Models of computation (MOD) 2016/17
Appello straordinario – April 5, 2018

[Ex. 1] Let us extend the syntax of arithmetic expressions with the term $a^\times$, whose operational semantics is defined by the rules

$$\frac{\langle a, \sigma\rangle \to n}{\langle a^\times, \sigma\rangle \to n} \qquad \frac{\langle a, \sigma\rangle \to n \quad \langle a^\times, \sigma\rangle \to m}{\langle a^\times, \sigma\rangle \to n \underline{\times} m}$$

1. Prove termination of extended expressions by structural induction.
2. Prove by rule induction that $\forall \sigma, n.\ P(\langle 1^\times, \sigma\rangle \to n)$, where

$$P(\langle 1^\times, \sigma\rangle \to n) \stackrel{\text{def}}{=} n = 1$$

3. Show a counterexample to determinacy of extended expressions.

[Ex. 2] Let $(D, \preceq)$ be the CPO with bottom such that $D = \mathbb{N} \cup \{\infty_1, \infty_2\}$ and $\preceq \cap (\mathbb{N} \times \mathbb{N}) = \leq$, ∞_2 is the top element and $x \preceq \infty_1$ iff $x \neq \infty_2$.

1. Consider the function $succ : D \to D$ such that $\forall n \in \mathbb{N}. succ(n) = n + 1$ and $succ(\infty_1) = succ(\infty_2) = \infty_2$.
   Prove that the function $succ$ is monotone but not continuous.
2. Let $\{d_i\}_{i \in \mathbb{N}}$ be a chain.
   Prove that if $\bigsqcup_{i \in \mathbb{N}} d_i = \infty_2$ then the chain is finite.
   Hint: Note that if ∞_1 or ∞_2 belong to the chain then it is finite.

[Ex. 3] Let us consider the CCS processes

$$p \stackrel{\text{def}}{=} \mathbf{rec}\ x.(\alpha.x + \beta.(\mathbf{rec}\ y.\ \beta.x + \alpha.y) + \gamma.\mathbf{nil}) \qquad r \stackrel{\text{def}}{=} \mathbf{rec}\ u.(\overline{\beta}.u)$$

$$q \stackrel{\text{def}}{=} \mathbf{rec}\ z.(\alpha.z + \beta.\beta.z + \gamma.\mathbf{nil})$$

1. Draw the LTSs of the processes $s \stackrel{\text{def}}{=} (p|r)\backslash\beta$ and $t \stackrel{\text{def}}{=} (q|r)\backslash\beta$.
2. Show that s and t are not strong bisimilar.
3. Prove that s and t are weak bisimilar.

[Ex. 4] The President of the Big Nation tells person P_1 her intention to run or not to run in the next election. Then P_1 relays the news to P_2, who in turn relays the message to P_3, and so forth, always to some new person. We assume that there is a probability p that a person will change the answer from yes to no when transmitting it to the next person and a probability q that will change it from no to yes.

1. Model the system as a DTMC.
2. What is the probability that the n-th exchanged message is *yes* for n large enough?
3. If P_n says *yes*, what it the probability that P_{n+2} says *yes* too?