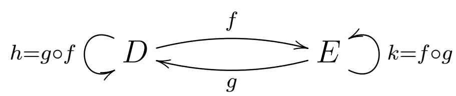
Models of computation (MOD) 2014/15
Mid-term exam – May 28, 2015

[**Ex. 1**] Let D, E be two CPO$_\perp$ and $f : D \to E$, $g : E \to D$ be two continuous functions between them. Their compositions $h = g \circ f : D \to D$ and $k = f \circ g : E \to E$ are known to be continuos and thus have least fixpoints.



Let $e_0 = \mathrm{fix}\ k \in E$. Prove that $g(e_0) = \mathrm{fix}\ h \in D$ by showing that:

1. $g(e_0)$ is a fixpoint for h, and
2. that $g(e_0)$ is the least pre-fixpoint for h.

[**Ex. 2**] Recall that a weak bisimulation is a relation R such that:

$$\forall p, q. \quad p\,R\,q \text{ implies } \begin{cases} \forall \mu, p'. \quad p \xrightarrow{\mu} p' \quad \text{implies} \quad \exists q'.\, q \overset{\mu}{\Rightarrow} q' \quad \text{and} \quad p'\,R\,q' \\ \forall \mu, q'. \quad q \xrightarrow{\mu} q' \quad \text{implies} \quad \exists p'.\, p \overset{\mu}{\Rightarrow} p' \quad \text{and} \quad p'\,R\,q' \end{cases}$$

Let us define a *loose bisimulation* to be a relation R such that:

$$\forall p, q. \quad p\,R\,q \text{ implies } \begin{cases} \forall \mu, p'. \quad p \overset{\mu}{\Rightarrow} p' \quad \text{implies} \quad \exists q'.\, q \overset{\mu}{\Rightarrow} q' \quad \text{and} \quad p'\,R\,q' \\ \forall \mu, q'. \quad q \overset{\mu}{\Rightarrow} q' \quad \text{implies} \quad \exists p'.\, p \overset{\mu}{\Rightarrow} p' \quad \text{and} \quad p'\,R\,q' \end{cases}$$

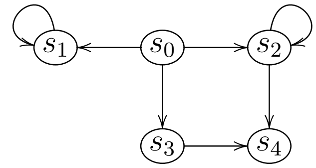
Prove that weak bisimilarity is the largest loose bisimulation by showing that:

1. any loose bisimulation is a weak bisimulation; and
2. any weak bisimulation is a loose bisimulation.

Hint: For (2) prove first, by mathematical induction on $n \geq 0$, that for any weak bisimulation R, any two processes $p\,R\,q$, and any sequence of transitions $p \xrightarrow{\tau} p_1 \xrightarrow{\tau} p_2 \cdots \xrightarrow{\tau} p_n$ there exists q' with $q \overset{\tau}{\Rightarrow} q'$ and $p_n\,R\,q'$.

[**Ex. 3**] Let us consider the μ-calculus formula $\phi = \nu x.\Diamond x$.

1. Compute the semantics $[\![\phi]\!]\,\rho$ of the formula, by spelling out what are the states that satisfy ϕ.
2. Evaluate the formula on the transition system below:



[Ex. 4] A machine can be described as being in three different states: (R) under repair, (W) waiting for a new job, (O) operating.

- While the machine is operating the probability to break is $\frac{1}{20} = 0.05$ and the probability to get finished (go to waiting) is $\frac{1}{10} = 0.1$.
- If the machine is under repair there is a $\frac{1}{10} = 0.1$ probability to get repaired, and then the machine will become waiting.
- A broken machine is never brought directly (in one step) to operation.
- If the machine is waiting there is a $\frac{9}{10} = 0.9$ probability to get into operation.
- A waiting machine does not break.

1. Describe the system as a DTMC, draw the corresponding transition system and define the transition probability matrix. Is it ergodic?
2. Assume that the machine is waiting at time t. What is the probability to be operating at time $t + 1$? Explain.
3. What is the probability that the machine is operating after a long time? Explain.