

### Esercitazione #5

HOFL, inferenza di tipi e semantica operazionale

#### [Ex. 1] Determinare il tipo del termine HOFL

$$t \stackrel{\text{def}}{=} \mathbf{rec} \ x. \ ((\lambda y. \ \mathbf{if} \ y \ \mathbf{then} \ 0 \ \mathbf{else} \ 0) \ x).$$

Poi calcolare la sua forma canonica (lazy).

#### [Ex. 2] Determinare il tipo del termine

$$map \stackrel{\text{def}}{=} \lambda f. \ \lambda x. \ ((f \ \mathbf{fst}(x)), (f \ \mathbf{snd}(x)))$$

Poi calcolare le forme canoniche (lazy) dei termini seguenti.

$$t_1 \stackrel{\text{def}}{=} map \ (\lambda z. \ 2 \times z) \ (1,2) \qquad t_2 \stackrel{\text{def}}{=} \mathbf{fst} \ (map \ (\lambda z. \ 2 \times z) \ (1,2))$$

## Teoria dei domini

[Ex. 3] Let  $(D, \sqsubseteq_D)$  be a CPO and  $f : D \to D$  be a continuous function. Prove that the set of fixpoints of f is itself a CPO (ordered by  $\sqsubseteq_D$ ).

## HOFL semantica denotazionale

[Ex. 4] (Test for convergence) We would like to modify the denotational semantics of HOFL assigning to the construct

#### if t then $t_0$ else $t_1$

- the semantics of  $t_1$  if the semantics of t is  $\perp_{\mathbb{Z}_{\perp}}$ , and
- the semantics of  $t_0$  otherwise.
- Is it possible? If not, why?

[Ex. 5] (Strict conditional) Modify the operational semantics of HOFL by taking the following rules for conditionals:

 $\frac{t \to 0 \quad t_0 \to c_0 \quad t_1 \to c_1}{\text{if } t \text{ then } t_0 \text{ else } t_1 \to c_0} \qquad \qquad \frac{t \to n \quad n \neq 0 \quad t_0 \to c_0 \quad t_1 \to c_1}{\text{if } t \text{ then } t_0 \text{ else } t_1 \to c_1}.$ 

Without changing the denotational semantics, prove that:

- 1. for any term t and canonical form c, we have  $t \to c \Rightarrow \forall \rho$ .  $\llbracket t \rrbracket \rho = \llbracket c \rrbracket \rho$ ;
- 2. in general  $t \Downarrow \not\Rightarrow t \downarrow$  (exhibit a counterexample).

# [Ex. 6] Determine the type of the HOFL term $t \stackrel{\text{def}}{=} \mathbf{rec} \ f. \ (\ \lambda x.1 \ , \ \mathbf{fst}(f) \ 0 \ )$

Then, compute the (lazy) denotational semantics of t.