

Models of computation (MOD) 2015/16

Exam – Jan. 20, 2017

[Ex. 1] Let the transition relation \rightarrow^n with $n \in \mathbb{N}$ be defined by the following inference rules

$$\frac{}{\langle c, \sigma \rangle \rightarrow^0 \sigma} \quad \frac{\langle c, \sigma \rangle \rightarrow \sigma'' \quad \langle c, \sigma'' \rangle \rightarrow^n \sigma'}{\langle c, \sigma \rangle \rightarrow^{n+1} \sigma'}$$

1. Prove the determinacy of \rightarrow^n (for any $n \in \mathbb{N}$).
2. Prove that for all c, σ, σ', n :

$$\langle c, \sigma \rangle \rightarrow^n \sigma' \Rightarrow \forall k < n. \exists \sigma''. \langle c, \sigma \rangle \rightarrow^k \sigma''.$$

[Ex. 2] Let S be a non-empty, finite set. A *multiset* over S is a function $M : S \rightarrow \mathbb{N}$ that assigns to each element $s \in S$ its multiplicity $M(s)$. Let $\mu(S)$ denote the set of all multisets over S , ordered by the relation

$$M \sqsubseteq M' \stackrel{\text{def}}{=} \forall s \in S. M(s) \leq M'(s)$$

1. Prove that $(\mu(S), \sqsubseteq)$ is a partial order with bottom.
2. Exhibit a set S and a chain $\{M_i\}_{i \in \mathbb{N}}$ in $(\mu(S), \sqsubseteq)$ that has no lub.
3. Let $\delta : (\mu(S), \sqsubseteq) \rightarrow (\mathbb{N}, \leq)$ be defined by $\delta(M) \stackrel{\text{def}}{=} |\{s \in S \mid M(s) > 0\}|$. Prove that δ is monotone.

[Ex. 3] Consider the HOFL term

$$t \stackrel{\text{def}}{=} \mathbf{rec} \ f. \ \lambda x. \ \mathbf{if} \ \mathbf{fst}(x) \ \mathbf{then} \ \mathbf{snd}(x) \ \mathbf{else} \ (f \ (\mathbf{snd}(x), \mathbf{fst}(x)))$$

1. Under which hypothesis is t typable?
2. Compute the (lazy) denotational semantics of t .

[Ex. 4] Consider the CCS processes

$$p \stackrel{\text{def}}{=} \mathbf{rec} \ x. (\alpha.\mathbf{nil} + \tau.\mathbf{rec} \ y. (\beta.y + \tau.x)) \quad q \stackrel{\text{def}}{=} \mathbf{rec} \ z. (\alpha.\mathbf{nil} + \beta.z)$$

1. Draw the LTSs of p and q .
2. Prove that p and q are weakly bisimilar.
3. Are p and q weakly observational congruent? (Explain)