

Algorithm Engineering

Q1

$$S = (1, 2, 4, 6, 7, 8, 9)$$

$$\begin{array}{ccc} \uparrow & & \uparrow \\ l=1 & S[m]=6 & r=7 \\ \text{low}=1 & \text{"} & \text{hi}=9 \\ & 4 & \end{array}$$

Encode 6 in the interval $\left\{ \begin{array}{l} \text{low} + m - l = 1 + 4 - 1 = 4 \\ \text{hi} + m - r = 9 + 4 - 7 = 6 \end{array} \right.$

This means to encode $6 - 4 = 2$ in $\lceil \log_2 6 - 4 + 1 \rceil = 2$ bits $\Rightarrow 10$

After that the two recursive subsequences are:

$(1, 2, 4)$ and $(7, 8, 9)$

$l=1, r=3, \text{low}=1, \text{hi}=5$

\Downarrow no bits emitted

$S[m]=2$ which is encoded in the interval $[1+2-1, 5-3+2]$
 $[2, 4]$

using $\lceil \log_2 4 - 2 + 1 \rceil = 2$ bits

We therefore encode $2 - 2 = 0$ in 2 bits $\Rightarrow 00$

For the encoding of Elias-Fano we use 4 bits, where the

low part consists of $\lceil \log_2 \frac{N}{n} \rceil = \lceil \log_2 \frac{10}{7} \rceil = 1$ bit

- 1 = 0001
- 2 = 0010
- 4 = 0100
- 6 = 0110
- 7 = 0111
- 8 = 1000
- 9 = $\frac{100}{\text{hi}}, \frac{1}{\text{low}}$

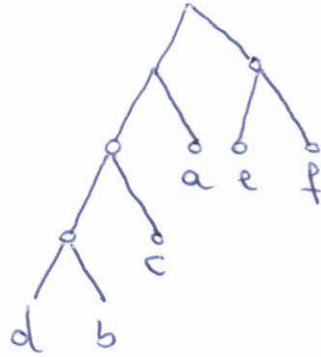
$L = 1000101$

$H = \frac{10}{0} \frac{10}{1} \frac{10}{2} \frac{110}{3} \frac{110}{4} \frac{000}{5} \frac{000}{6} \frac{000}{7}$

Q 2

Symbol = 1	//
2	a, e, f
3	c
4	d, b

FC		ew
1	2	//
2	1	01
3	1	001
4	0	0000



compressed file = 10001

Canonical tree

- $v = 1, l = 1 \rightarrow v < fc[l] = 2, l++ = 2$
- $v = 10, l = 2 \rightarrow v > fc[l] = 1 \Rightarrow$ decode $symbol[l, 1] = e$
- $v = 0, l = 1 \rightarrow v < fc[l] = 2, l++ = 2$
- $v = 00, l = 2 \rightarrow v < fc[l] = 1, l++ = 3$
- $v = 001, l = 3 \rightarrow v \geq fc[l] = 1 \Rightarrow$ decode $symbol[l, 0] = c$

Q 3



$P = 110$ reaches node u
 choose arbitrary leaf, say 2nd
 compare P vs 10010
 $lcp = 1$
 go up to right child of the root
 and move to the rightmost
 leaf.

Q4

$$S = \frac{a, b, c, d, e, f}{\frac{1}{2}, \frac{1}{10}, \frac{3}{4}, \frac{3}{4}, 0, 1} \quad \begin{array}{l} n=6 \\ m=2 \end{array}$$

we extract a random number p and compare it with $\frac{m-s}{n-j+1}$ where $s=0$ initially

$$J=1, s=0 \rightarrow \frac{2}{6} = \frac{1}{3} < \frac{1}{2} = p \quad \text{no pick}$$

$$J=2, s=1 \rightarrow \frac{1}{5} > \frac{1}{10} = p \quad \text{pick } b$$

$$J=3, s=1 \rightarrow \frac{1}{4} < \frac{3}{4} = p \quad \text{no pick}$$

$$J=4, s=1 \rightarrow \frac{1}{3} < \frac{3}{4} = p \quad \text{no pick}$$

$$J=5, s=1 \rightarrow \frac{1}{2} > 0 = p \quad \text{pick } e$$

stop