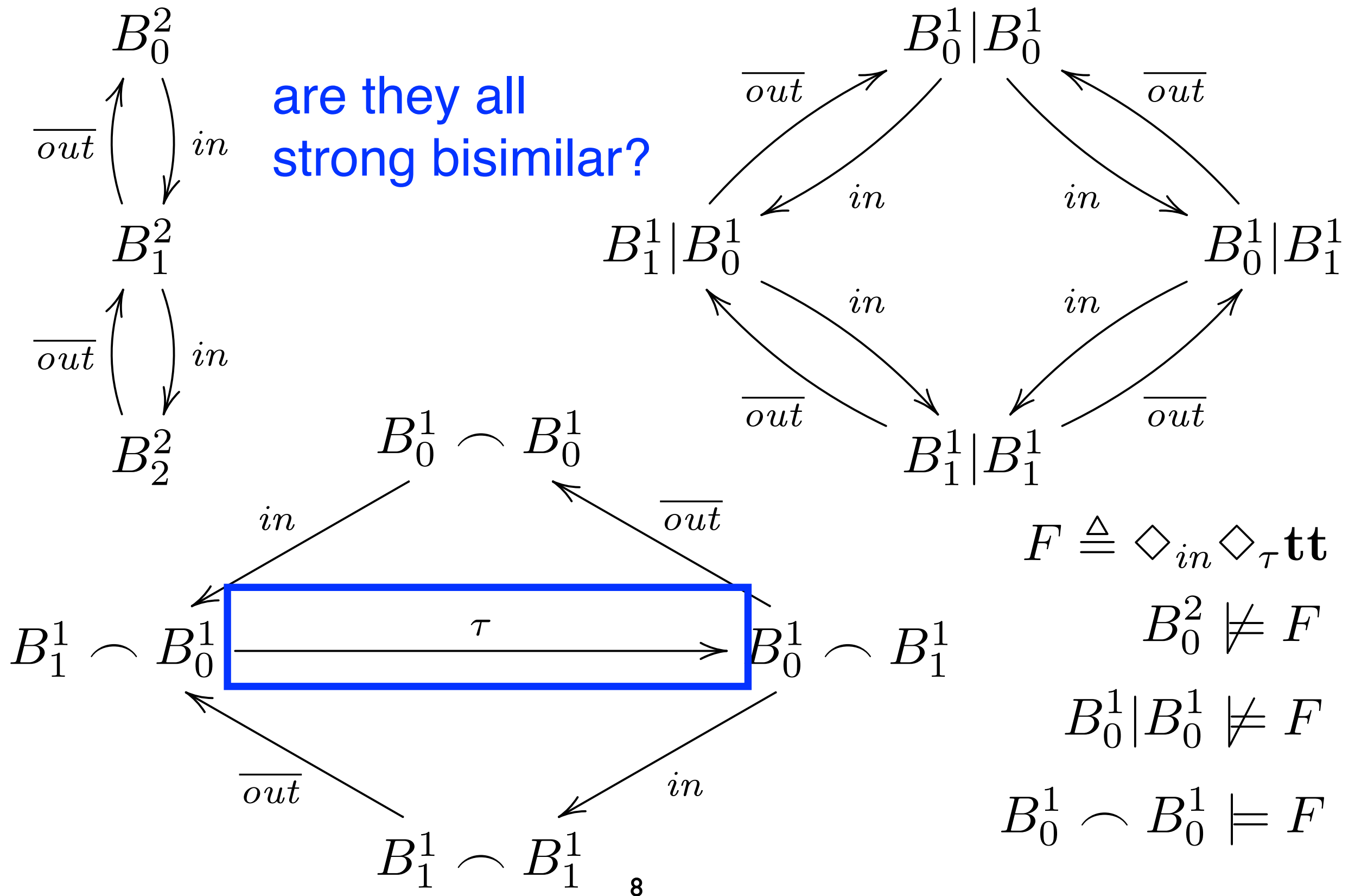
$$p \frown q \triangleq (p[\eta] \parallel q[\phi]) \setminus c$$

$$B_1^1 \triangleq \overline{out}.B_0^1 \quad \phi(in) = c$$



Comparing buffers

are they all
strong bisimilar?



Silent transitions



τ -transitions are silent, non observable

they represent internal steps of the system

they can be used just for bookkeeping

can we abstract away from them?

can we find a broader equivalence?

necessary to relate an abstract specification (little use of τ)

with a concrete implementation (lots/tons of τ)

Weak bisimulation game

coarser equivalence: more power to the defender!

Alice picks a process and an ordinary transition

Bob replies possibly using many additional silent transitions

arbitrarily many, but finitely many

such sequences are called *weak* transitions

$$p \xRightarrow{\mu} q$$

what if Alice picks a silent transition?

Bob can just leave the other process idle

i.e. can choose not to move

Weak transitions

$$p \xRightarrow{\tau} q \quad \text{iff} \quad p (\xrightarrow{\tau})^* q$$

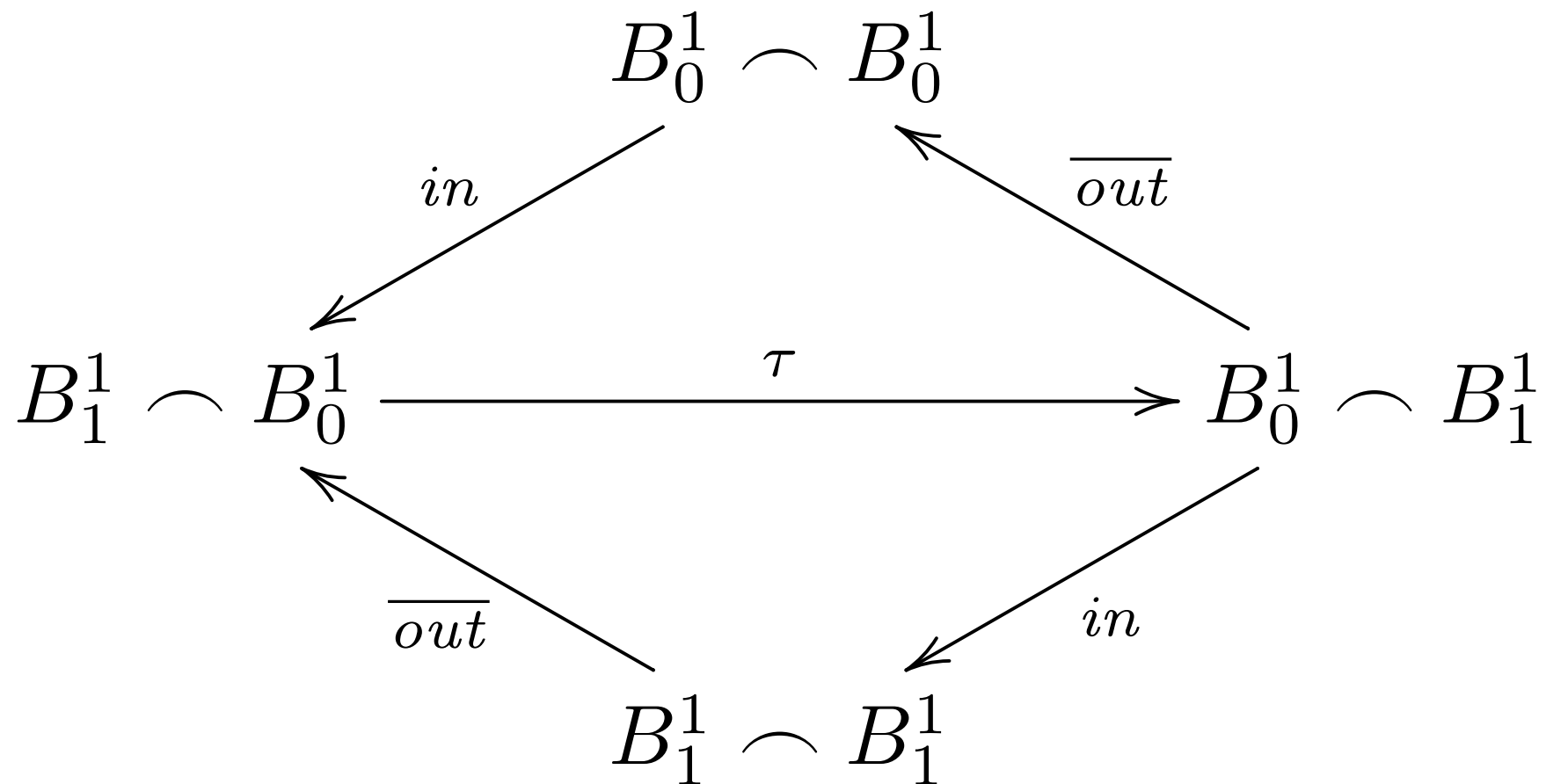
$$p = q \quad \vee \quad p \xrightarrow{\tau} \dots \xrightarrow{\tau} q$$

p can reach q via a (possibly empty) finite sequence of τ -transitions

$$p \xRightarrow{\lambda} q \quad \text{iff} \quad \exists p', q'. p \xRightarrow{\tau} p' \xrightarrow{\lambda} q' \xRightarrow{\tau} q$$

p can reach q via a λ -transition possibly preceded and followed by empty/finite sequences of τ -transitions

Example



$$B_0^1 \frown B_0^1 \xRightarrow{\tau} B_0^1 \frown B_0^1$$

$$B_0^1 \frown B_0^1 \xRightarrow{in} B_0^1 \frown B_1^1$$

$$B_1^1 \frown B_0^1 \xRightarrow{\overline{out}} B_0^1 \frown B_0^1$$

CCS

weak bisimulation

Weak bisimulation

$\mathbf{R}$ is a *weak* bisimulation if

$$\forall p, q. (p, q) \in \mathbf{R} \Rightarrow \left\{ \begin{array}{l} \forall \mu, p'. p \xrightarrow{\mu} p' \Rightarrow \exists q'. q \xRightarrow{\mu} q' \wedge p' \mathbf{R} q' \\ \wedge \text{ Alice plays} \\ \forall \mu, q'. q \xrightarrow{\mu} q' \Rightarrow \exists p'. p \xRightarrow{\mu} p' \wedge p' \mathbf{R} q' \end{array} \right.$$

Bob replies

weak transitions

Weak bisimilarity

weak bisimilarity:

$p \approx q$ iff $\exists \mathbf{R}$ a weak bisimulation with $(p, q) \in \mathbf{R}$

TH. weak bisimilarity is an equivalence relation

TH. any strong bisimulation is a weak bisimulation

Cor. strong bisimilarity implies weak bisimilarity

Weaker bisimilarity?

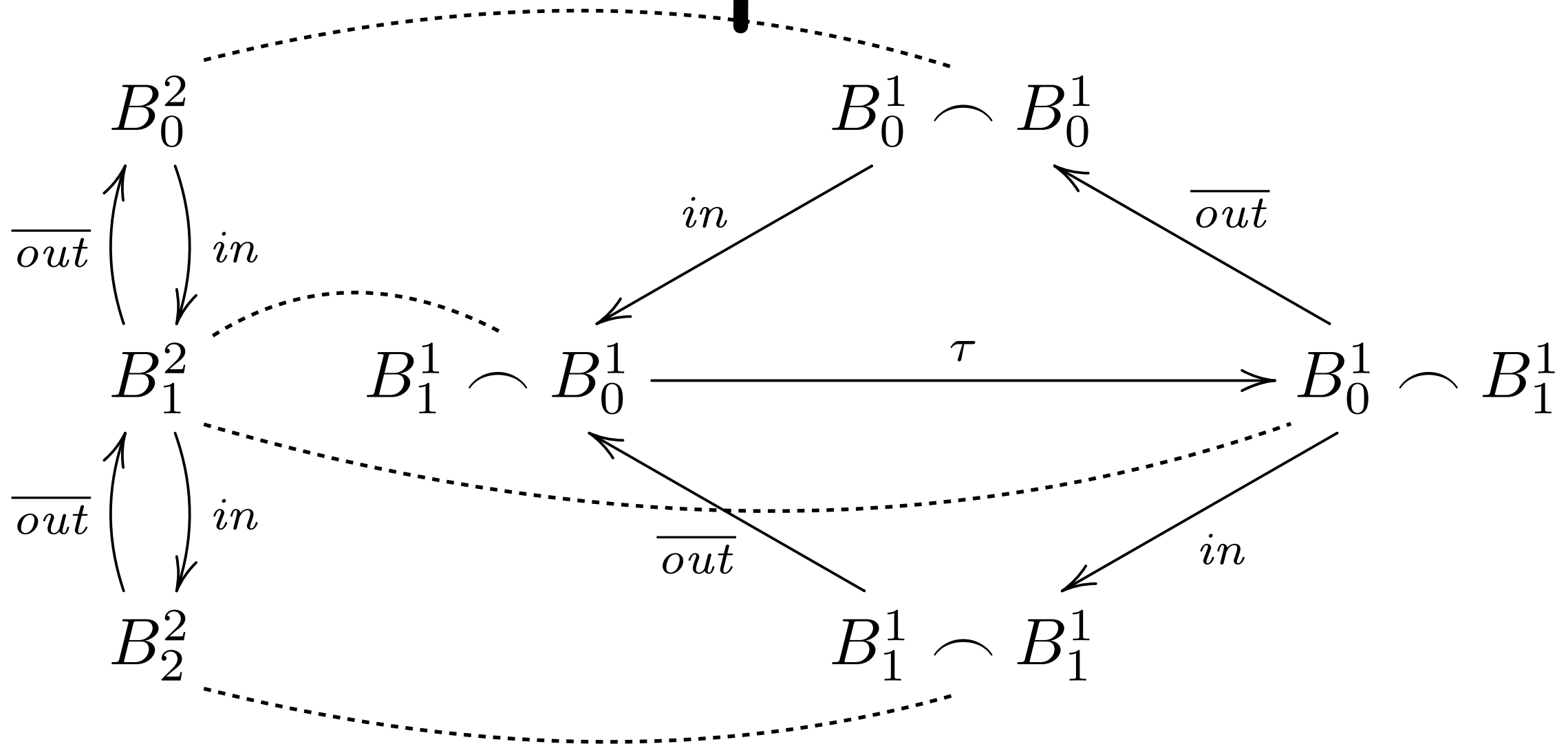
what if we give extra power to Alice as well?

$$\forall p, q. (p, q) \in \mathbf{R} \Rightarrow \left\{ \begin{array}{ll} \forall \mu, p'. p \xRightarrow{\mu} p' & \Rightarrow \quad \exists q'. q \xRightarrow{\mu} q' \wedge p' \mathbf{R} q' \\ \wedge & \text{Alice plays} \quad \text{Bob replies} \\ \forall \mu, q'. q \xRightarrow{\mu} q' & \Rightarrow \quad \exists p'. p \xRightarrow{\mu} p' \wedge p' \mathbf{R} q' \end{array} \right.$$

weak transitions

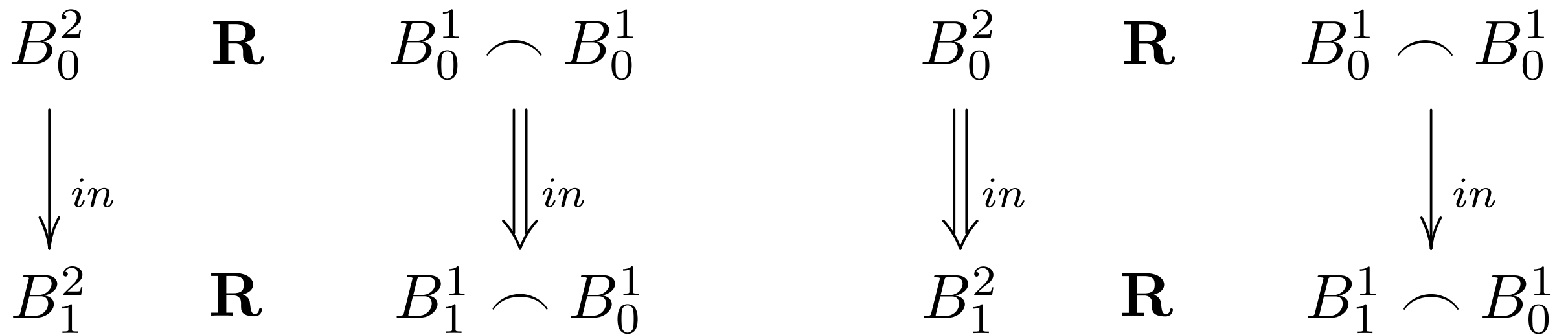
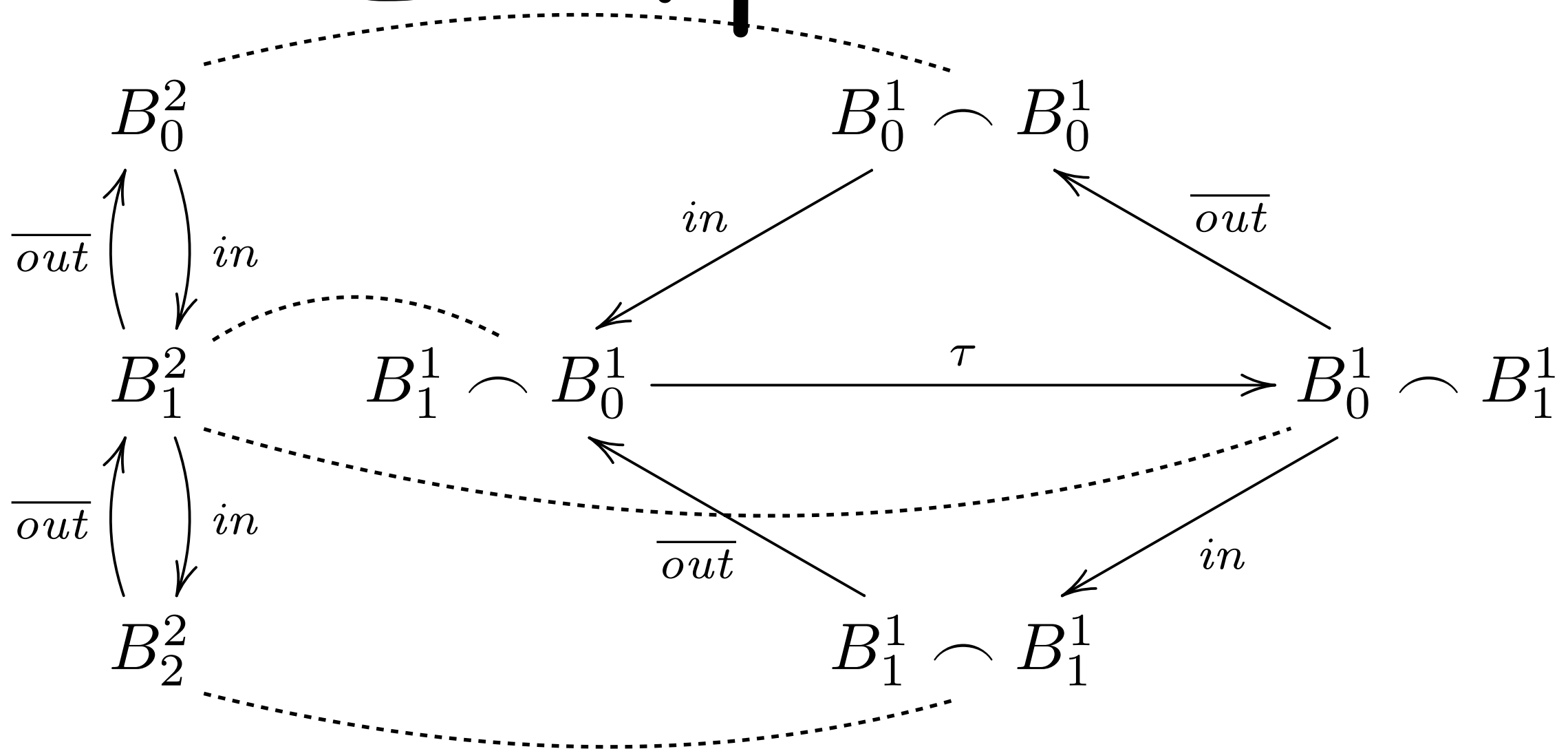
nothing changes: we still get the same weak bisimilarity

Example

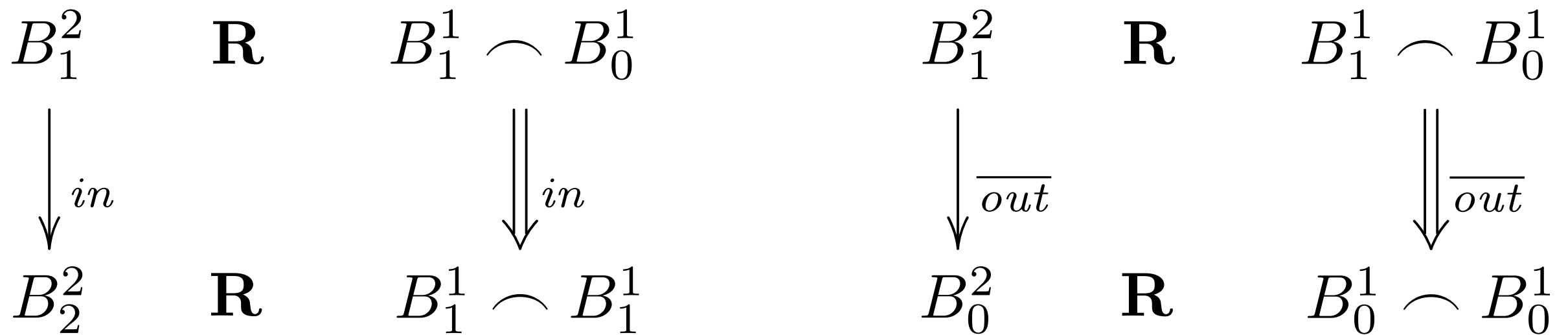
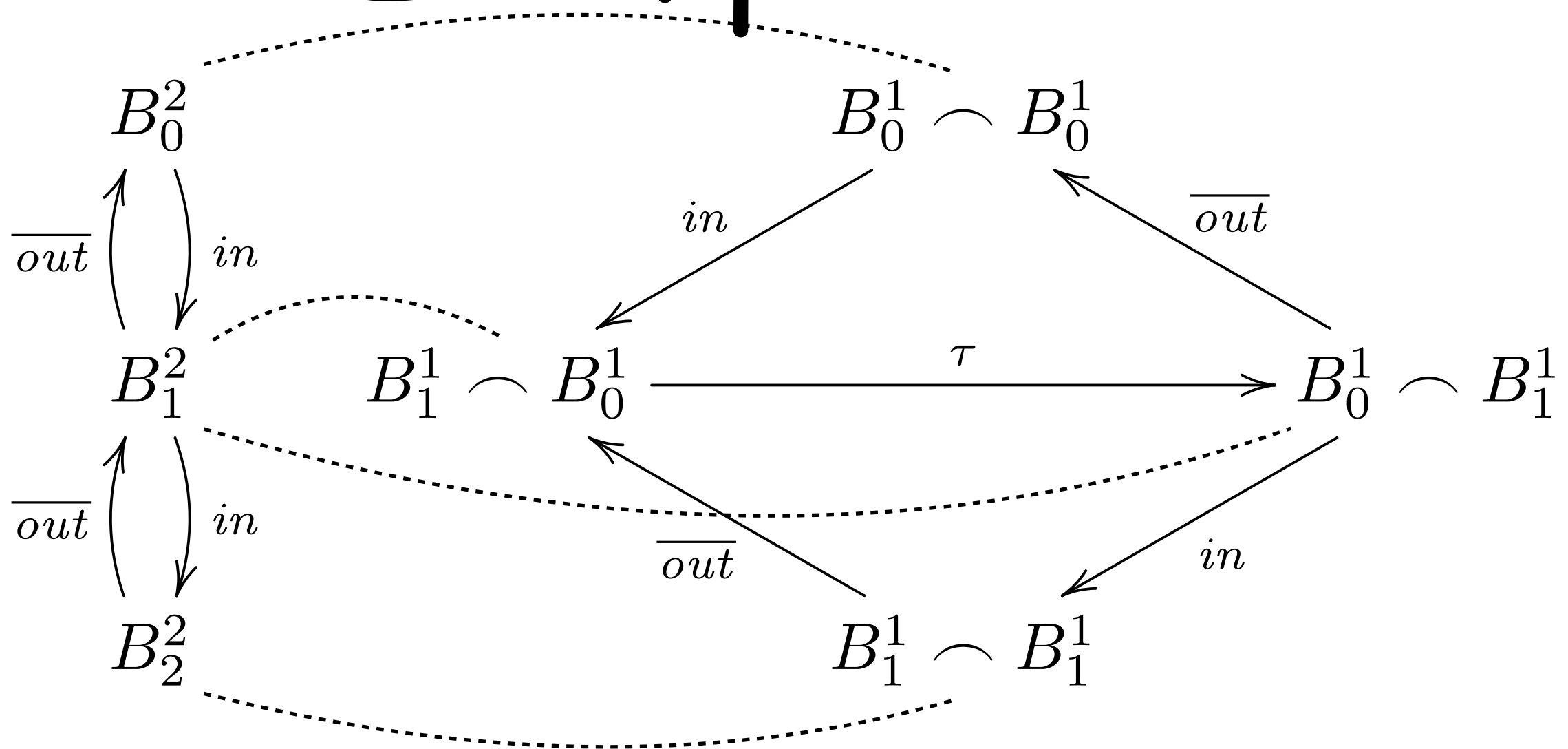


$\mathbf{R} \triangleq \left\{ \begin{array}{l} (B_0^2, B_0^1 \frown B_0^1), \\ (B_1^2, B_1^1 \frown B_0^1), \\ (B_1^2, B_0^1 \frown B_1^1), \\ (B_2^2, B_1^1 \frown B_1^1) \end{array} \right\}$ is a weak bisimulation relation

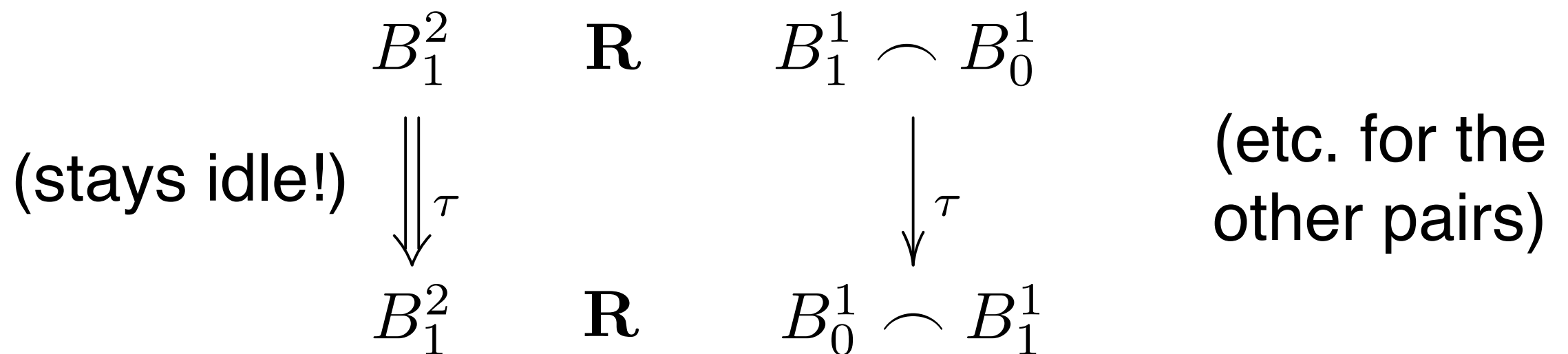
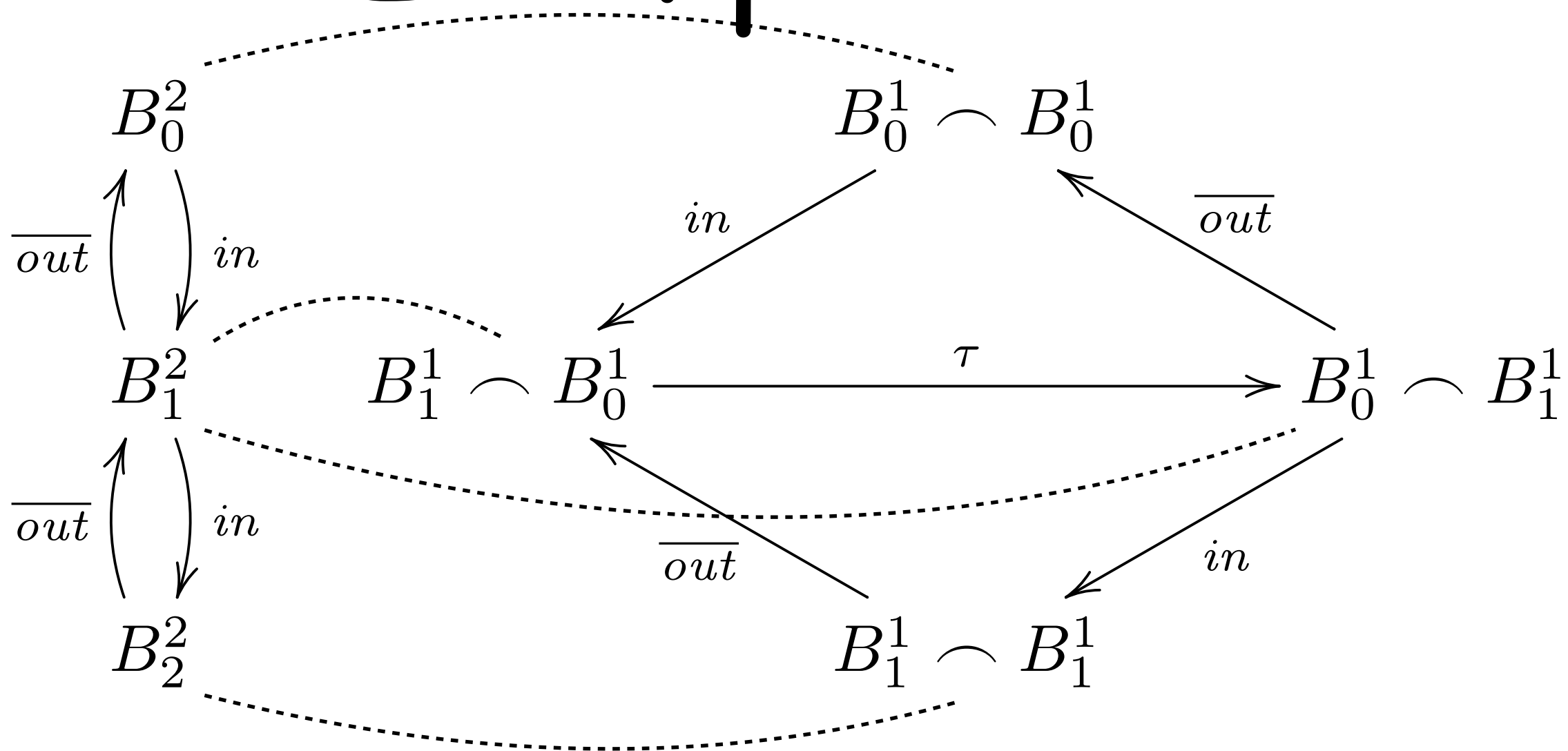
Example



Example



Example



Weak bis as a fixpoint

$$\Psi(\mathbf{R}) \triangleq \left\{ (p, q) \left| \begin{array}{l} \forall \mu, p'. p \xrightarrow{\mu} p' \Rightarrow \exists q'. q \xRightarrow{\mu} q' \wedge p' \mathbf{R} q' \\ \wedge \\ \forall \mu, q'. q \xrightarrow{\mu} q' \Rightarrow \exists p'. p \xRightarrow{\mu} p' \wedge p' \mathbf{R} q' \end{array} \right. \right\}$$

$$\Psi : \wp(\mathcal{P} \times \mathcal{P}) \rightarrow \wp(\mathcal{P} \times \mathcal{P})$$

maps relations to relations

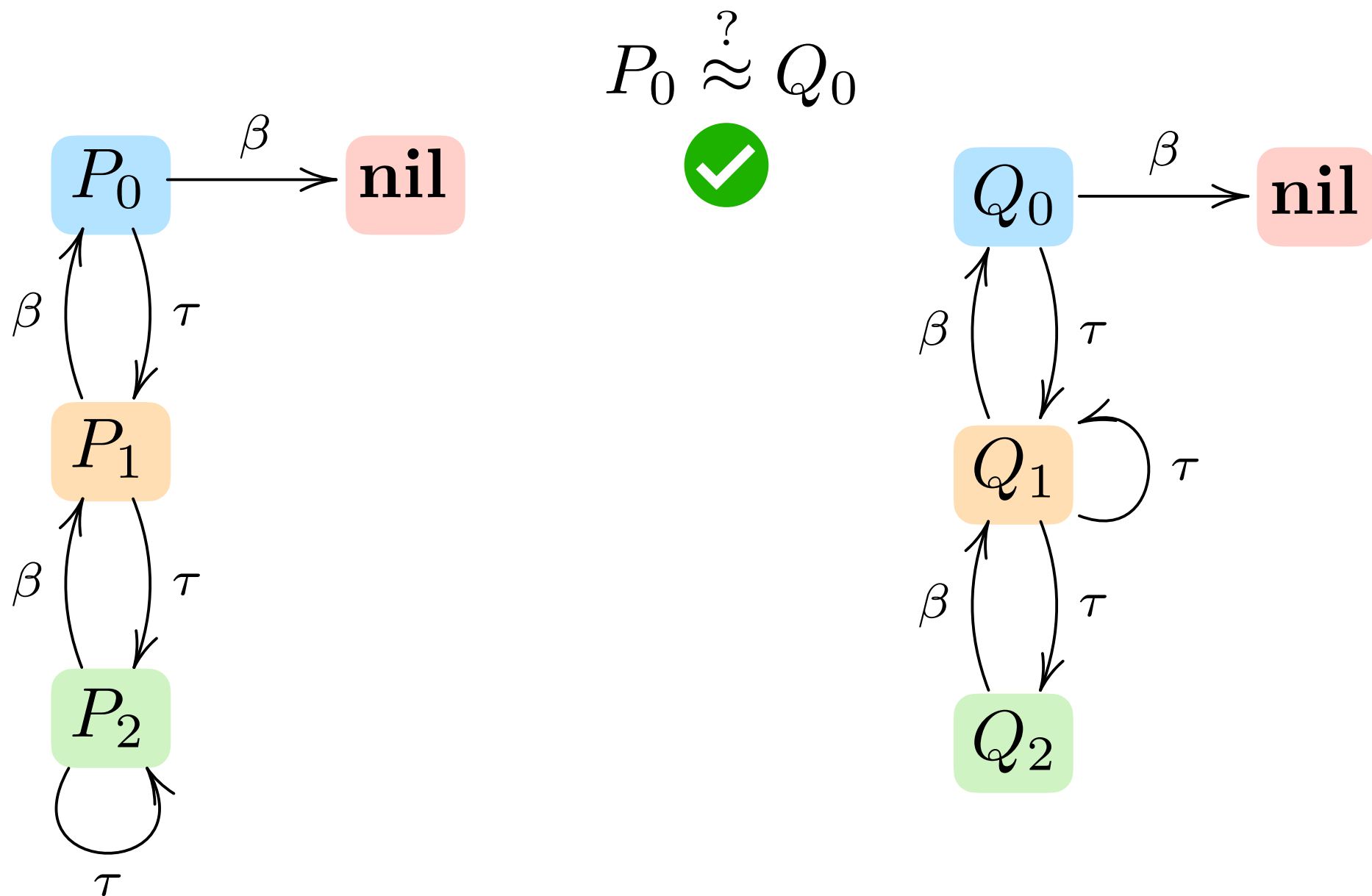
$$\mathbf{R} \subseteq \Psi(\mathbf{R})$$

a weak bisimulation

$$\approx = \Psi(\approx)$$

weak bisimilarity is a fixpoint

Exercise



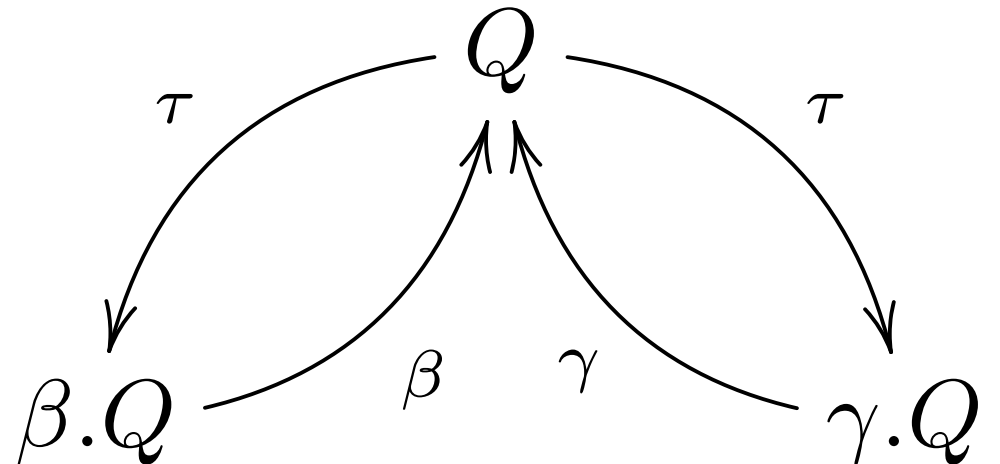
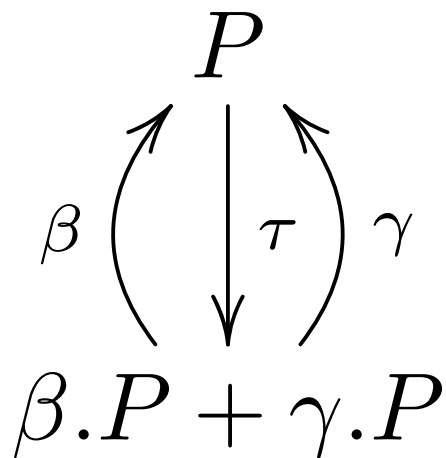
is a weak bisimulation!

$$\mathbf{R} = \{ \{P_0, Q_0\} , \{P_1, Q_1\} , \{P_2, Q_2\} , \{\mathbf{nil}\} \}$$

Exercise

$$P \triangleq \tau.(\beta.P + \gamma.P) \qquad P \stackrel{?}{\approx} Q \qquad Q \triangleq \tau.\beta.Q + \tau.\gamma.Q$$





$$Q \xrightarrow{\tau} \beta.Q$$

$$P \xRightarrow{\tau} P$$

$$P \xrightarrow{\tau} \beta.P + \gamma.P$$

$$\beta.Q \xRightarrow{\tau} \beta.Q$$

$$\beta.P + \gamma.P \xrightarrow{\gamma} P$$

$$\beta.Q \not\xRightarrow{\gamma}$$

Alice wins!

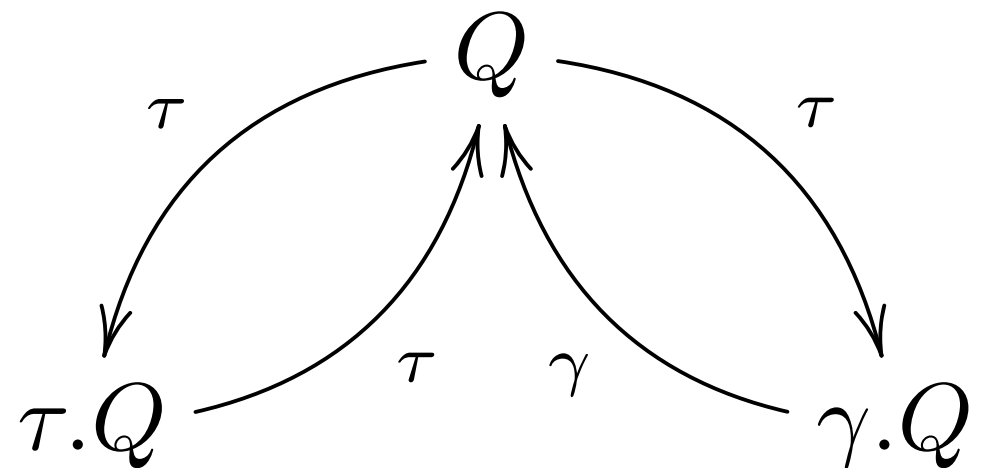
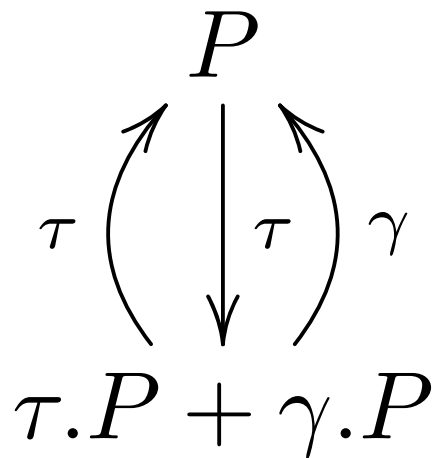


Exercise

$$P \triangleq \tau.(\tau.P + \gamma.P)$$

$$P \stackrel{?}{\approx} Q$$

$$Q \triangleq \tau.\tau.Q + \tau.\gamma.Q$$



$$\mathbf{R}_0 = \{ \{ P , Q , \tau.P + \gamma.P , \tau.Q , \gamma.Q \} \}$$

is a weak bisimulation!

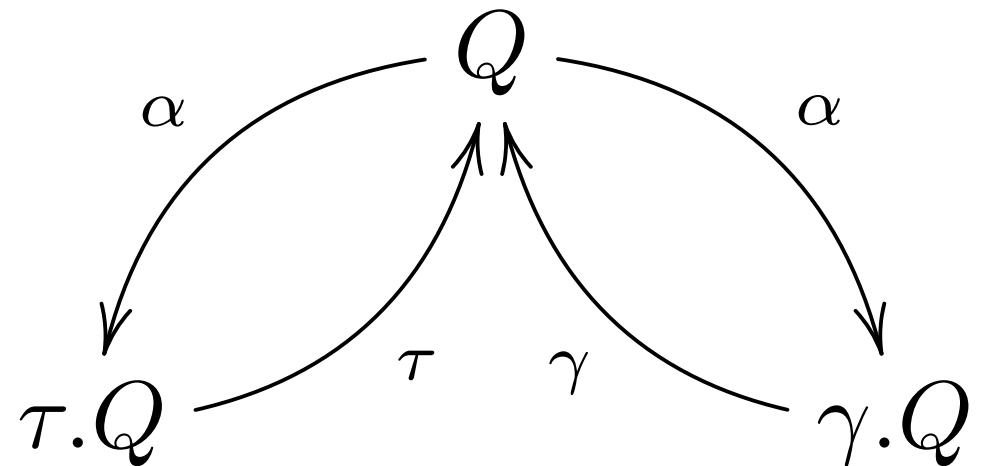
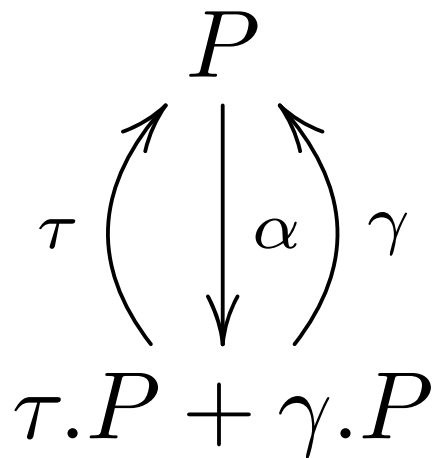


Exercise

$$P \triangleq \alpha.(\tau.P + \gamma.P)$$

$$P \stackrel{?}{\approx} Q$$

$$Q \triangleq \alpha.\tau.Q + \alpha.\gamma.Q$$



$$Q \xrightarrow{\alpha} \gamma.Q$$

$$P \xRightarrow{\alpha} \tau.P + \gamma.P$$

$$\tau.P + \gamma.P \xrightarrow{\tau} P$$

$$\gamma.Q \xRightarrow{\tau} \gamma.Q$$

$$P \xrightarrow{\alpha} \tau.P + \gamma.P$$

$$\gamma.Q \not\xRightarrow{\alpha}$$

Alice wins!

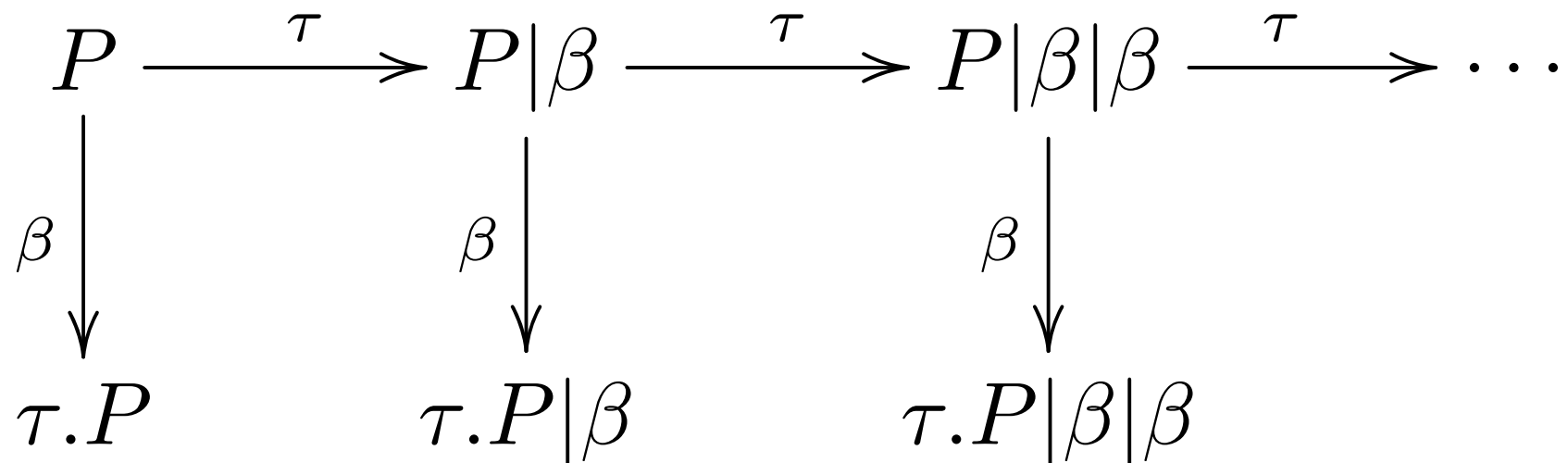
CCS

problems with weak semantics

Problems with weak bis

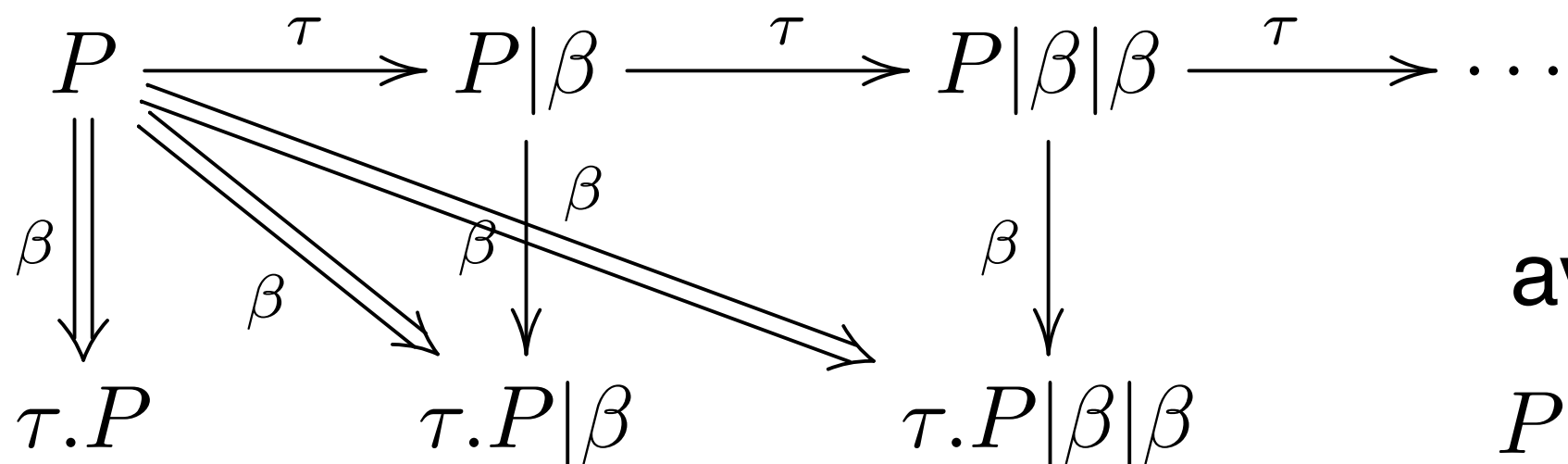
with respect to weak transitions,
guarded processes can have infinitely branching LTS

$$P \triangleq \mathbf{rec} \ x. \ \tau.x|\beta$$



many arrows omitted

assume $p|\mathbf{nil} = p$



avoid τ -prefixes?

$$P \triangleq \mathbf{rec} \ x. \ (\alpha.x|\bar{\alpha}|\beta)\backslash\alpha$$

Problems with weak bis

weak bisimilarity is not a congruence (w.r.t. +)

take $P \triangleq \alpha$ $Q \triangleq \tau.\alpha$

if $P \xrightarrow{\alpha} \text{nil}$ then $Q \xRightarrow{\alpha} \text{nil}$

if $Q \xrightarrow{\tau} \alpha$ then $P \xRightarrow{\tau} P$

$$P \approx Q \qquad \mathbb{C}[P] \not\approx \mathbb{C}[Q]$$

take the context $\mathbb{C}[\cdot] \triangleq [\cdot] + \beta$

$$\mathbb{C}[P] \triangleq \alpha + \beta$$

$$\mathbb{C}[Q] \triangleq \tau.\alpha + \beta$$

Alice plays $\mathbb{C}[Q] \xrightarrow{\tau} \alpha$

Bob can only reply $\mathbb{C}[P] \xRightarrow{\tau} \mathbb{C}[P]$

Alice plays $\mathbb{C}[P] \xrightarrow{\beta} \text{nil}$

Bob cannot reply $\alpha \not\xRightarrow{\beta}$

Alice wins!

Problems with weak bis

cannot distinguish between deadlock
and silent divergence

$$\mathbf{rec}\ x.\ \tau.x \approx \mathbf{nil}$$

$$\mathbf{rec}\ x.\ \tau.x \xrightarrow{\tau} \mathbf{rec}\ x.\ \tau.x \qquad \mathbf{nil} \xRightarrow{\tau} \mathbf{nil}$$

CCS

weak observational congruence

Weak obs congruence

$$p \cong q \quad \text{iff} \quad p \approx q \wedge \forall r. p + r \approx q + r$$

Equivalently

$$p \cong q \quad \text{iff} \quad \left\{ \begin{array}{ll} \forall p'. p \xrightarrow{\tau} p' & \Rightarrow \exists q', q''. q \xrightarrow{\tau} q'' \xRightarrow{\tau} q' \wedge p' \approx q' \\ \forall \lambda, p'. p \xrightarrow{\lambda} p' & \Rightarrow \exists q'. q \xRightarrow{\lambda} q' \wedge p' \approx q' \\ \text{and vice versa} & \end{array} \right.$$

not a recursive definition!
(refers to weak bisimilarity)

at the level of bisimulation game:

Bob is not allowed to use an idle move at the very first turn
(at the following turns, ordinary weak bisimulation game)

TH. $\cong$ is the largest congruence contained in $\approx$

Weak obs congruence

Note: $\cong$ is not a weak bisimulation!

$$P \triangleq \alpha$$

$$Q \triangleq \tau.\alpha$$

$$\beta.P$$

$$\cong$$

$$\beta.Q$$

$$\beta \downarrow$$

$$P$$

$$\approx$$

$$\beta \downarrow$$

$$Q$$

$$P \not\approx Q$$

$$\cong \not\subseteq \Psi(\cong)$$

Weak obs congruence

All the laws for strong bisimilarity are still valid

Additionally: Milner's τ -laws

$$p + \tau.p \cong \tau.p$$

$$\mu.(p + \tau.q) \cong \mu.(p + \tau.q) + \mu.q$$

$$\mu.\tau.p \cong \mu.p$$