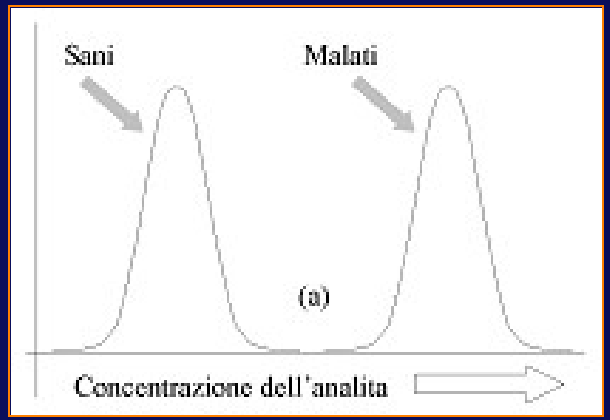
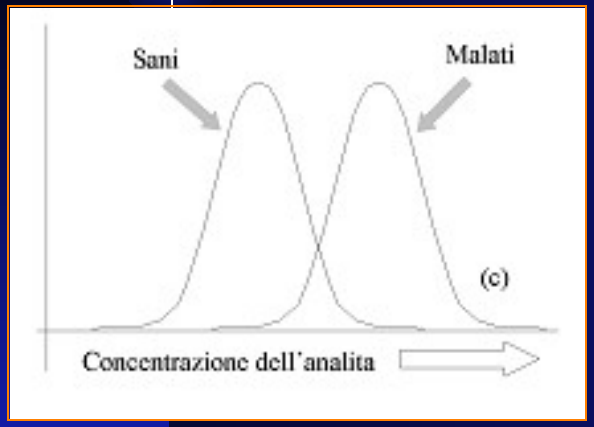
Bayes Theorem in Medicine

The ideal test should have confidence level 100%, classifying a patient as healthy or sick with absolute certainty. The ideal test would give answers like in the picture on the right. X-axis is test outcome, Y-axis is frequency of the outcome.





Real test have non-negligible error margin. The outcome is positive for some healthy people (*false positive*) and negative for some sick people (*false negatives*).

In other words, being positive does not mean being necessarily sick, and vice versa.

The Bayes theorem help us in evaluating the probability that a piece of evidence (the test outcome) is really due to a possible cause (the presence of the illness).

Let us consider this example

A certain illness has a 2% diffusion in the overall population.

A clinical test discovers the illness in 80% of cases (20% *false negatives*), while in 10% of cases it is positive even when the patient is healthy (10% *false positives*).

My test is positive. What is the probability I am ill?

Almost all physicians say " about 80%”.

Reality is very different!

Phisicians error is confusing the probability that the test is positive if the patient is ill with teh probability that teh patient is ill if teh test is positive.

We know that $\mathbf{P(T+ \mid M) = 80\%}$; but we are really interested in $\mathbf{P(M \mid T+)}$.

We can find it with the Bayes theorem.

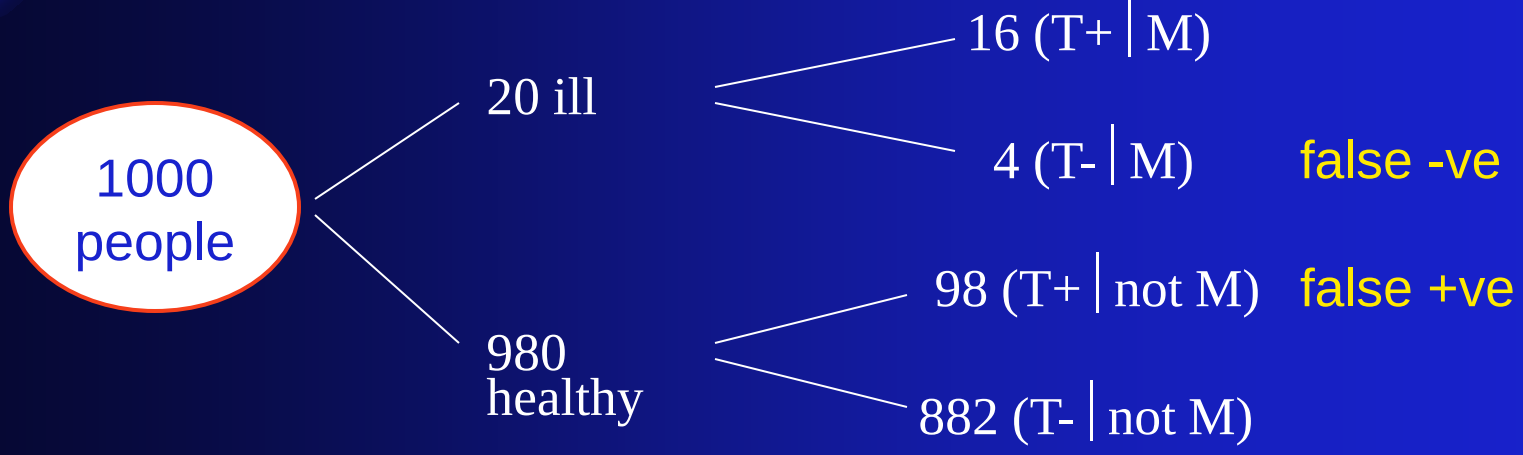
Let us apply the Bayes theorem:

$$P(M|T+) = \frac{P(T+|M) \cdot P(M)}{P(T+)} = \frac{0.8 \cdot 0.02}{0.8 \cdot 0.02 + 0.1 \cdot 0.98} \approx 14\%$$

The probability of being really ill is low, even though the test is positive.

The point can be easier to catch in another perspective.

Let us split teh sample of 1,000 people who underwent the test.

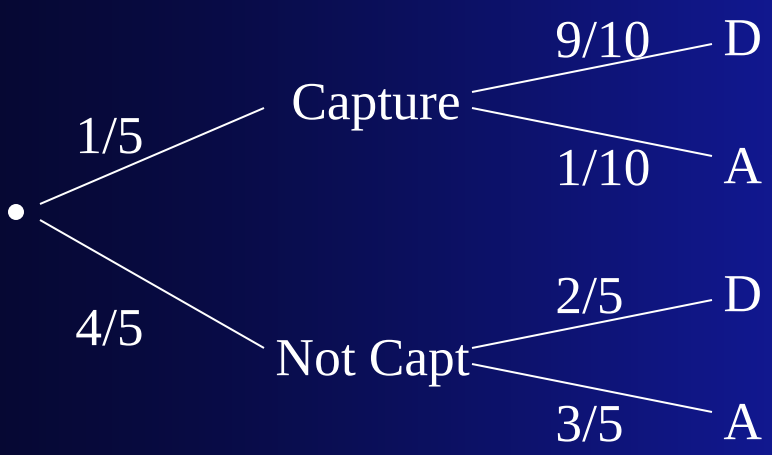


Out of these 1,000 people, 114 (16 + 98) are positive: but only 16 are really ill. The probability is only 14/116 = 14% circa.

A dog is chasing a cat. If the dog catches the cat, it kills that with probability 90%, but it manages to catch the cat only in 20% cases. The cat runs away very quickly and has probability 40% to be killed by a car.

We find the cat dead. What is the probability that the dog killed it?

First, we compute the probability the cat dies:



$P(C) \cdot P(D \mid C) = 1/5 \cdot 9/10 = 9/50$

$P(NC) \cdot P(D \mid NC) = 4/5 \cdot 2/5 = 8/25$

P(D) = 9/50 + 8/25 = 1/2

Bayes Theorem:

$$P(C|D) = \frac{P(C) \cdot P(D|C)}{P(D)} = \frac{9/50}{1/2} = \frac{9}{25} = 36\%$$