Models of computation (MOD) 2014/15

Exam – Sept. 9, 2015

[Ex. 1] Let b be a boolean expression and c a command. Consider the IMP command

$$w \stackrel{\text{def}}{=} \textbf{while } b \textbf{ do } c$$

1. Prove that: $\forall \sigma, \sigma'.\ \langle w, \sigma \rangle \to \sigma' \ \Rightarrow \langle b, \sigma' \rangle \to \mathit{false}$.

   Hint: Proceed by rule induction.

2. Exploit the property above to prove that $w \sim (w; w)$, i.e., that the commands w and $w; w$ are operationally equivalent.

[Ex. 2] A relation $R \subseteq X \times Y$ is *surjective* if $\forall y \in Y.\ \exists x \in X.\ (x, y) \in R$. Consider the set $\mathbf{SR} \subseteq \wp(\omega \times \omega)$ of surjective relations over natural numbers, ordered by inclusion.

1. Is $(\mathbf{SR}, \subseteq)$ a partial order?

2. Is it complete?

3. Does it have a bottom element?

Explain your answers carefully.

[Ex. 3] Let us consider the HOFL term

$$t \stackrel{\text{def}}{=} \textbf{rec } f.\ \lambda x.\ \textbf{if } f(x-1) \textbf{ then } 0 \textbf{ else } x + x$$

1. Find the principal type of t.

2. Compute the denotational semantics of t.

[Ex. 4] Consider the CCS processes

$$\begin{array}{llllll} p & \stackrel{\text{def}}{=} & \textbf{rec } X.\ a.(b.b'.X \ + \ c.c'.X) & \qquad r & \stackrel{\text{def}}{=} & \textbf{rec } Z.\ (\bar{b}.Z + \bar{c}.Z) \\ q & \stackrel{\text{def}}{=} & \textbf{rec } Y.\ a.b.(b'.Y \ + \ c'.Y) & \qquad s & \stackrel{\text{def}}{=} & (\textbf{rec } V.\ \bar{b}.V) \mid (\textbf{rec } W.\ \bar{c}.W) \end{array}$$

1. Are the processes r and s strong bisimilar?

2. Draw the LTS for the processes $(p|r)\backslash b\backslash c$ and $(q|s)\backslash b\backslash c$.

3. Are the processes $(p|r)\backslash b\backslash c$ and $(q|s)\backslash b\backslash c$ weak bisimilar?