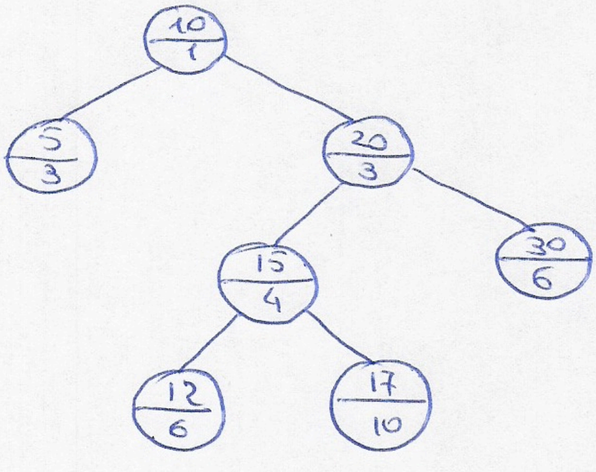
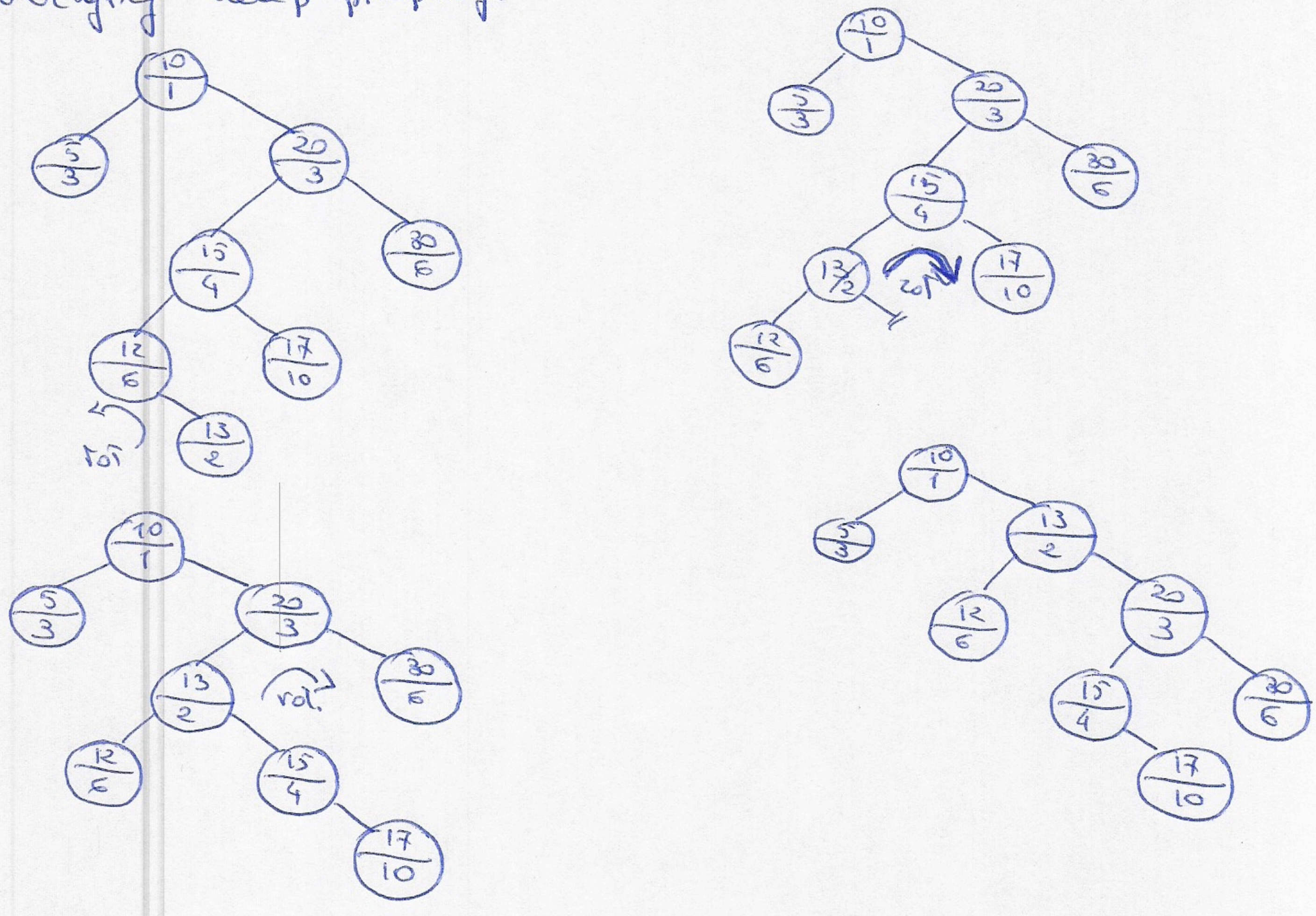
Algorithm Engineering 16/12/2020

Q 1. The keys, and their priorities, are given in a way that rotations are not needed to guarantee the Treap property.



Then it is required to insert in this Treap the pair <13, 2>. So the first step is to find the correct position to insert (13), then it starts a sequence of rotations that re-establish the underlying heap property.



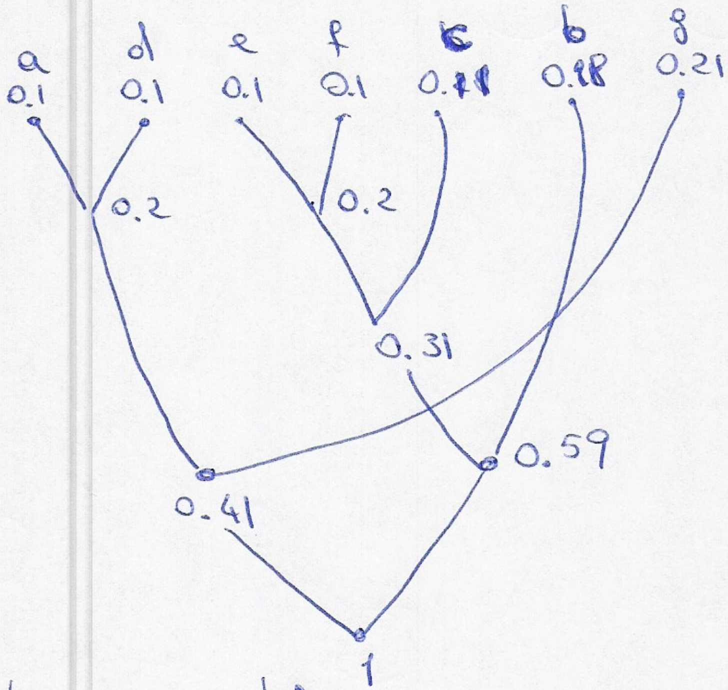
Q2.

1	2
a	b

c	d	e	f
3	1	4	2

Since $m = 2$, and the values of c, e are greater than they are not inserted. Conversely, d is inserted in position 1 and f is inserted in position 2, so the final configuration is | d | f |.

Q3.



Σ	level
a	3
b	2
c	3
d	3
e	4
f	4
g	2

num	1	2	3	4
	0	2	3	2

symb		fc
1	//	2
2	b, g	2
3	a, c, d	1
4	e, f	0

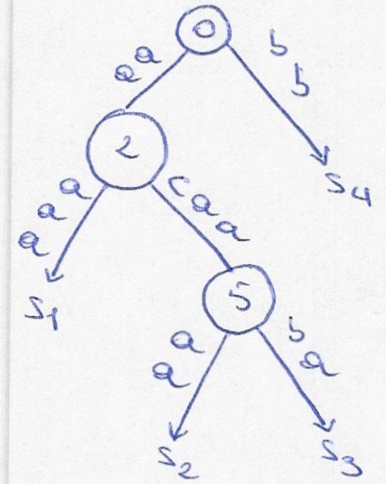
- $v = 1 < fc[1] = 2 \longrightarrow v = (11)_2 = 3 > fc[2] = 2$
  output $symb[2, v - fc[2]] = symb[2,1] = g$
  $v = (001)_2 \geq fc[3] = 1$
- $v = 0 < fc[1] = 2 \longrightarrow v = (00)_2 < fc[2] = 2 \longrightarrow$
  $\longrightarrow symb[3, v - fc[3]] = symb[3,0] = a$

Q4. We know that the length in bit compressed via Arithmetic is given by the formula $d = \lceil \log_2 \frac{2}{s} \rceil$, where s is $\prod_i p(T[i])$

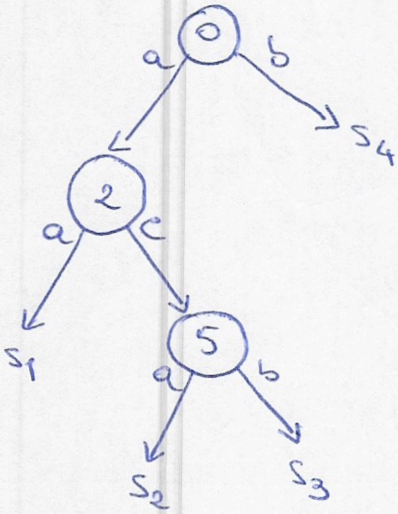
$$= [p(a)]^4 \cdot [p(b)]^2 = [1/8]^4 \cdot [1/4]^2 = \left(\frac{1}{2^3}\right)^4 \cdot \left(\frac{1}{2^2}\right)^2 = \frac{1}{2^{16}}$$

$$d = \lceil \log_2 2 \cdot 2^{16} \rceil = \lceil \log_2 2^{17} \rceil = 17 \text{ bit}$$

Q5.



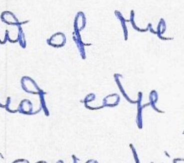
Patricia Trie is obtained from the compacted Trie to the left by keeping only the first characters of every edge.



① $p = \underset{0}{\underline{a}}\, a\, \underset{2}{\underline{c}}\, b\, b\, \underset{5}{\underline{a}} \rightarrow s_2$ is selected

② compute $lcp(p, s_2) = 3$ $\underset{p}{\boxed{b}}$ $\underset{s_2}{\boxed{a}}$

③ We percolate upward the Patricia Trie until we reach the edge



and since $\underset{p}{\boxed{b}} > \underset{s_2}{\boxed{a}}$ we have that P is to the right of the subtree descending from that edge, and thus P is lexicographically between s_3 and s_4.