

Algorithm Engineering – Exercises

11 July 2024

Name and Surname:

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Question #1 [rank 3] Simulate the Reservoir Sampling algorithm by drawing $m=3$ items from a sequence of length $n=9$: [a, b, c, d, e, f, g, h, i], assuming that the random integers extracted by the algorithm are [2, 4, 1, 2, 3, 1].

Question #2 [score 4+4+3]. Given the set S of pairs {<C,4>, <A,6>, <I, 10>, <B,3>, <H,7>, <F,2>}, where the first component is the key, and the second component is the priority.

- Build a TREAP data structure by inserting the pairs in that order (you can assume that it is a MIN heap).
- Show the execution of the SPLIT operation on the key D.
- Show the execution of the MERGE operation on the two TREAPS formed by the previous SPLIT.

Question #3 [rank 4+4+4] Given the set of strings $S = \{abaa, abca, abma, baa, bbb\}$, build a Patricia trie and show the steps executed by a lexicographic search for the strings $P1 = bbc$, and for the string $P2 = abb$.

Question #4 [rank 4] Given the string $T = aba$, and the probabilities $P(a) = \frac{3}{4}$ and $P(b) = \frac{1}{4}$, show the result of the Arithmetic Coding algorithm applied on T .
(*hint*: Please work with the dyadic fractions, not change them into reals.)

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Questions

- **[7 points]** Given a sequence S of items and an integer $m < n/2$, show the pseudo-code of the *reservoir sampling* algorithm.
- **[3 points]** Prove the correctness of the *reservoir sampling* algorithm (i.e., prove that it does a uniform sampling).
- **[7 points]** Describe the LSD algorithm to sort n items of b bits each.
- **[7 points]** Given two sorted lists of integers, say $L1$ and $L2$ of lengths n and m respectively, describe the “mutual-partition” algorithm to compute their intersection.
- **[3 points]** Write the algorithm to execute $\text{Access}(i)$ over an integer sequence encoded via Elias-Fano