

Consiglio Nazionale  
delle Ricerche

# Mobility Patterns

# Content of this lesson

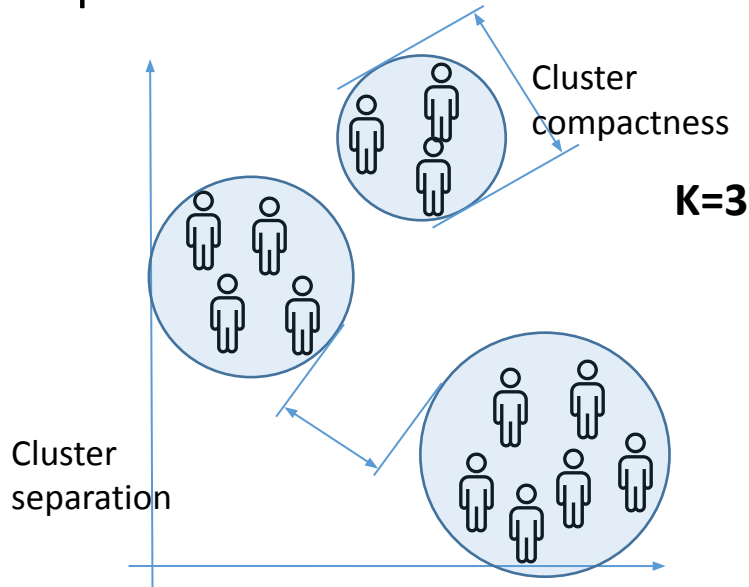
- Global patterns: Clustering
  - Trajectory distances
  - Trajectory clustering
- Local patterns
  - Flocks, Convoys & Swarms
  - Moving clusters
  - T-Patterns

# **Global Patterns**

# Clustering

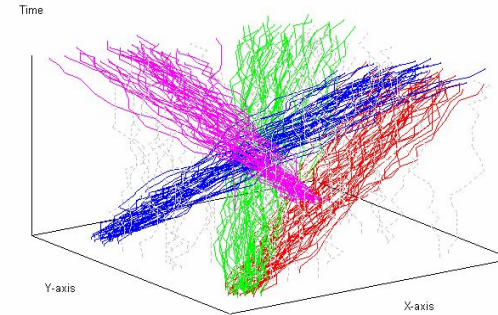
(sample K-means family)

- Find  $k$  subgroups that form compact and well-separated clusters



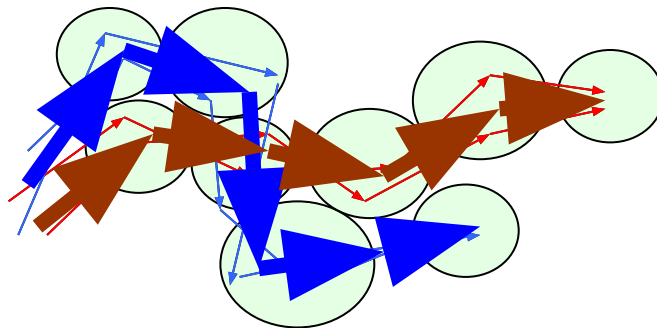
# Trajectory clustering

- Trajectories are grouped based on similarity



# Trajectory Clustering

- Questions:
  - Which distance between trajectories?
  - Which kind of clustering?
  - What is a cluster 'mean' in our case?
    - A representative trajectory?



# Trajectory Distances

# Families of Trajectory Distances

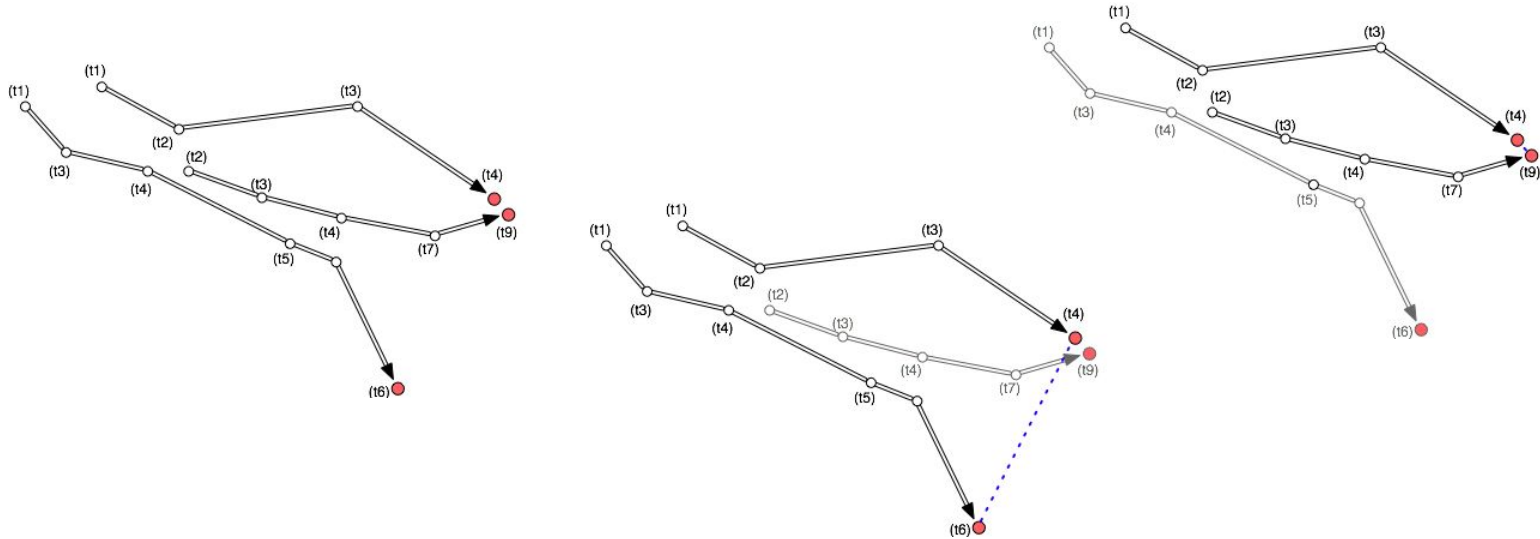
- Trajectory as **set** of points
  - Single-point approaches
  - Hausdorff distance
- Trajectory as **sequence** of points
  - Fréchet distance
  - Time series distances: Euclidean, DTW & LCSS
- Trajectory as **time-stamped sequence** of points
  - Average Euclidean distance



# Reduce Trajectories to single points

## Common Destination

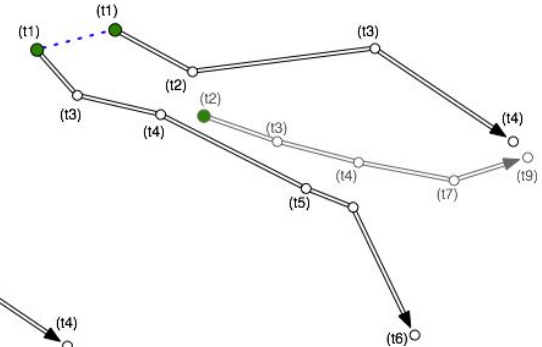
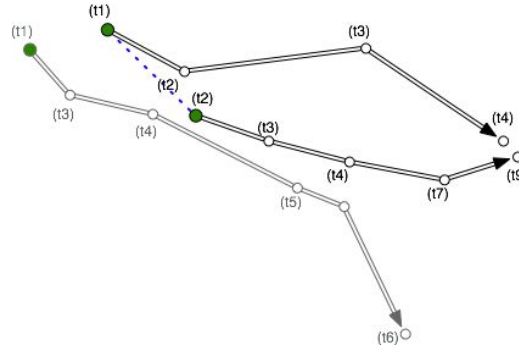
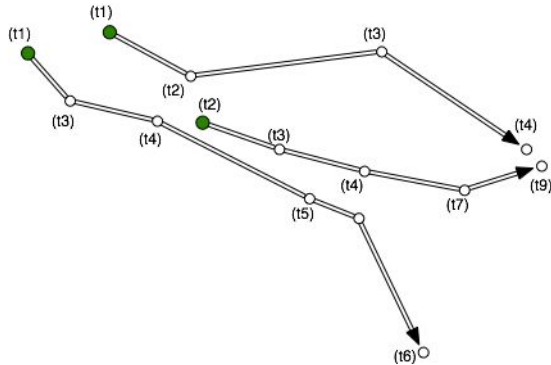
- ❑ Select last point  $P_{last}$  for each trajectory
- ❑  $D(T, T') = \text{Euclidean}(P_{last}, P'_{last})$



# Reduce Trajectories to single points

## Common Origin

- ❑ Select first point *Pfirst* for each trajectory
- ❑  $D(T, T') = \text{Euclidean}(P_{\text{first}}, P'_{\text{first}})$

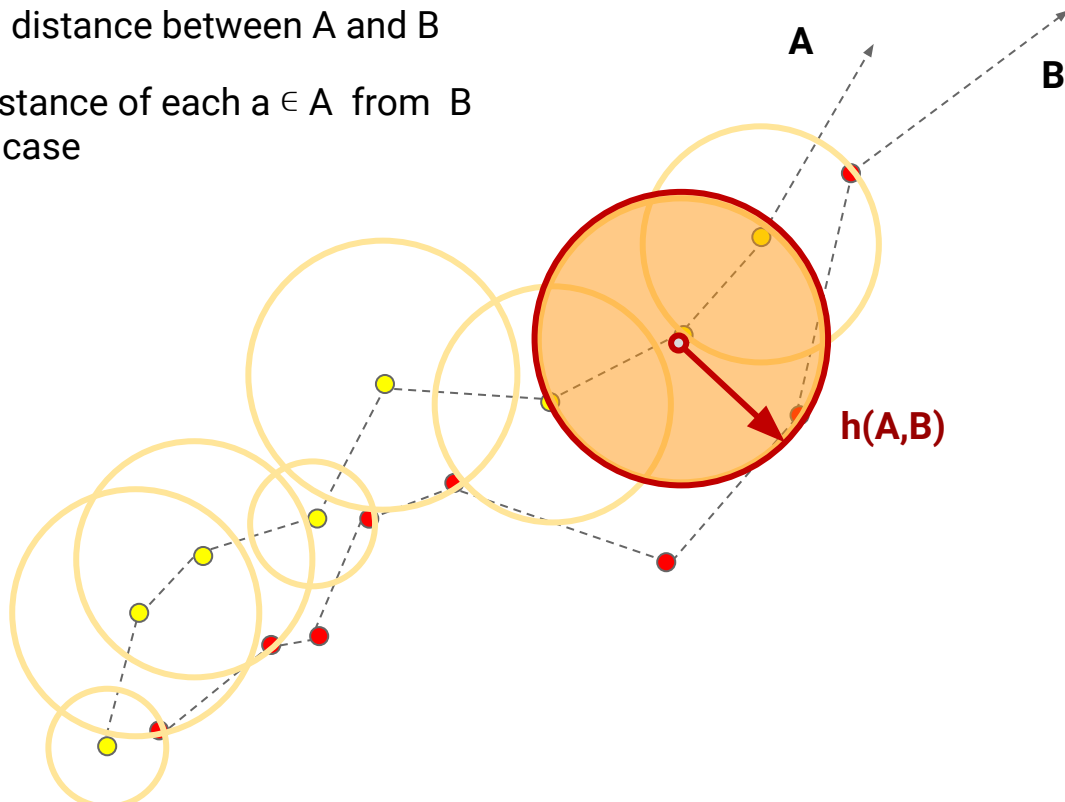


# Trajectory as set of points

## Hausdorff distance

Start from an example: distance between A and B

- Find minimum distance of each  $a \in A$  from B
- Return the worst case

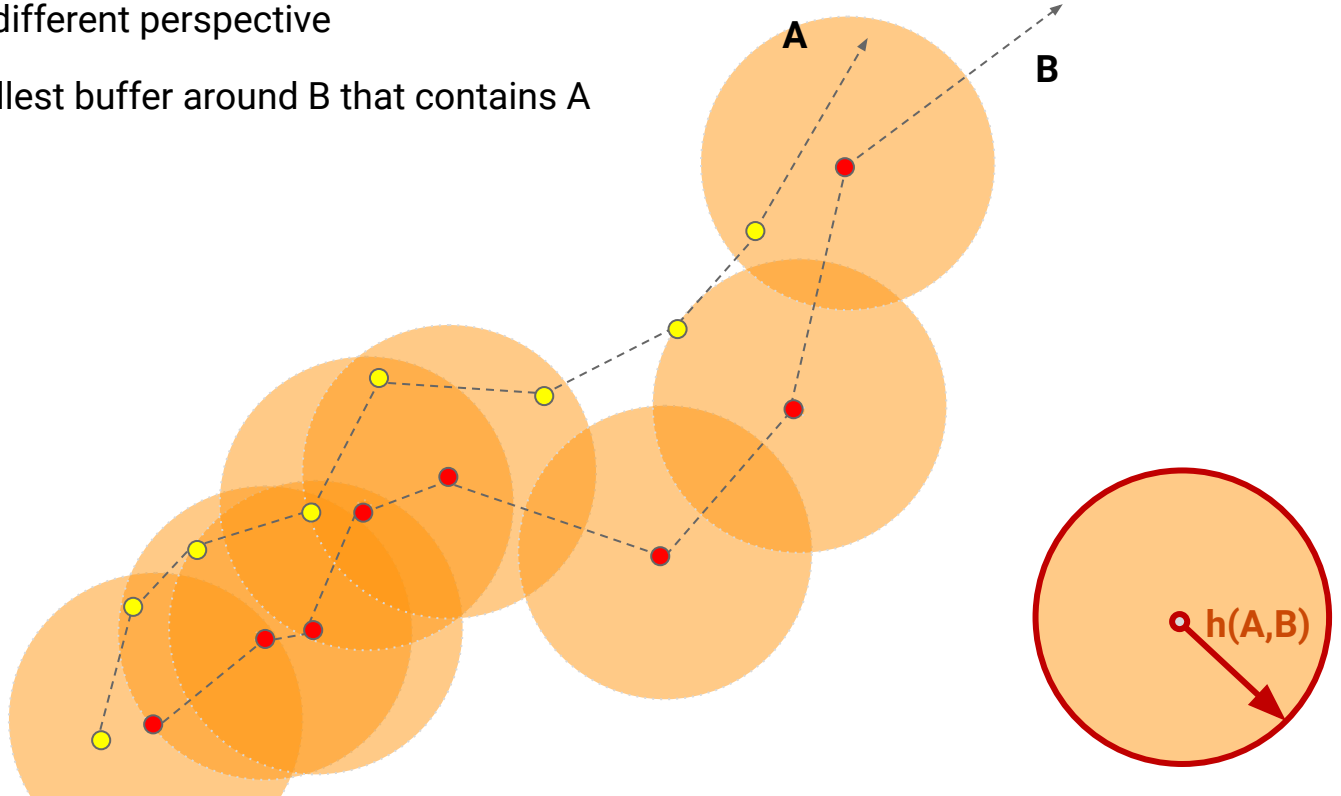


# Trajectory as set of points

## Hausdorff distance

Same concept, from a different perspective

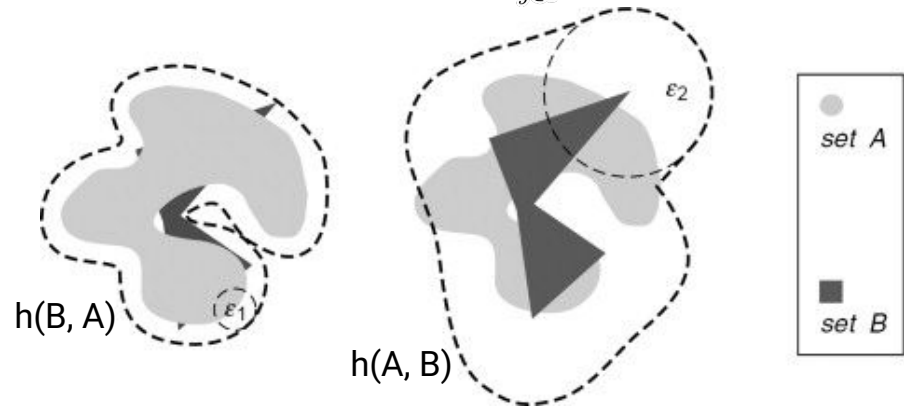
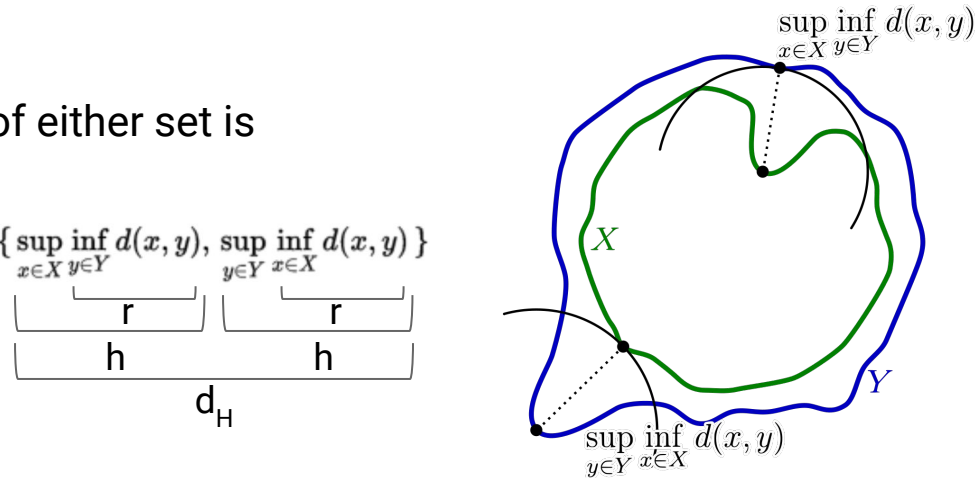
- $h(A,B)$  is the smallest buffer around B that contains A



# Trajectory as set of points

## Hausdorff distance

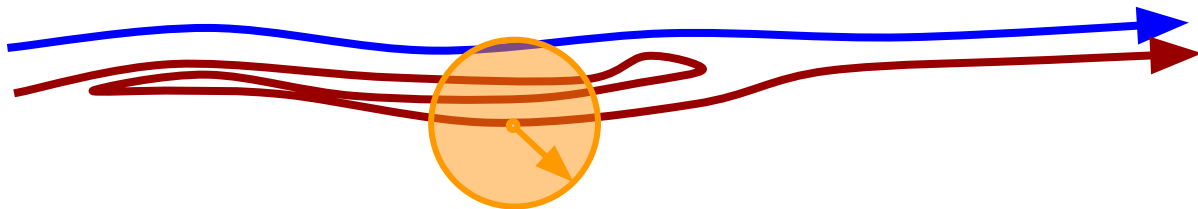
- Intuition: two sets are close if every point of either set is close to some point of the other set
- Formally, given sets A and B:  $d_H(X, Y) = \max\{ \sup_{x \in X} \inf_{y \in Y} d(x, y), \sup_{y \in Y} \inf_{x \in X} d(x, y) \}$ 
  - $r(x, B) = \inf \{d(x, b) : b \in B\}$
  - $h(A, B) = \sup\{r(a, B) : a \in A\}$
  - $d_H(A, B) = \max \{ h(A, B), h(B, A) \}$
- Equivalently:
  - $h(A, B)$  = minimum buffer radius around B that fully contains A
  - $d_H(A, B)$  = symmetric version of  $h()$



# Trajectory as sequence of points

## From Hausdorff to Fréchet distance

- Applied to trajectories, sometimes Hausdorff distance yields counter-intuitive results
- How far are these?



- Reasonable in a set-oriented view
- Wrong in terms of moving objects

# Trajectory as sequence of points

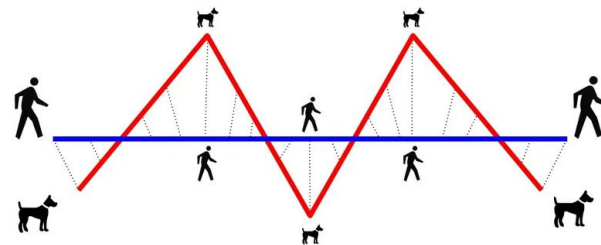
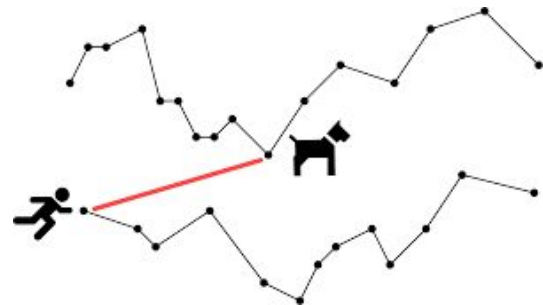
## Fréchet distance

- Intuition: equivalent of Dynamic Time Warping on continuous curves
- Formally:

$$F(A, B) = \inf_{\alpha, \beta} \max_{t \in [0, 1]} \left\{ d\left(A(\alpha(t)), B(\beta(t))\right) \right\}$$

$\alpha$  and  $\beta$  are non-decreasing mappings from  $[0, 1]$  to the points along A and B in forward order

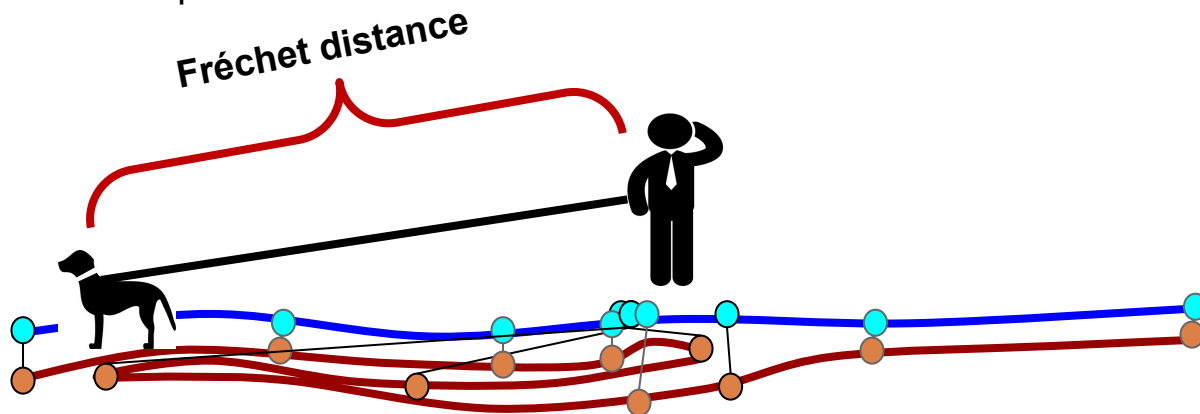
- Also described as “minimum leash length”:
  - What is the minimum length of a leash needed to stroll around the dog, given the owner’s and the dog’s trajectories?



# Trajectory as sequence of points

## Fréchet distance

- Back to our example

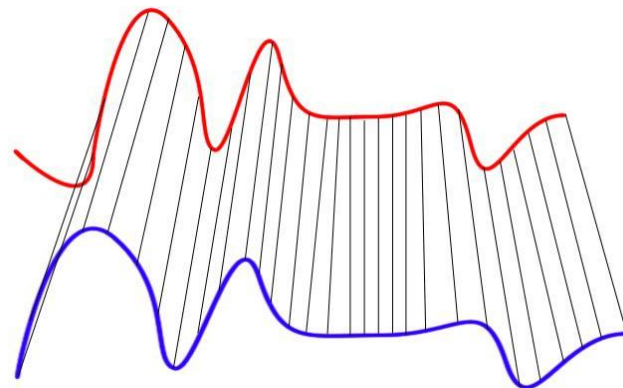




# Trajectory as sequence of points

## Time series distances

- Just replace “difference of two values” with “spatial distance of two points”
- Examples:
  - Dynamic Time Warping
    - Very similar to Fréchet!
  - Edit Distance with Real values
    - Similar to DTW, but can remove points



Dynamic Time Warping Matching

- IMPORTANT: most methods in this class assume constant sampling rates

# Trajectory as time-stamped sequence of points

## Average Euclidean distance

- The trajectory is seen as a continuous spatio-temporal curve
- Positions between input points (the GPS fixes) linearly interpolated

$$D(\tau_1, \tau_2) |_T = \frac{\int_T d(\tau_1(t), \tau_2(t)) dt}{|T|}$$

distance between moving objects  $\tau_1$  and  $\tau_2$  at time  $t$

- “Synchronized” behaviour distance
  - Similar objects = almost always in the same place at the same time
- Computed on the whole trajectory

# Clustering Algorithms

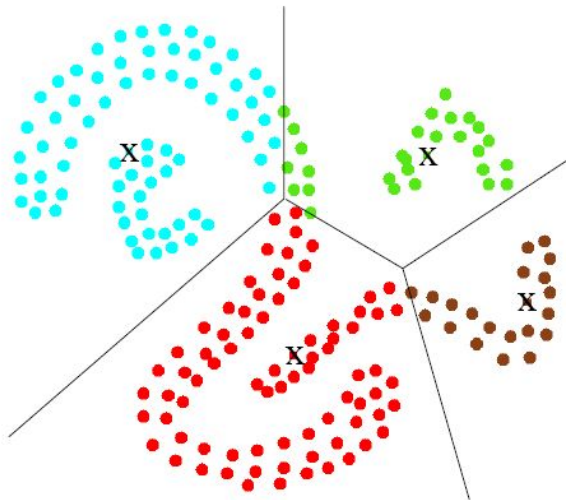
# Which kind of clustering method?

- In principle, any distance-based algorithm
- General requirements:
  - Non-spherical clusters should be allowed
    - E.g.: A traffic jam along a road = “snake-shaped” cluster
  - Tolerance to noise
  - Low computational cost
  - Applicability to complex, possibly non-vectorial data
- A suitable candidate: Density-based clustering
  - OPTICS (Ankerst et al., 1999)
  - Evolution of standard DBSCAN

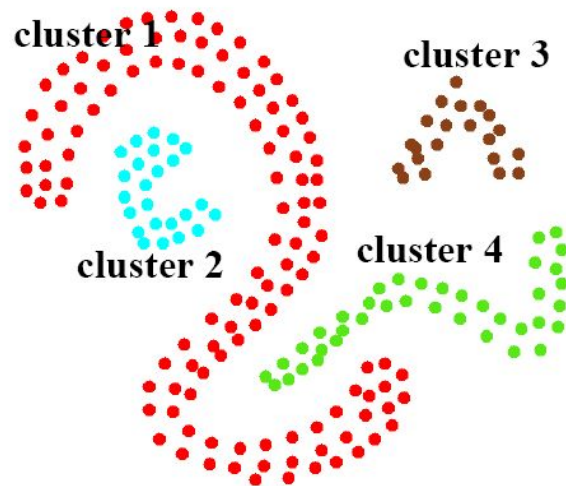
# Density Based Clustering

## A refresher

K-means



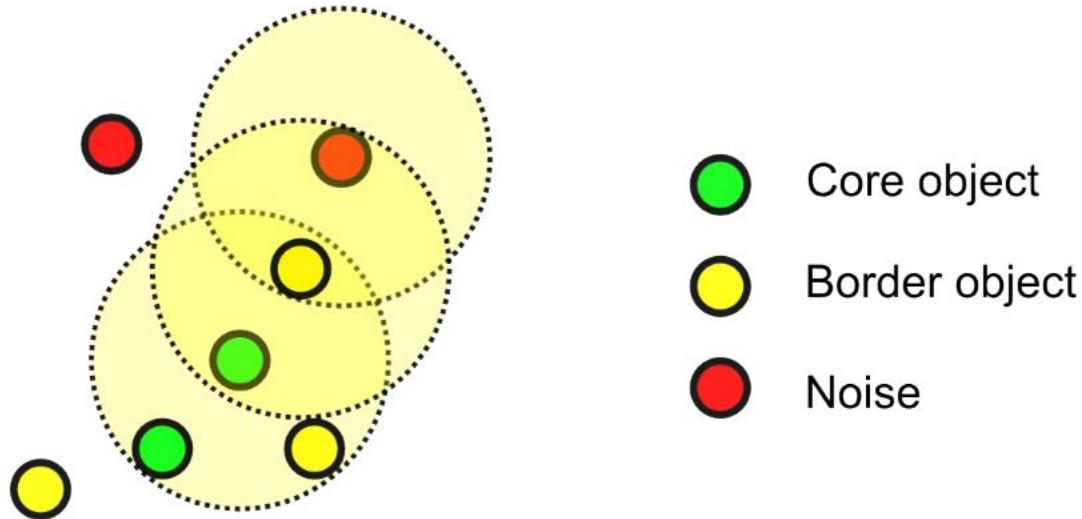
*Density-based*



# Density Based Clustering

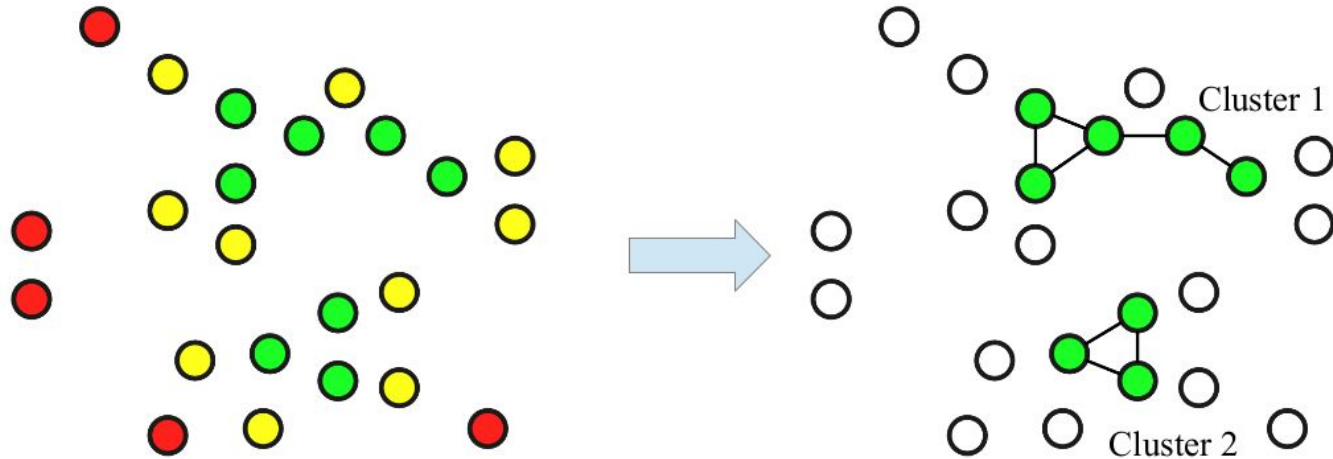
Step 1: label points as core (dense), border and noise

- Based on thresholds  $R$  (radius of neighborhood) and  $\text{min\_pts}$  (min number of neighbors)



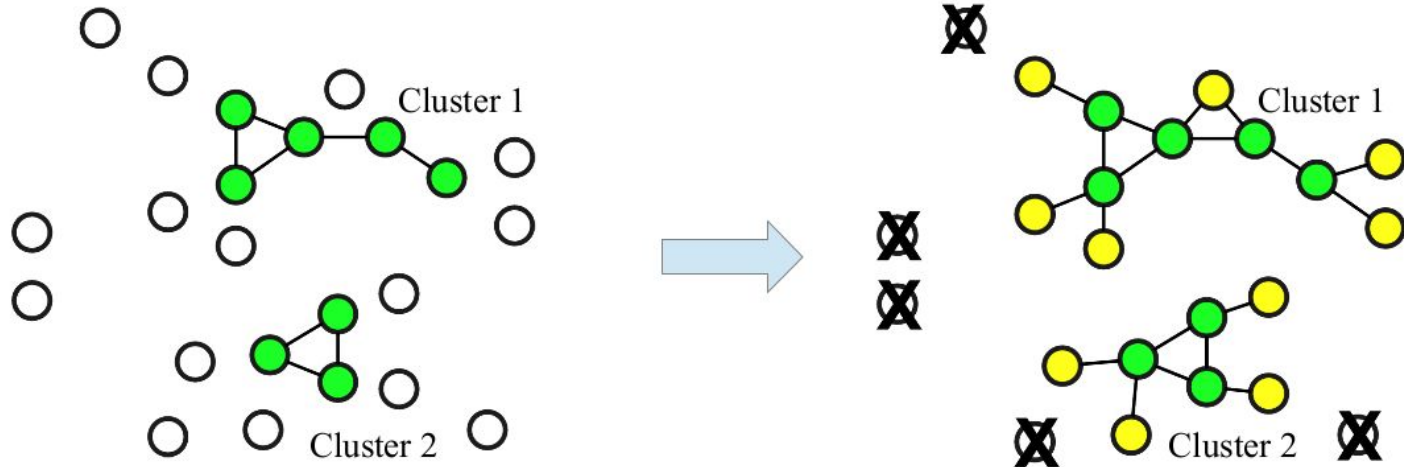
# Density Based Clustering

Step 2: connect core objects that are neighbors, and put them in the same cluster



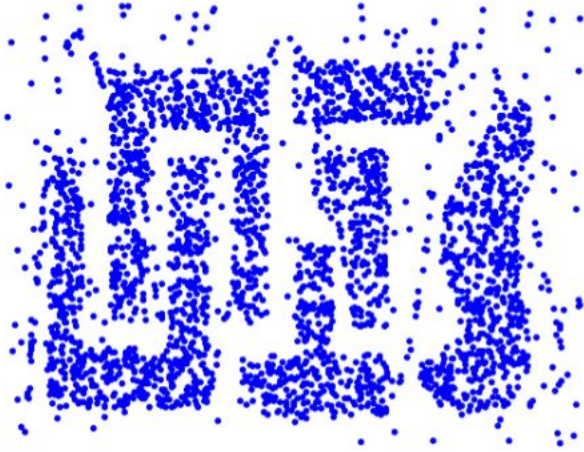
# Density Based Clustering

Step 3: associate border objects to (one of) their core(s), and remove noise

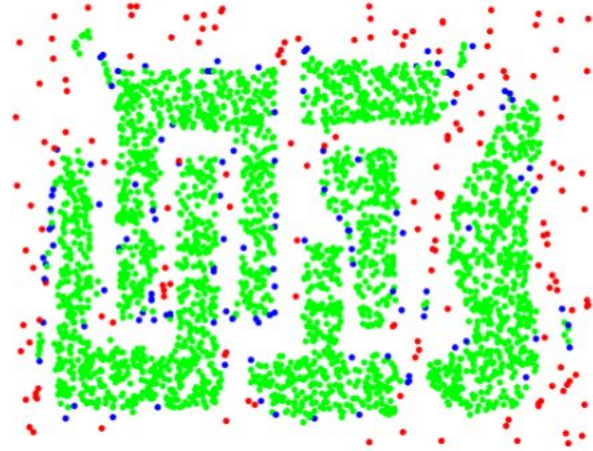




# Density Based Clustering

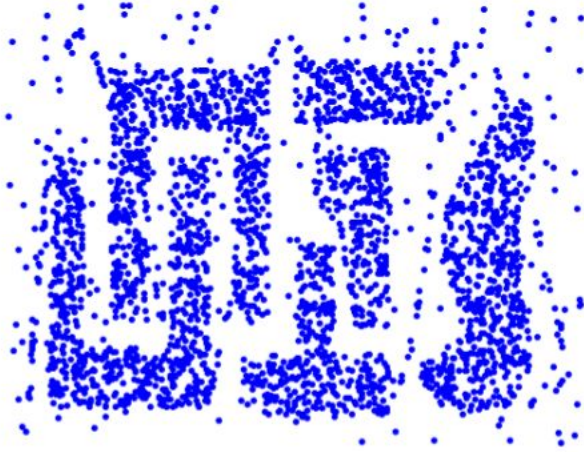


Original Points

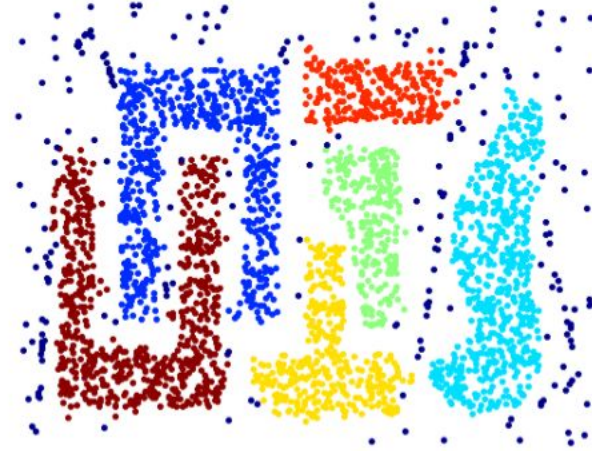


Point types: **core**,  
**border** and **noise**

# Density Based Clustering



Original Points

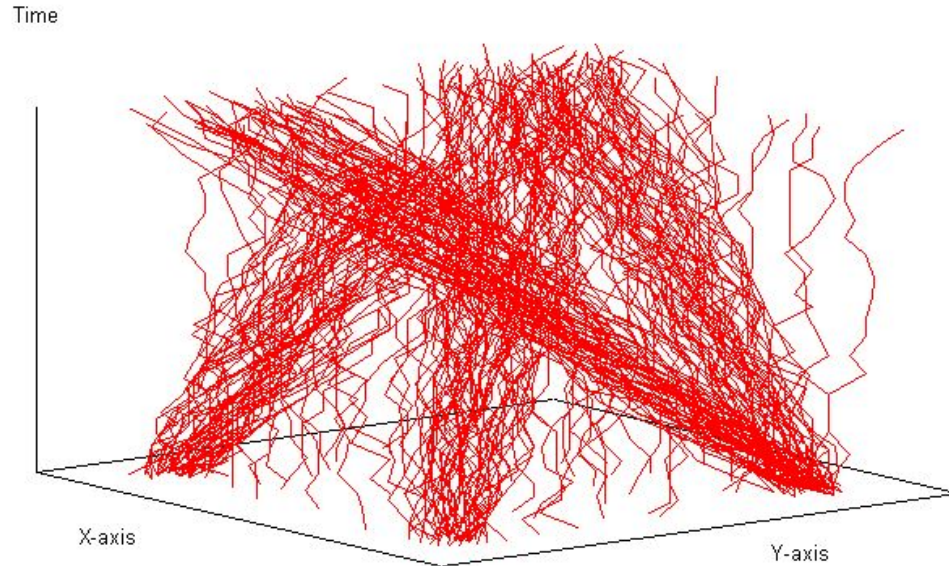


Clusters

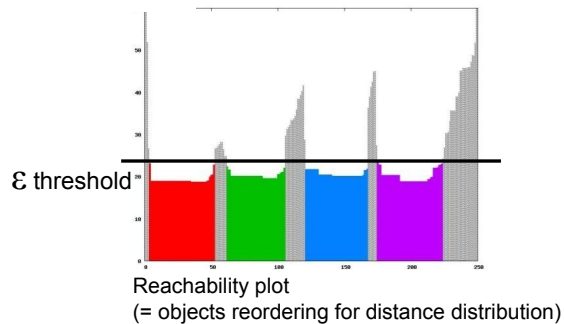
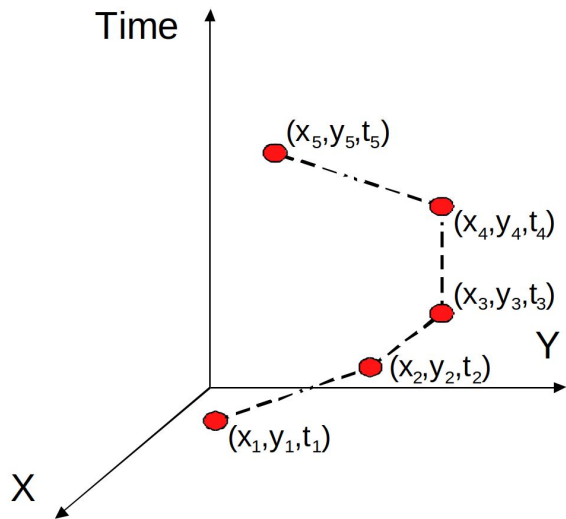
- Resistant to Noise
- Can handle clusters of different shapes and sizes

# A sample dataset

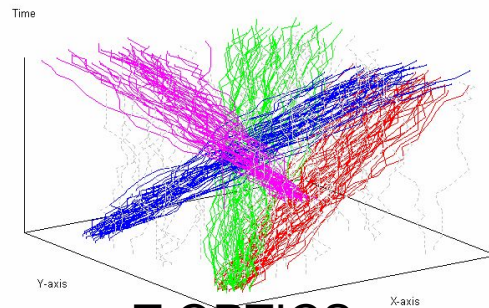
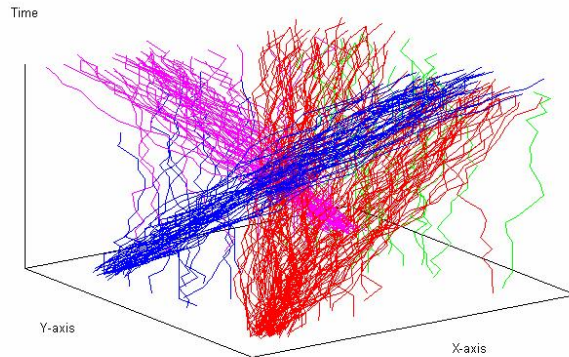
- A set of trajectories forming 4 clusters + noise (synthetic)



# T-OPTICS vs. K-means



## K-means

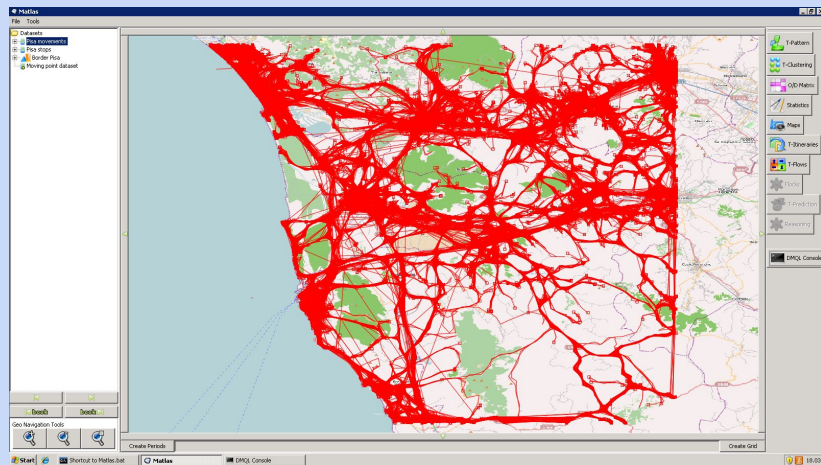


## T-OPTICS

# INTERVALLO

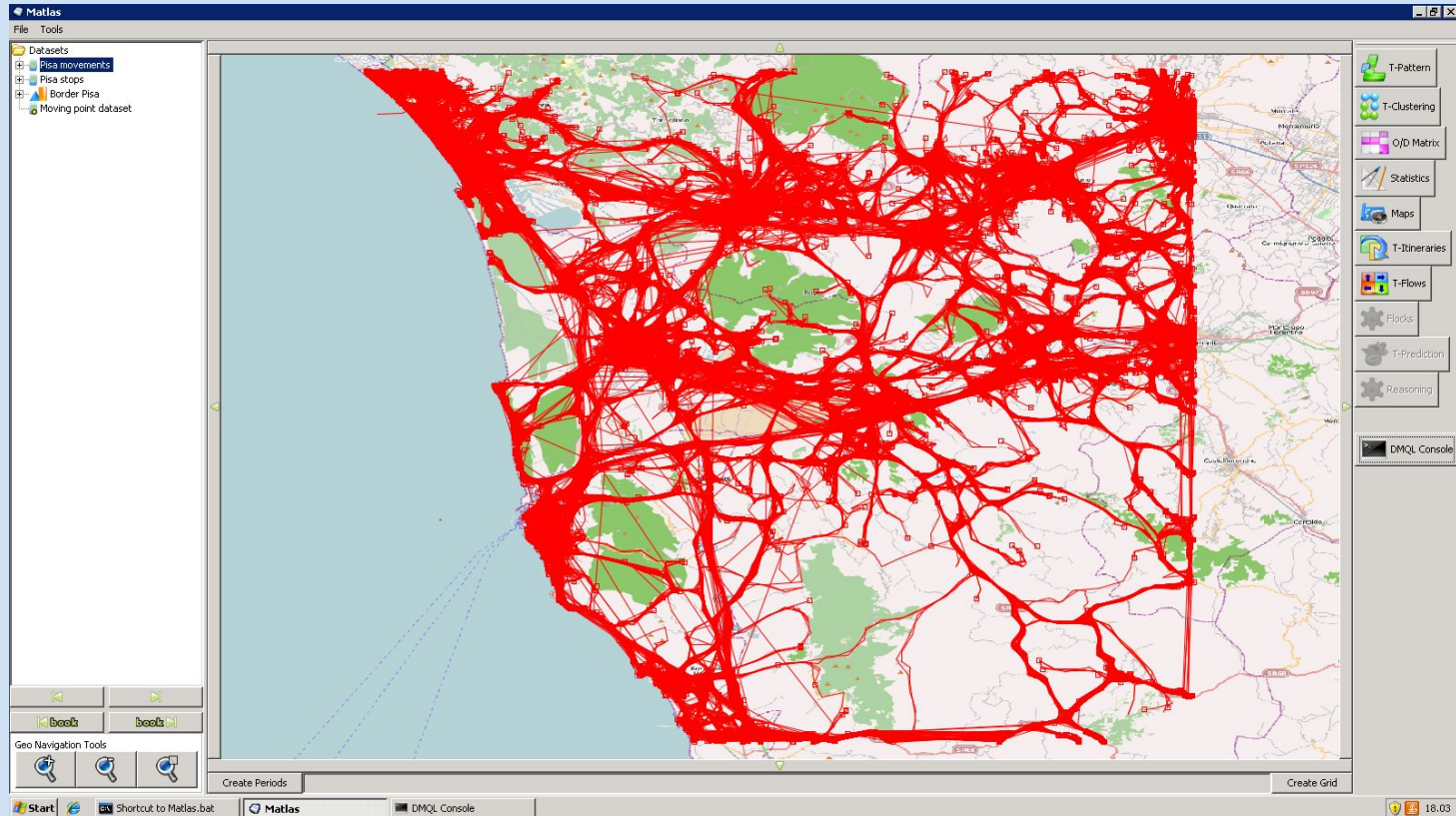
## What's the source of traffic in Pisa?

Trajectory clustering at work

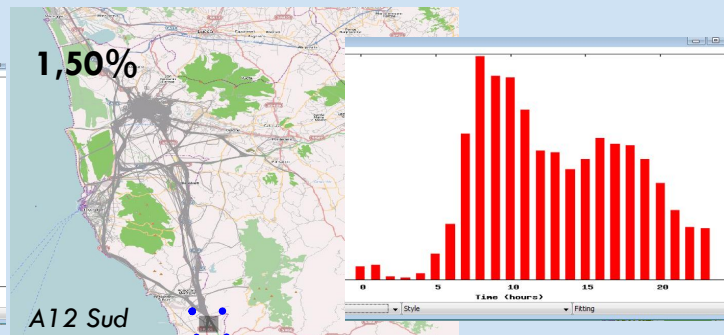
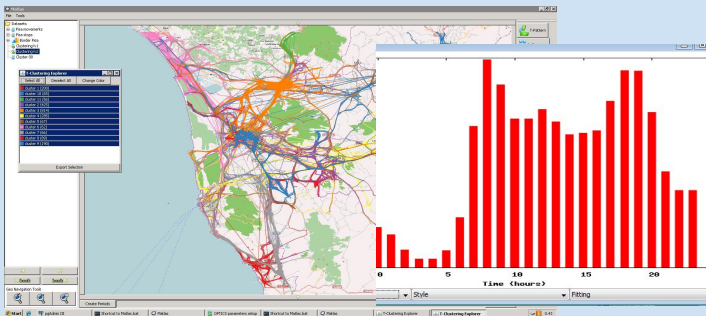




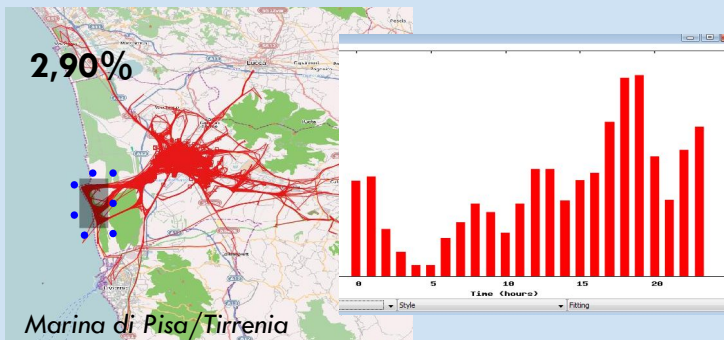
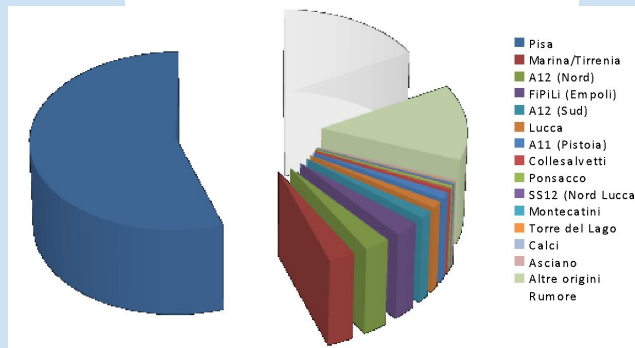
# Access patterns using T-clustering



# Characterizing the access patterns: origin & time



Origin distribution

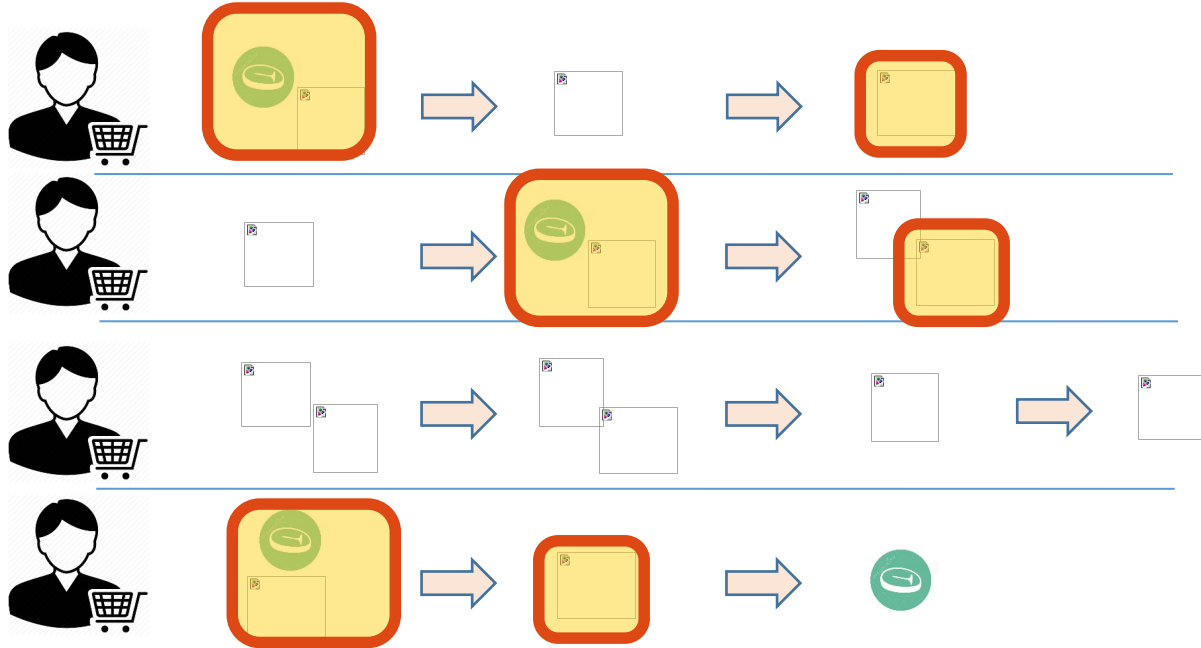


# Local Trajectory Patterns



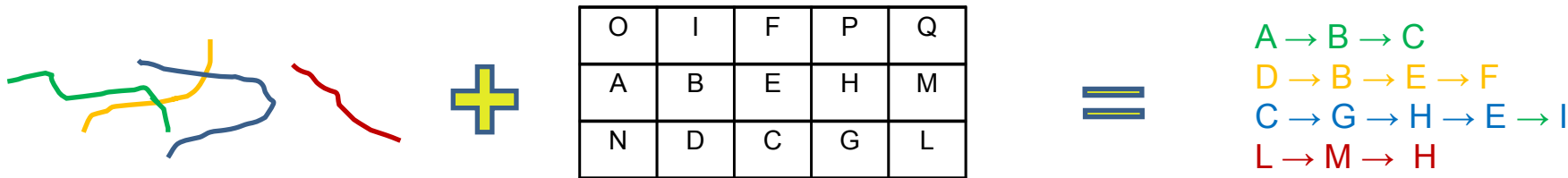
# Frequent patterns in sequences

- Frequent sequences (a.k.a. Sequential patterns)
- Input: sequences of events (or of groups)



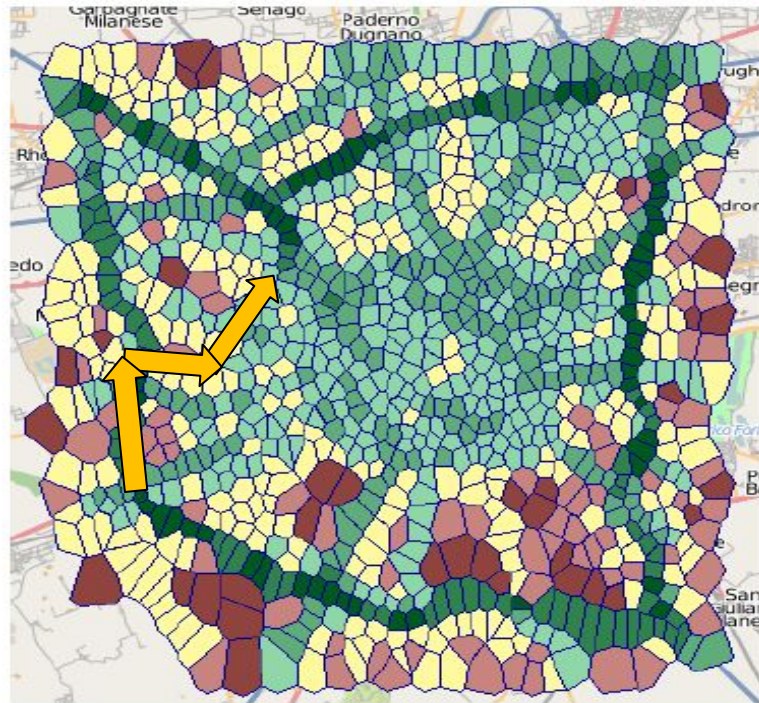
# From trajectories to sequential patterns: the easy way

- Map each trajectory to a sequence of areas
  - Predefined or driven by data

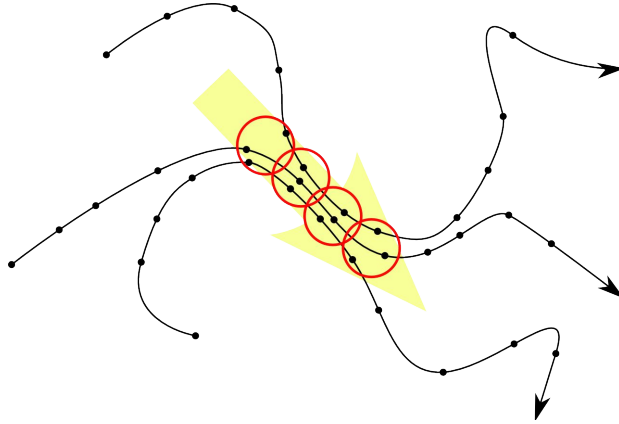


# From trajectories to sequential patterns: the easy way

- A “Trajectory frequent pattern” can be defined as sequential pattern over traversed areas



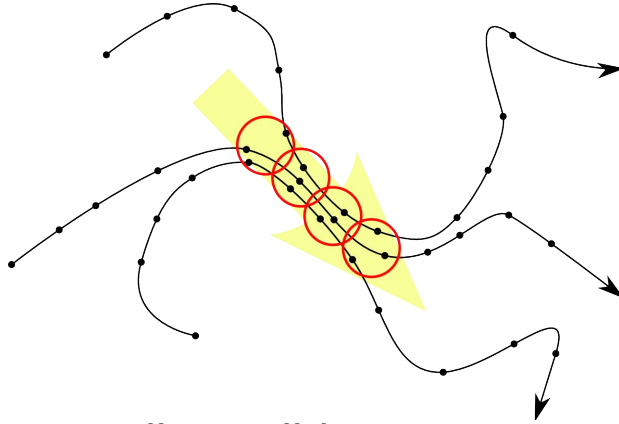
# Moving Trajectory Flocks



- Group of objects that move together (close to each other) for a time interval

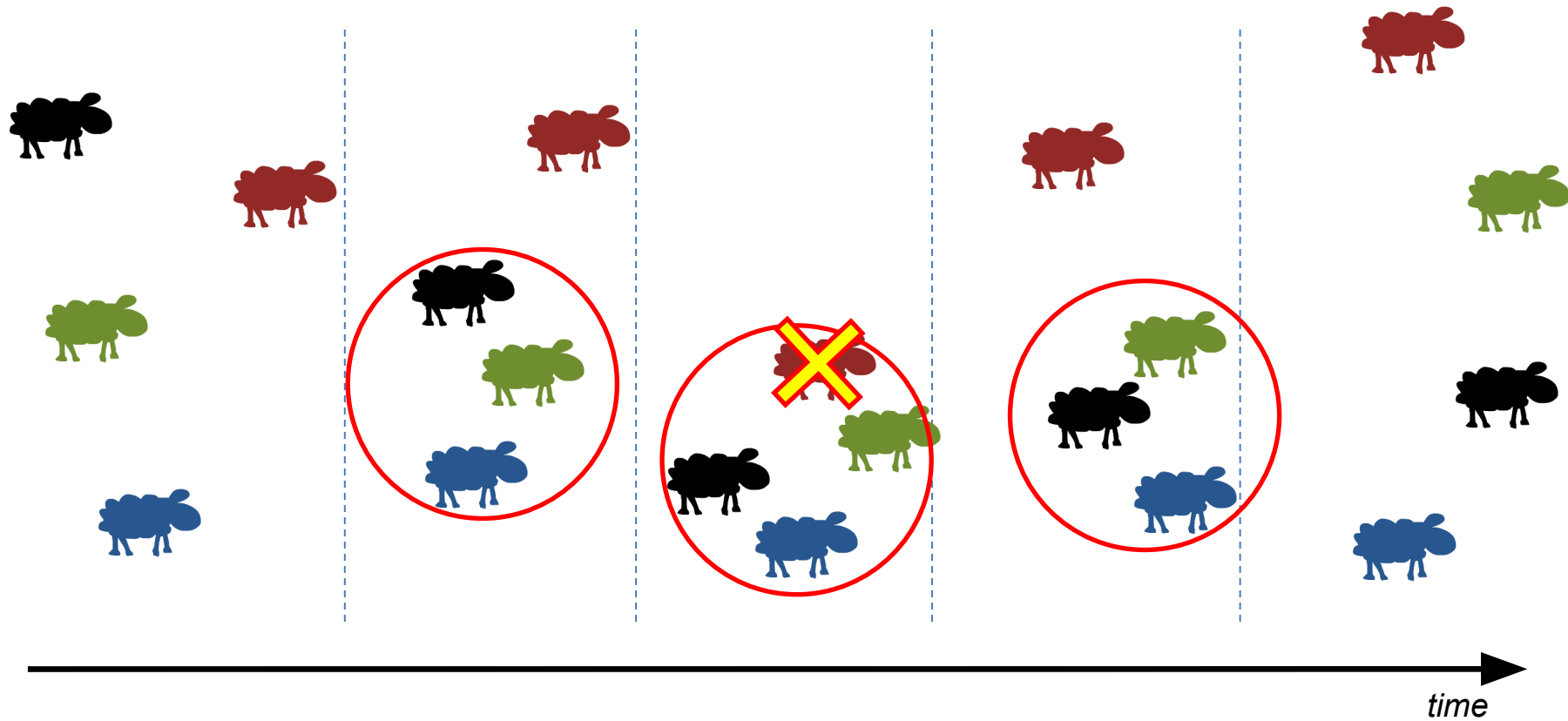


# Moving Trajectory Flocks



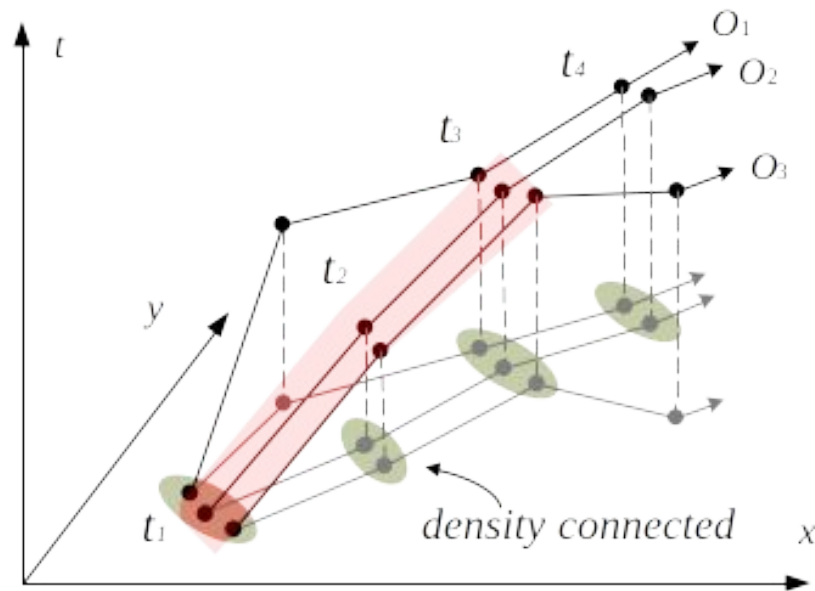
- Group of objects that move together (close to each other) for a time interval
- Discover all possible:
  - sets of objects  $O$ , with  $|O| > \text{min\_size}$  and
  - time intervals  $T$ , with  $|T| > \text{min\_duration}$
- such that for all timestamps  $t \in T$  the points in  $O|t$  are contained in a circle of radius  $r$

# Moving Trajectory Flocks



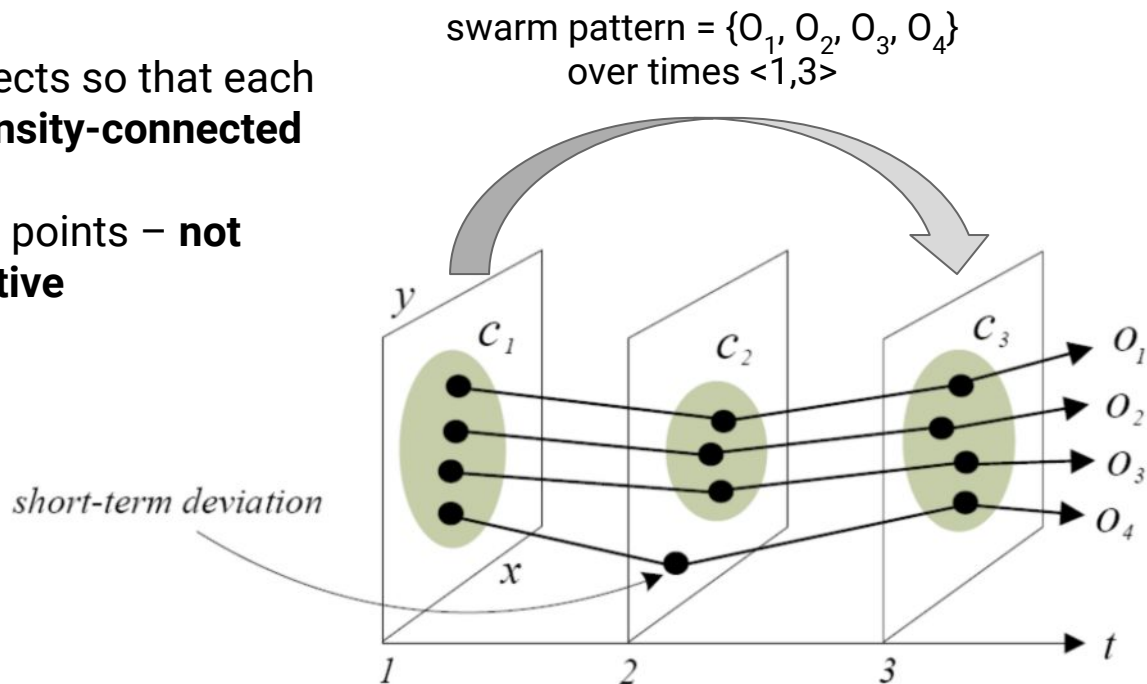
# From Flocks to Convoys

- Given radius  $r$ , size  $m$ , and time threshold  $k$ 
  - find all groups of objects so that each group consists of **density-connected objects** w.r.t.  $r$  and  $m$
  - during at least  $k$  consecutive time points
- Basically replace circles with DBSCAN clusters



# From Convoys to Swarms

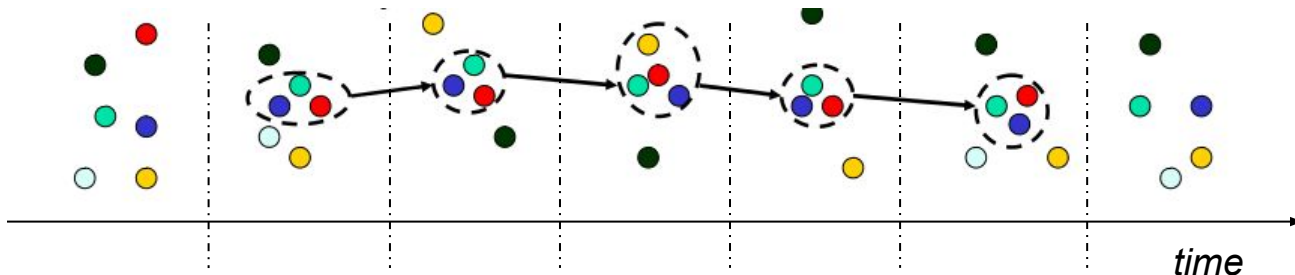
- Given radius  $r$ , size  $m$ , and time threshold  $k$ 
  - find all groups of objects so that each group consists of **density-connected objects** w.r.t.  $r$  and  $m$
  - during at least  $k$  time points – **not necessarily consecutive**





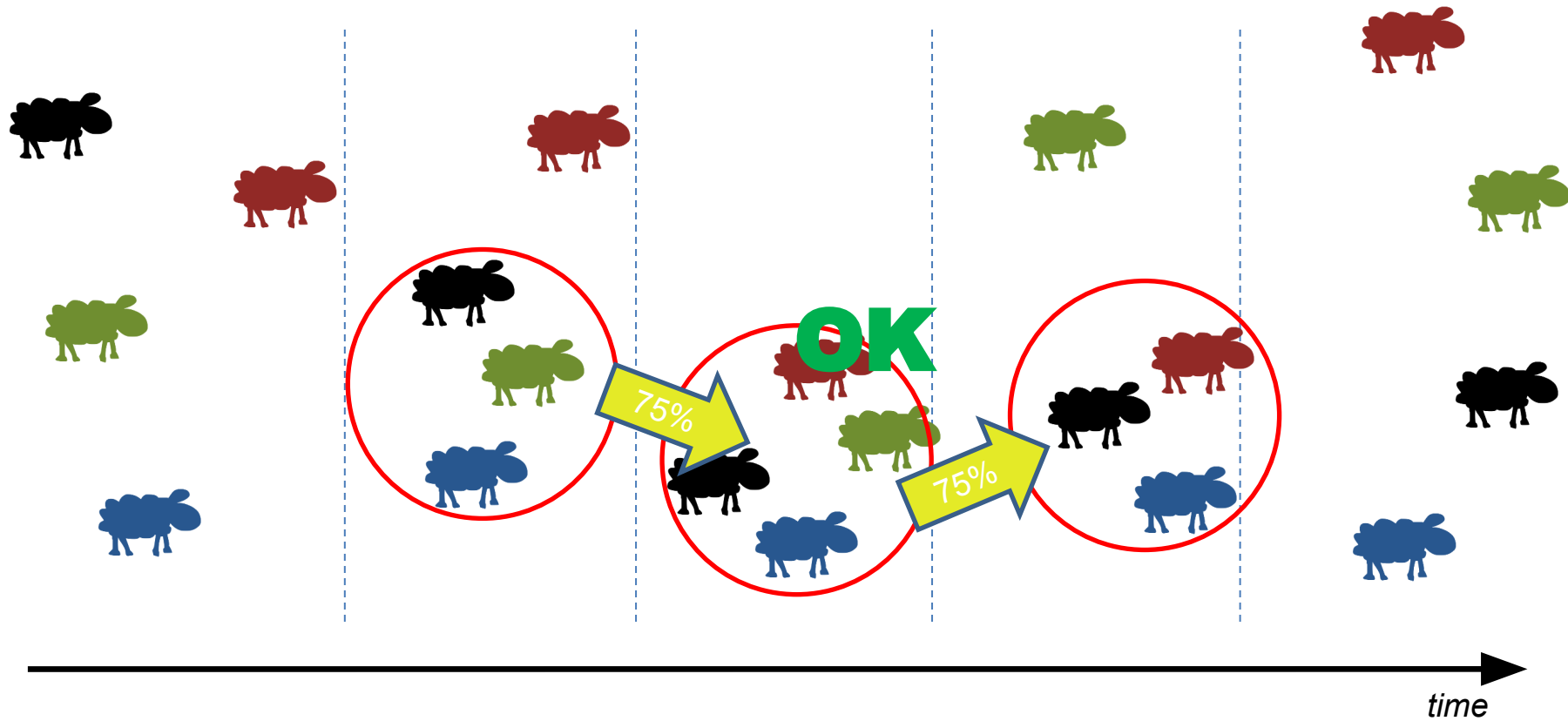
# Moving Clusters

- A **moving cluster** is a set of objects that move close to each other for a long time interval



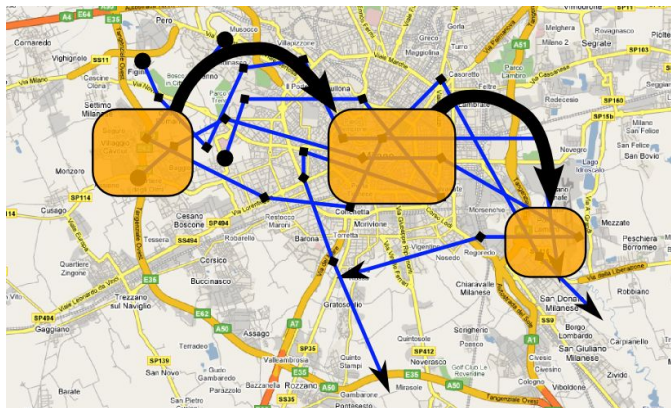
- Formal Definition [Kalnis et al., SSTD'05]:
  - A **moving cluster** is a sequence of (snapshot) clusters **c1, c2, ..., ck** such that for each timestamp  $i$  ( $1 \leq i < k$ ):  $\text{Jaccard}(c_i, c_{i+1}) \geq \theta$ 
    - $\text{Jaccard}(c_i, c_{i+1}) = |c_i \cap c_{i+1}| / |c_i \cup c_{i+1}|$
    - $0 < \theta \leq 1$
  - Clustering computed with density-based method (DBSCAN)

# Moving Clusters



# T-Patterns

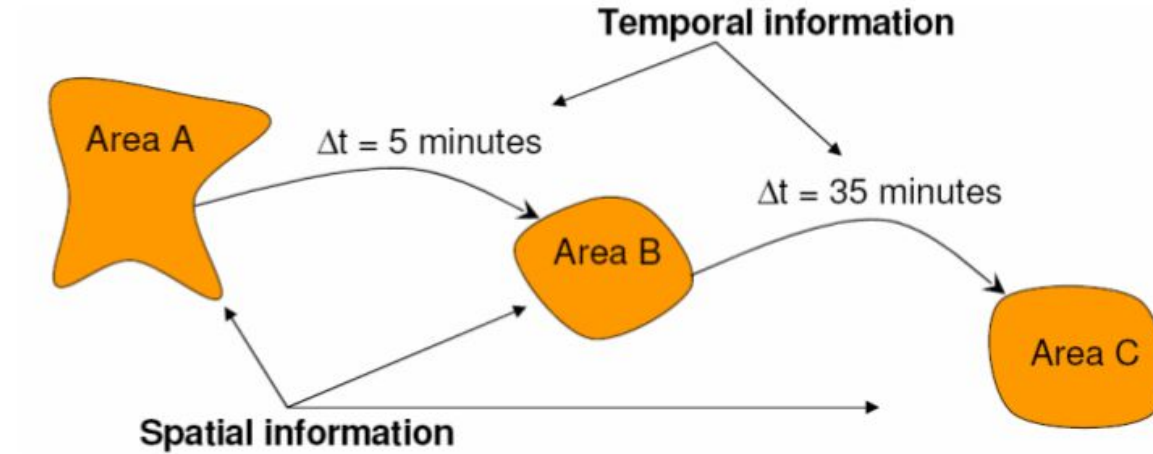
- A sequence of visited regions, **frequently** visited in the **specified order** with **similar transition times**



$$A_0 \xrightarrow{t_1} A_1 \xrightarrow{t_2} \dots A_{n-1} \xrightarrow{t_n} A_n$$

$t_i$  = transition time,  $A_i$  = spatial region

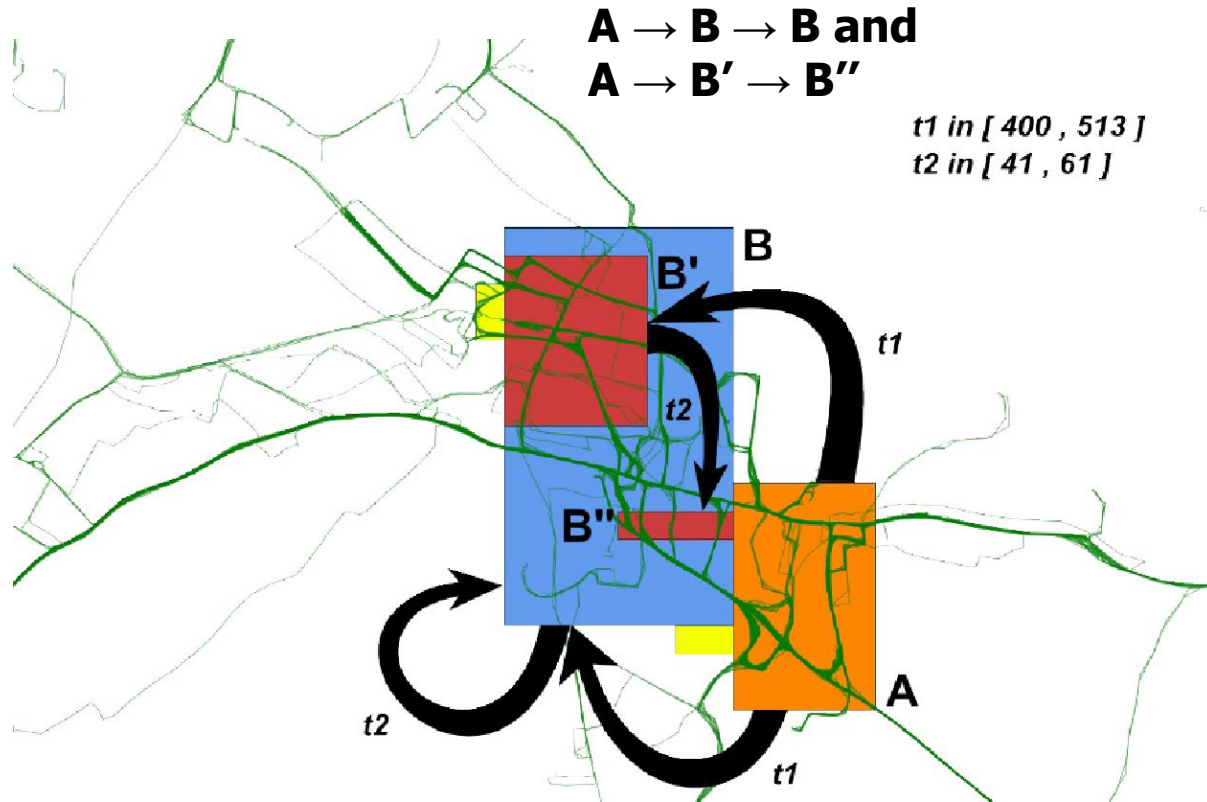
# T-Patterns



- Key features
  - Includes typical transition times in the output
  - Areas are automatically detected – not “the easy way”

# Sample Trajectory Pattern

Data Source: Trucks in Athens (273 trajectories)

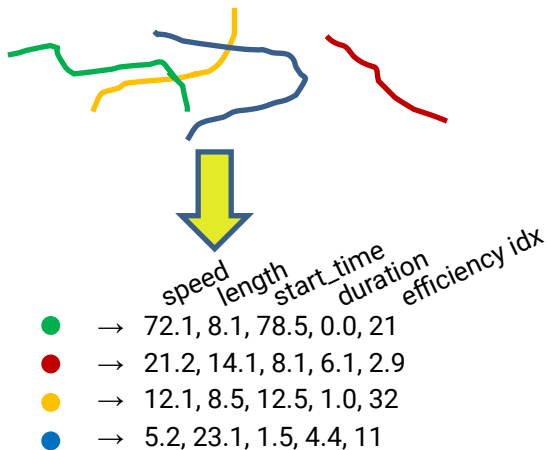


# **A quick peek into Deep Learning**

# Deep Learning approaches to Trajectory Clustering

## Traditional approach

- Preprocess the data to obtain features

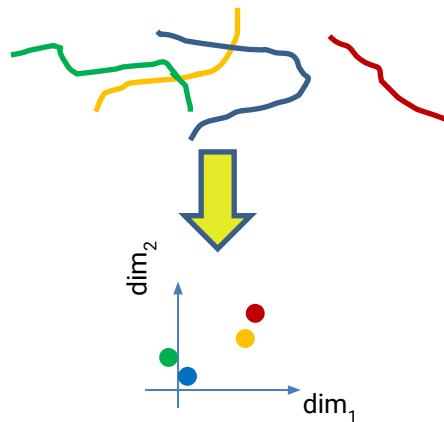


- Clustering over features



## Deep learning approach

- Learning a latent representation (or embeddings)

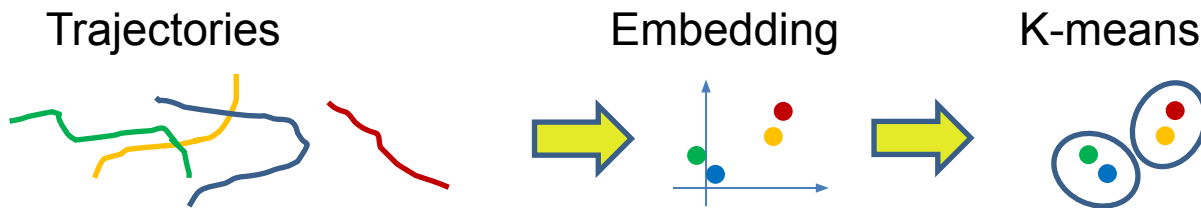


- Clustering over embeddings



# Deep Learning approaches to Trajectory Clustering

- Sample approach: DETECT: Deep Trajectory Clustering for Mobility-Behavior Analysis
- Basic idea:

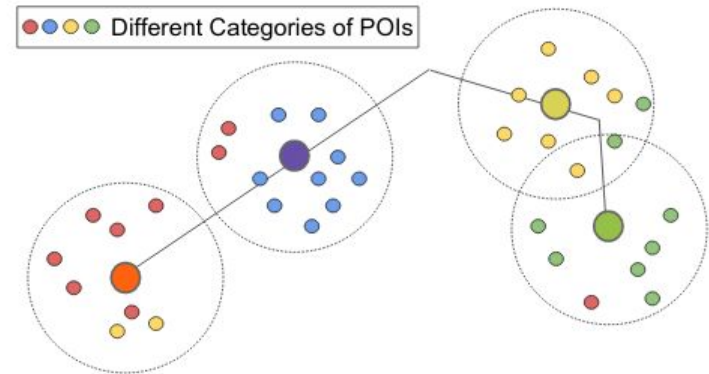
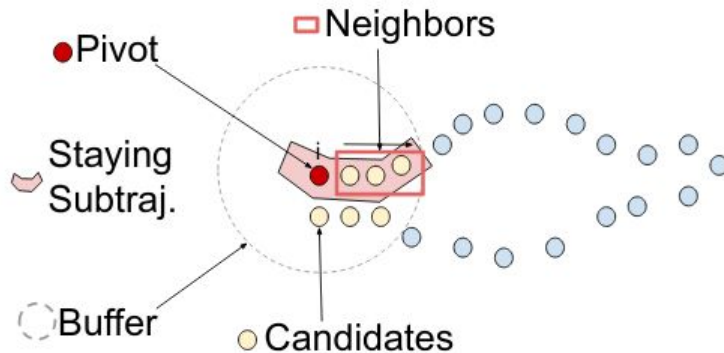


- Integrate the clustering step in the learning of embeddings
- Three steps:
  - Enrich trajectories with context
  - LSTM-based embedding of trajectories
  - Clustering on embeddings



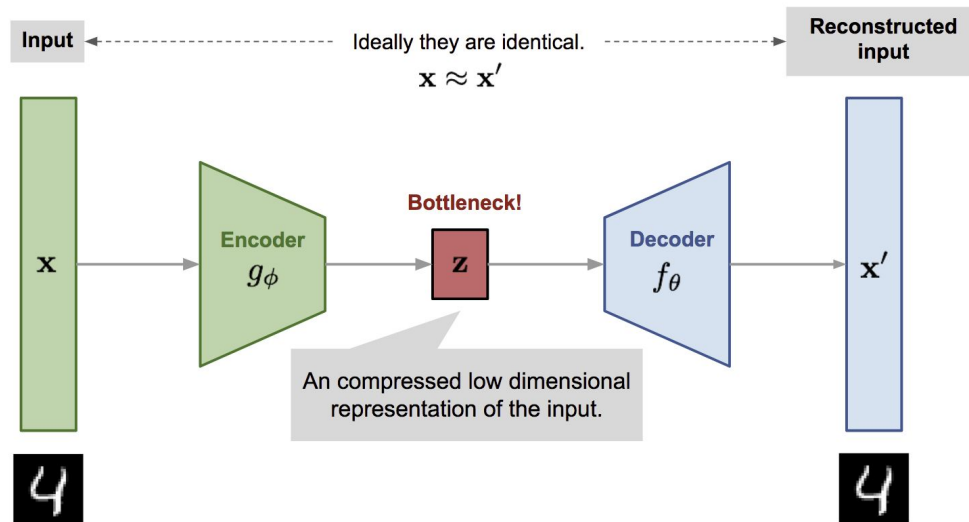
# DETECT / 1

- Enrich trajectories with context
  - Identify stay areas = segment of trajectory where there is no movement, basically a stop
  - Create a buffer around the area
  - Select all points-of-interest located there (hotels, shops, etc.)
  - Compute a feature vector, one feature per Pol category
- Output
  - $\text{Traj} = \langle (x,y,[f_1,\dots, f_n]), (x',y',[f'_1,\dots, f'_n]), \dots \rangle$



# DETECT / 2

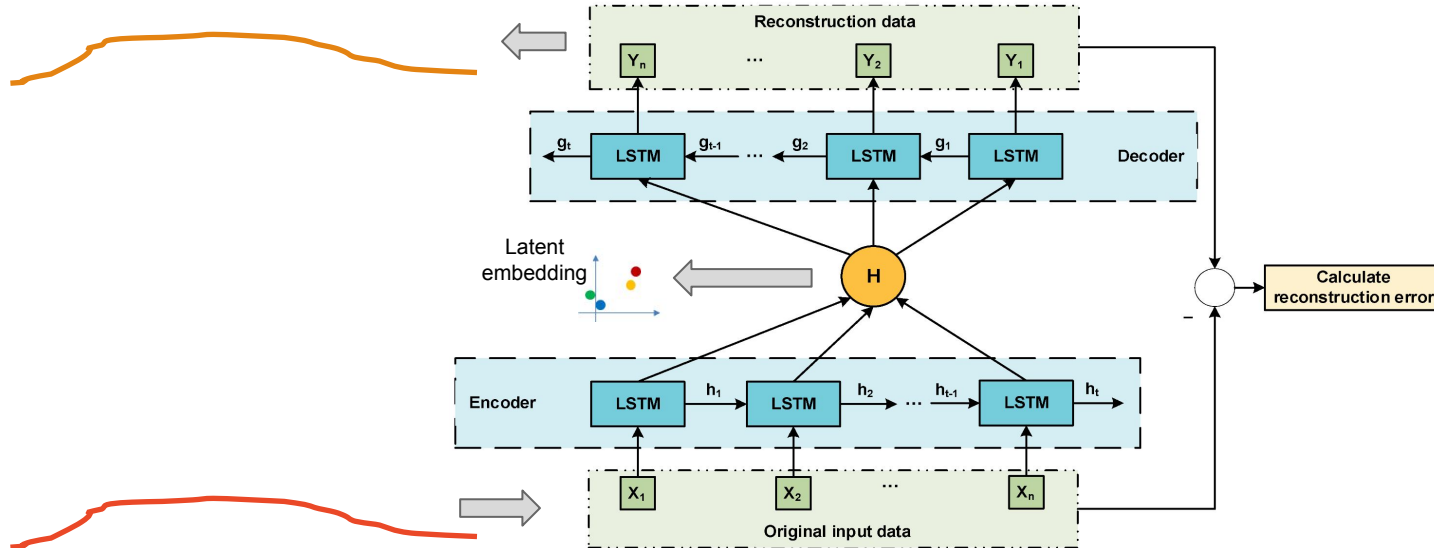
- LSTM-based embedding of trajectories
  - Apply an encoder-decoder schema to the enriched trajectories
  - Use LSTM as basic mechanism



- Objective: minimize the difference between the encoder input and the decoder output

# DETECT / 2

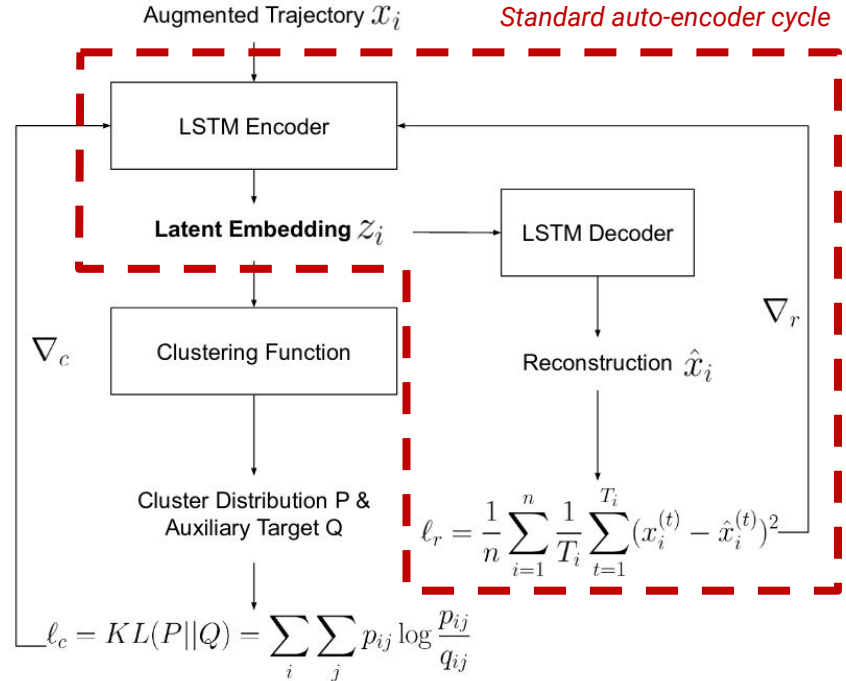
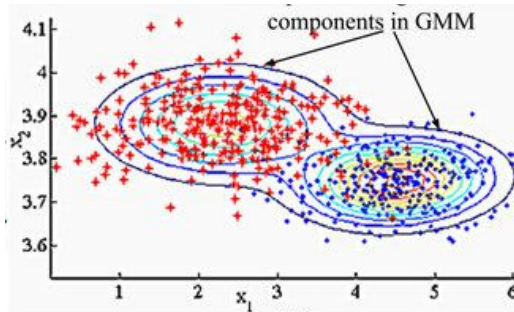
- LSTM-based embedding of trajectories
  - Apply an encoder-decoder schema to the enriched trajectories
  - Use LSTM as basic mechanism



- Objective: minimize the difference between the encoder input and the decoder output

# DETECT / 3

- Clustering on embeddings
- Clustering error becomes one term of the overall loss function
- P & Q = points distribution
  - P = real data (embedded)
  - Q = clusters (Student t-distribution around centers)



# Homeworks

# Food for thought

- **Hausdorff:** let interpret trajectory  $T = \langle p_1, p_2, \dots, p_n \rangle$  as a polyline, thus containing also the segments (= infinite sets of points) between each pair  $p_i, p_{i+1}$ . How can you compute the Hausdorff distance between two trajectories?
- **Local patterns in mobility:** can you find some examples in urban mobility where local patterns might be useful? Frequent sequences? Flocks?
- **Local or global:** we discover that 90% of vehicles in a city pass through the same road segment (maybe a bridge). Is that a local or global pattern? Is there really a difference?
- **The thin line between clusters and flows:** if you take all the trajectories that form a specific flow (same origin and same destination), how many clusters do you expect to find? What is the difference between a flow and a (trajectory) cluster?

# Material

- [paper] [Spatio-Temporal Trajectory Similarity Measures: A Comprehensive Survey and Quantitative Study](https://arxiv.org/abs/2303.05012v2), Danlei Hu et al., arXiv:  
<https://arxiv.org/abs/2303.05012v2>
  - Sections 1, 2, 3 (only the measures seen in these slides)
- [paper] [Computing longest duration flocks in trajectory data](https://dl.acm.org/doi/10.1145/1183471.1183479), Joachim Gudmundsson and Marc van Kreveld (2006), GIS '06,  
<https://dl.acm.org/doi/10.1145/1183471.1183479>
  - Section 1 (definitions)
- [paper] [Discovery of Convoys in Trajectory Databases](https://arxiv.org/abs/1002.0963v1), Hoyoung Jeung et al., VLDB 2008, <https://arxiv.org/abs/1002.0963v1>
  - Section 3 (definitions)

# Material

- [paper] [On Discovering Moving Clusters in Spatio-temporal Data](#), Kalnis, P., Mamoulis, N., Bakiras, S. SSTD 2005. [https://doi.org/10.1007/11535331\\_21](https://doi.org/10.1007/11535331_21)
  - Sections 1, 2, 4.1 (definitions and basic algorithm)
- [paper] [Trajectory pattern mining](#), Giannotti, Nanni, Pedreschi, Pinelli. KDD 2007. <https://dl.acm.org/doi/10.1145/1281192.1281230>
  - Section 3 (definitions)
- [paper] [DETECT: Deep Trajectory Clustering for Mobility-Behavior Analysis](#), M. Yue et al. Big Data 2019. <https://arxiv.org/abs/2003.0135>
  - Section II (focus on definitions and overall approach, not details)