

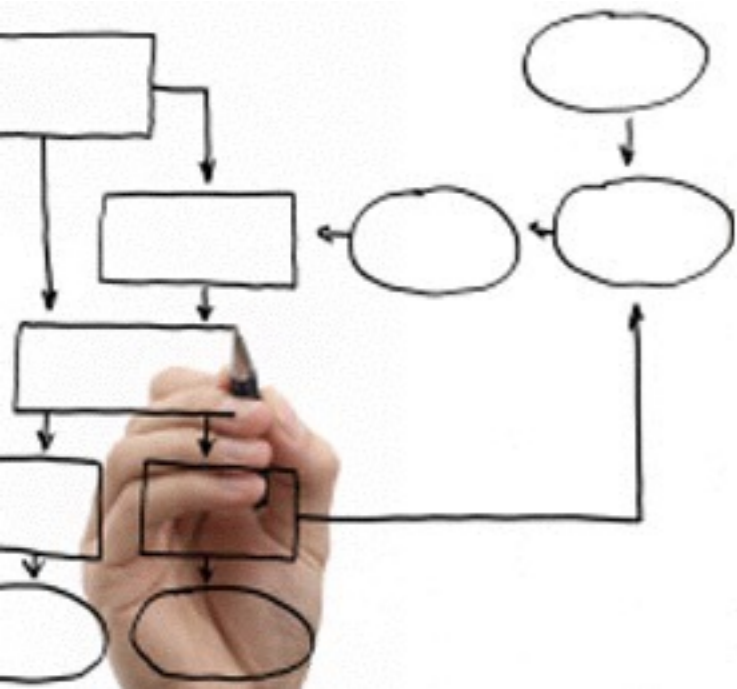
Methods for the specification and verification of business processes

MPB (6 cfu, 295AA)

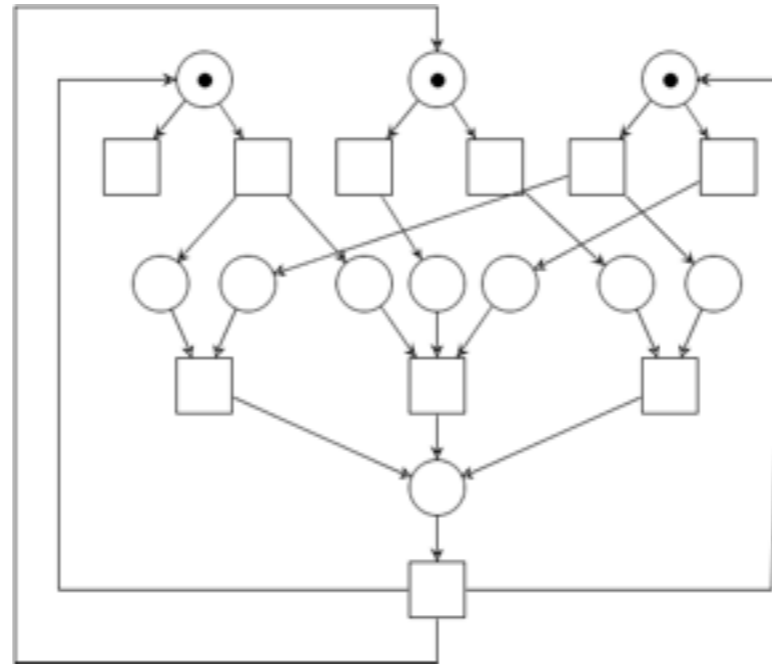
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<http://www.di.unipi.it/~bruni>

18 - Free-choice nets



Object



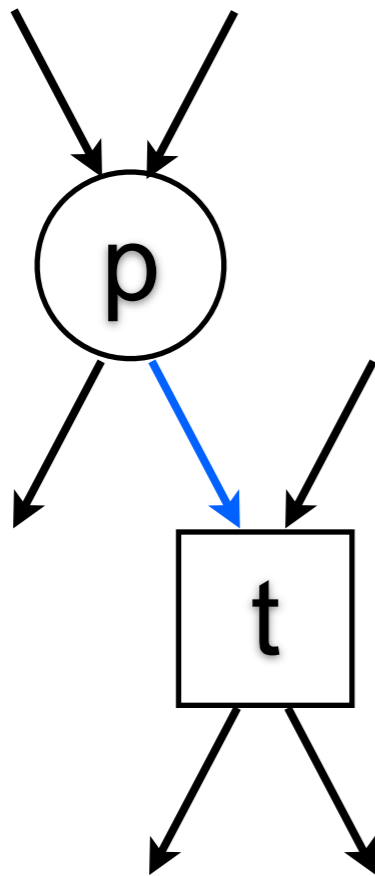
We study some “good” properties of free-choice nets

Free Choice Nets (book, optional reading)

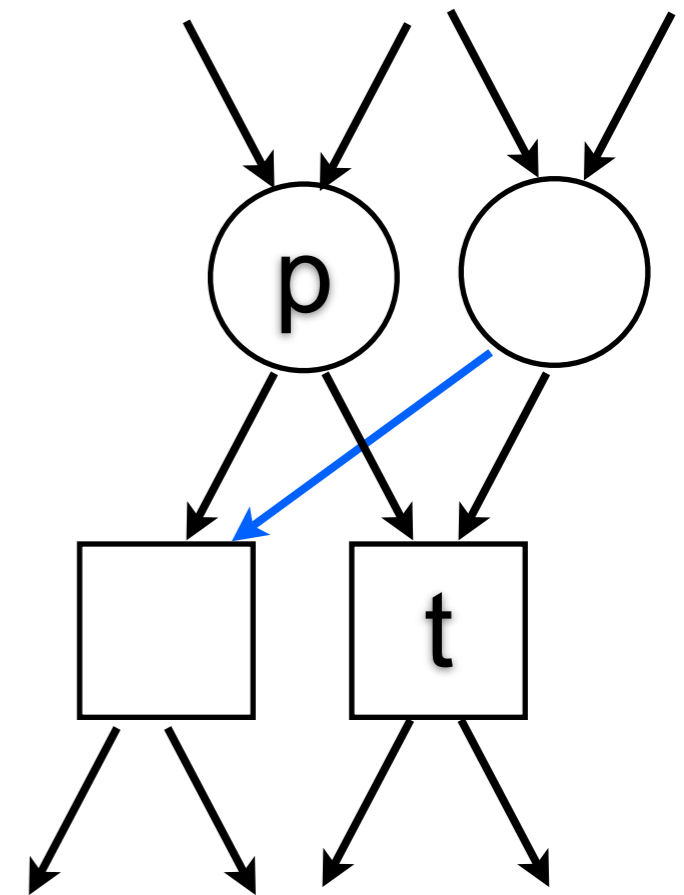
<https://www7.in.tum.de/~esparza/bookfc.html>

Free-choice net

Definition: We recall that a net N is **free-choice** if whenever there is an arc (p,t) , then there is an arc from any input place of t to any output transition of p



implies



Free-choice net: alternative definitions

Proposition: All the following definitions of free-choice net are equivalent.

1) A net (P, T, F) is free-choice if:

$$\forall p \in P, \forall t \in T, (p, t) \in F \text{ implies } \bullet t \times p \bullet \subseteq F.$$

2) A net (P, T, F) is free-choice if:

$$\forall p, q \in P, \forall t, u \in T, \{(p, t), (q, t), (p, u)\} \subseteq F \text{ implies } (q, u) \in F.$$

3) A net (P, T, F) is free-choice if:

$$\forall p, q \in P, \text{ either } p \bullet = q \bullet \text{ or } p \bullet \cap q \bullet = \emptyset.$$

4) A net (P, T, F) is free-choice if:

$$\forall t, u \in T, \text{ either } \bullet t = \bullet u \text{ or } \bullet t \cap \bullet u = \emptyset.$$

Free-choice net: my favourite definition

4) A net (P, T, F) is free-choice if:
 $\forall t, u \in T$, either $\bullet t = \bullet u$ or $\bullet t \cap \bullet u = \emptyset$.

Free-choice system

Definition: A system (N, M_0) is **free-choice** if N is free-choice

Example

$$\bullet t_1 = \{p_1, p_3\}$$

$$\bullet t_2 = \{p_3\}$$

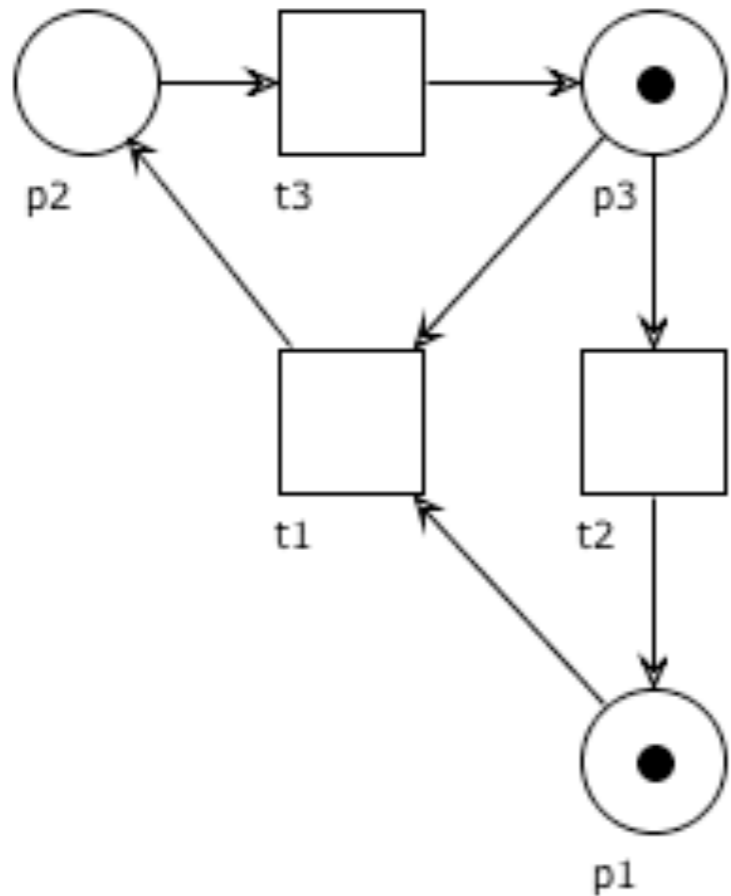
$$\bullet t_1 \neq \bullet t_2$$

$$\bullet t_1 \cap \bullet t_2 = \{p_3\} \neq \emptyset$$

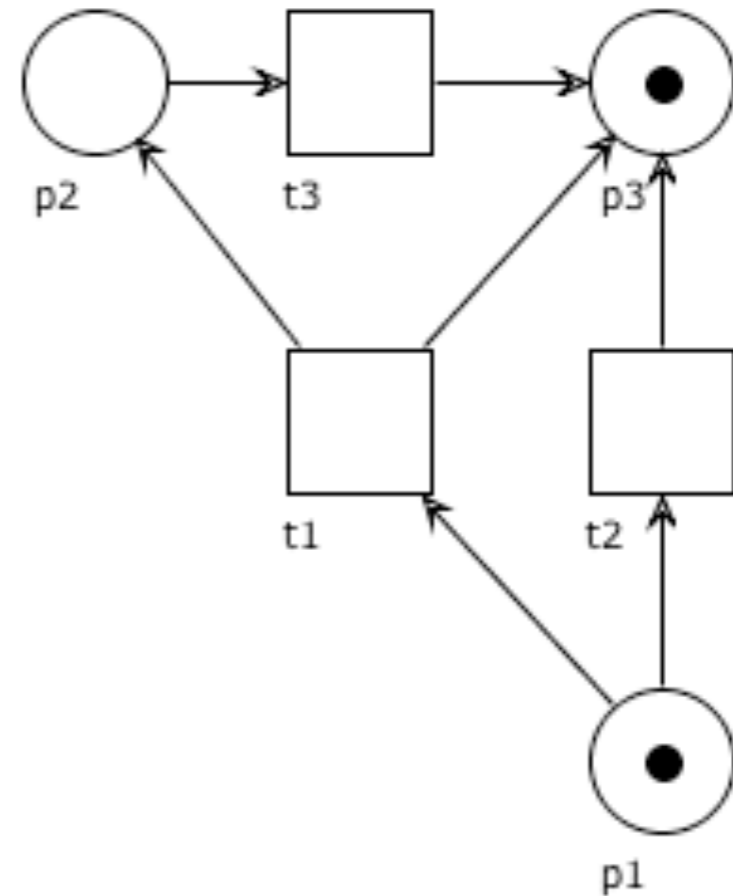
$$\bullet t_1 = \bullet t_2$$

$$\bullet t_1 \cap \bullet t_3 = \emptyset$$

$$\bullet t_2 \cap \bullet t_3 = \emptyset$$



non free-choice



free-choice

Fundamental property of free-choice nets

Proposition: Let (P, T, F, M_0) be free-choice.

If $M \xrightarrow{t}$ and $t \in p\bullet$, then $M \xrightarrow{t'}$ for every $t' \in p\bullet$.

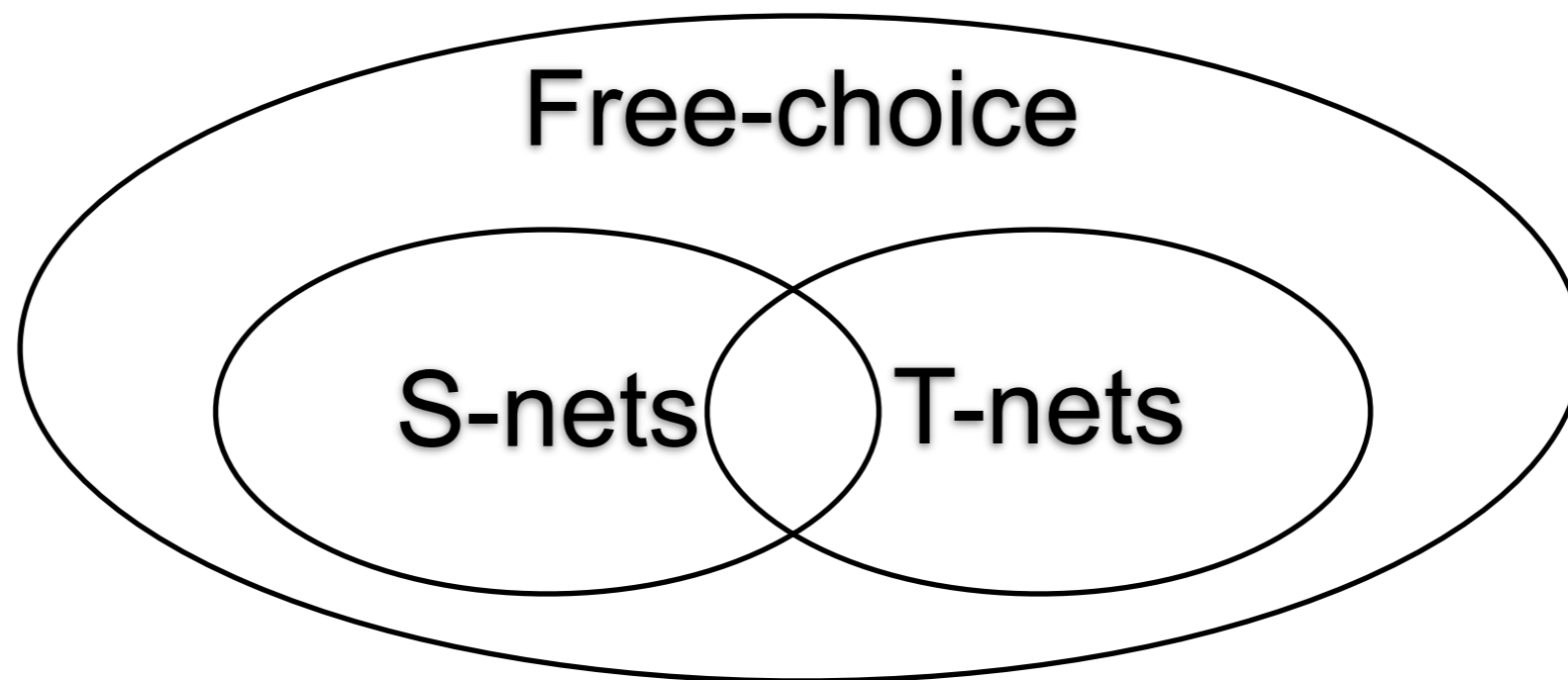
The proof is trivial, by definition of free-choice net

Exercises

Prove that every S-net is free-choice

Prove that every T-net is free-choice

Show a free-choice net that is neither an S-net nor a T-net



Free-choice N^*

Proposition: A workflow net N is free-choice
iff N^* is free-choice

N and N^* differ only for the reset transition,
whose pre-set (o) is disjoint
from the pre-set of any other transition

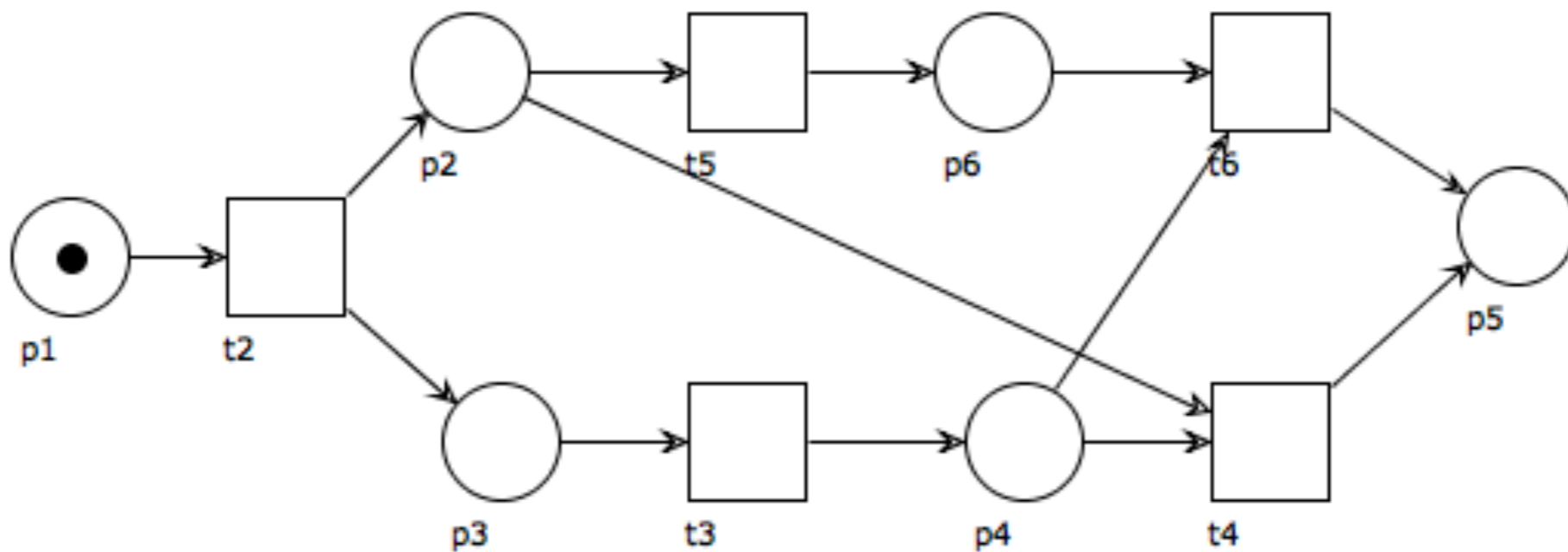
Free-Choice vs Soundness

Note that free-choice is orthogonal to soundness:

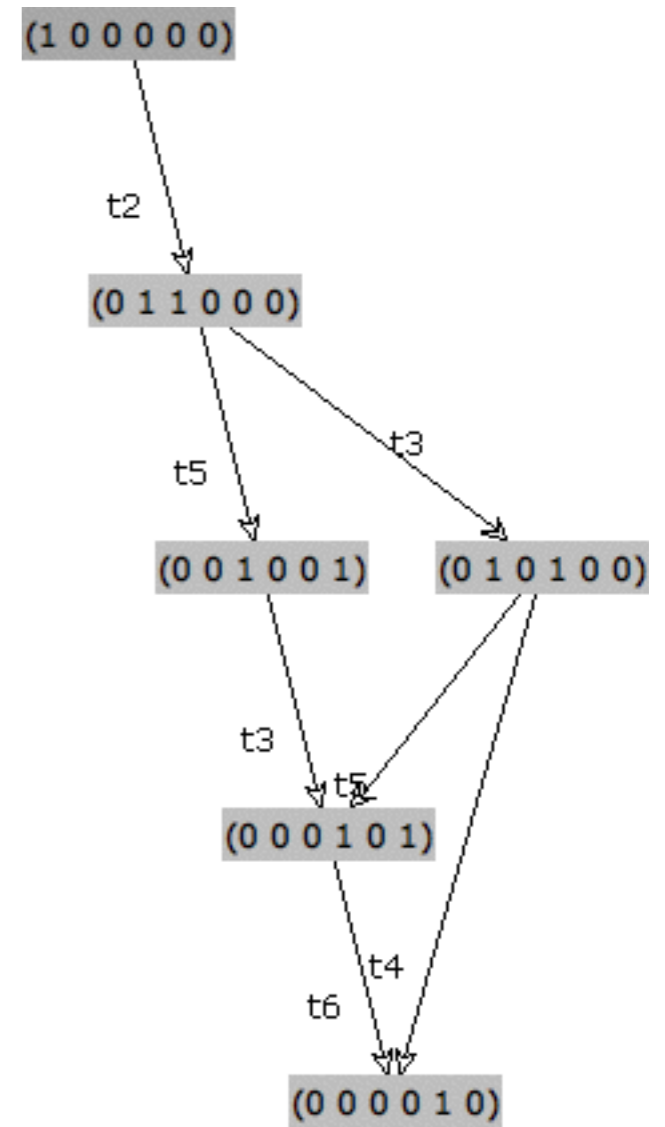
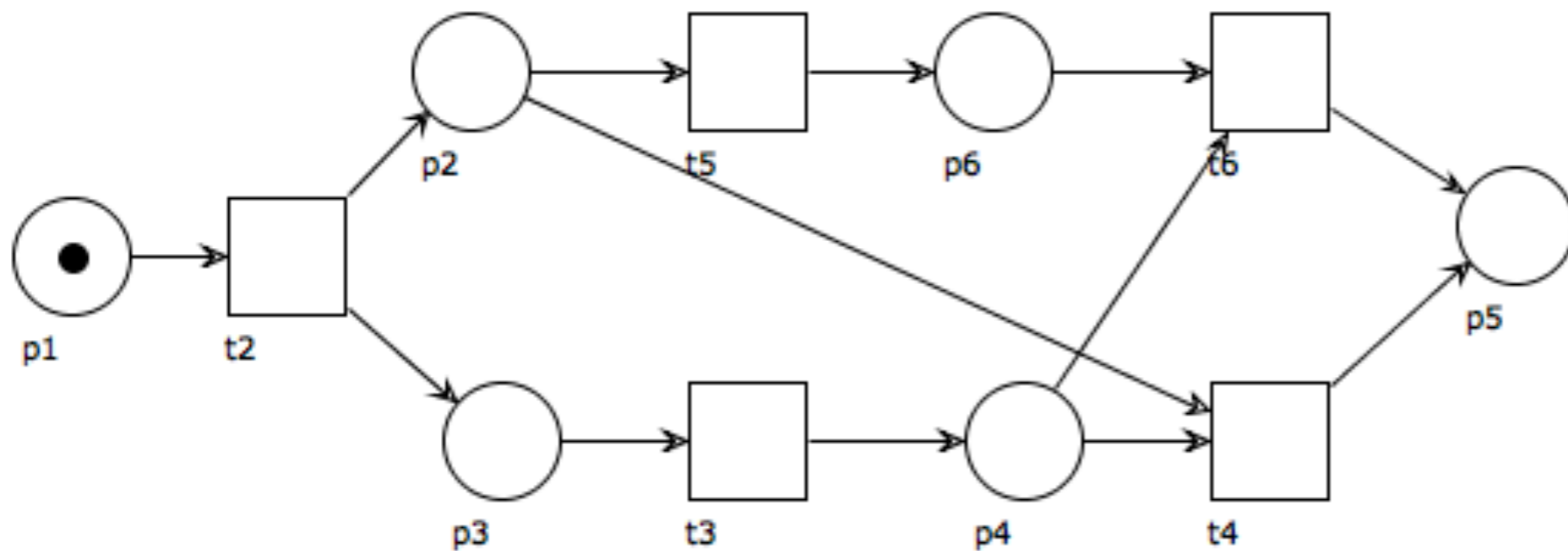
there exists WF-nets that are free-choice but not sound

there exists WF-nets that are sound but not free-choice

Example: sound but not free-choice



Example: sound but not free-choice



Exercise

Draw a workflow net that is free-choice but not sound

Rank Theorem

(main result, proof omitted)

Theorem:

A free-choice system (P, T, F, M_0) is live and bounded
iff

1. it has at least one place and one transition
2. it is connected
3. M_0 marks every proper **siphon**
4. it has a positive S-invariant
5. it has a positive T-invariant
6. $\text{rank}(N) = |C_N| - 1$

(where C_N is the set of **clusters**)

Clusters

Cluster

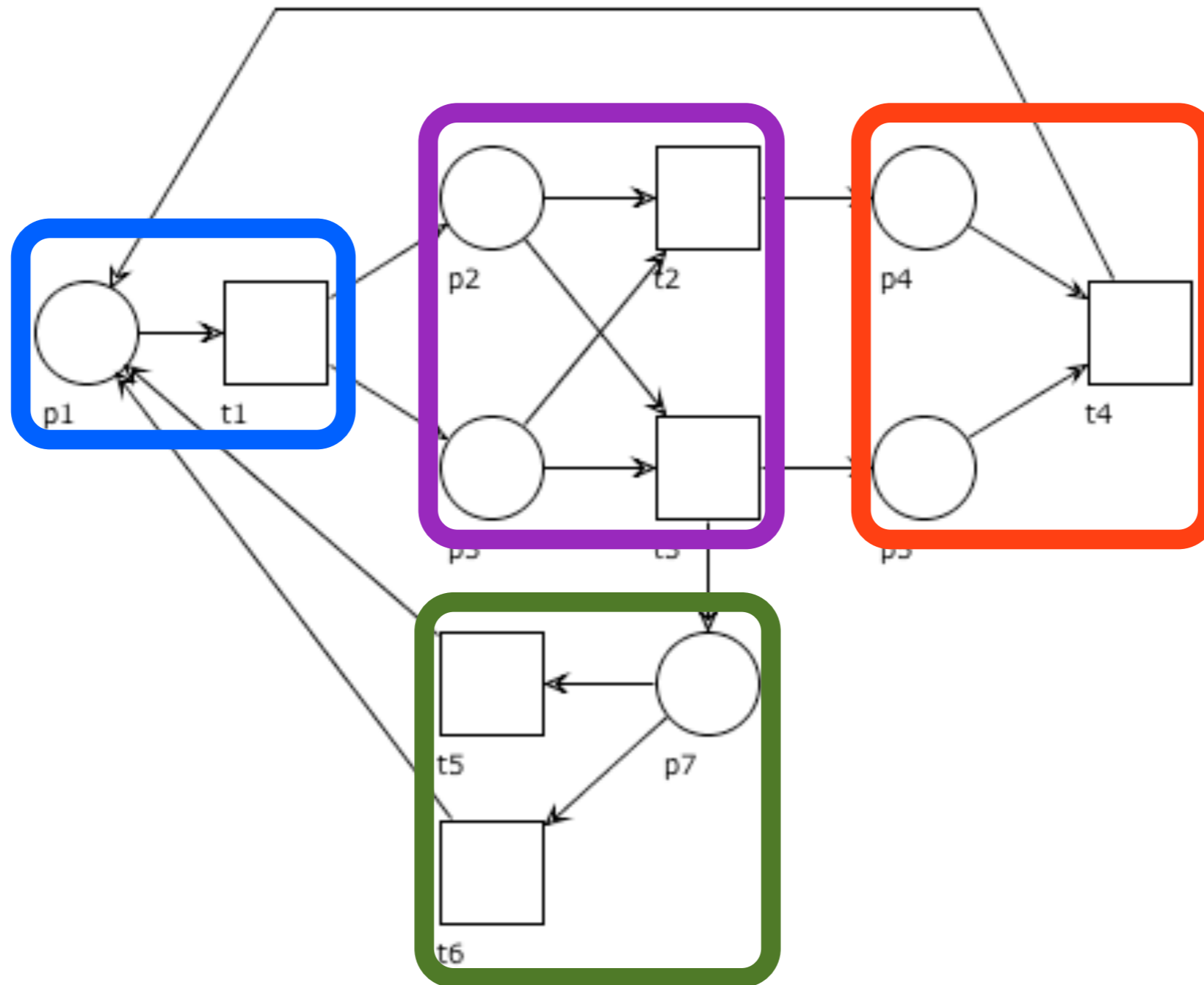
Let x be the node of a net $N = (P, T, F)$
(not necessarily free-choice)

Definition:

The **cluster** of x , written $[x]$, is the least set s.t.

1. $x \in [x]$
2. if $p \in [x] \cap P$ then $p \bullet \subseteq [x]$ (if a place p is in the cluster, then all transitions in the post-set of p are in the cluster)
3. if $t \in [x] \cap T$ then $\bullet t \subseteq [x]$ (if a transition t is in the cluster, then all places in the pre-set of t are in the cluster)

Cluster: example



Observation

Every place belongs to exactly one cluster

Every transition belongs to exactly one cluster

The set $\{ [x] \mid x \in P \cup T \}$ is a partition of $P \cup T$

Fundamental property of clusters in f.c. nets

Proposition:

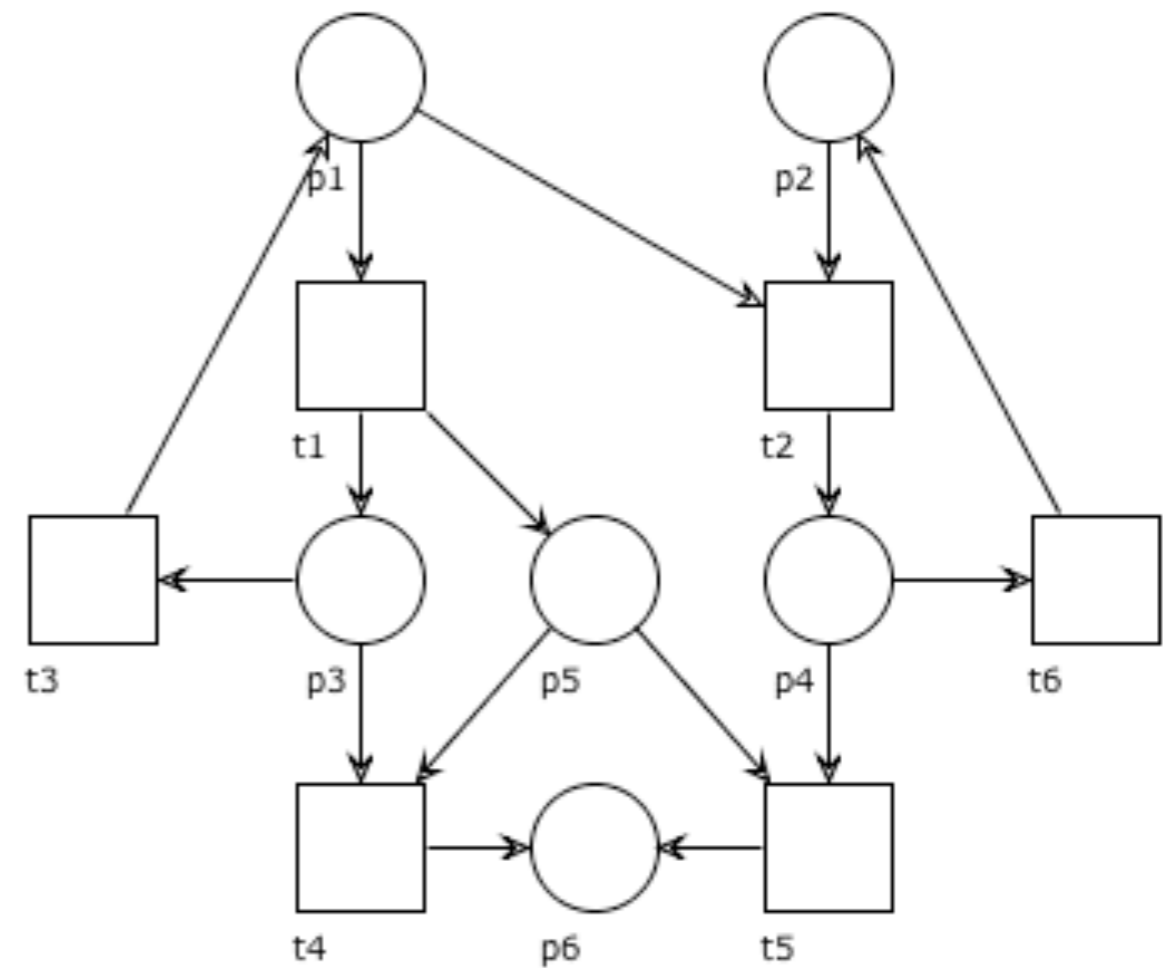
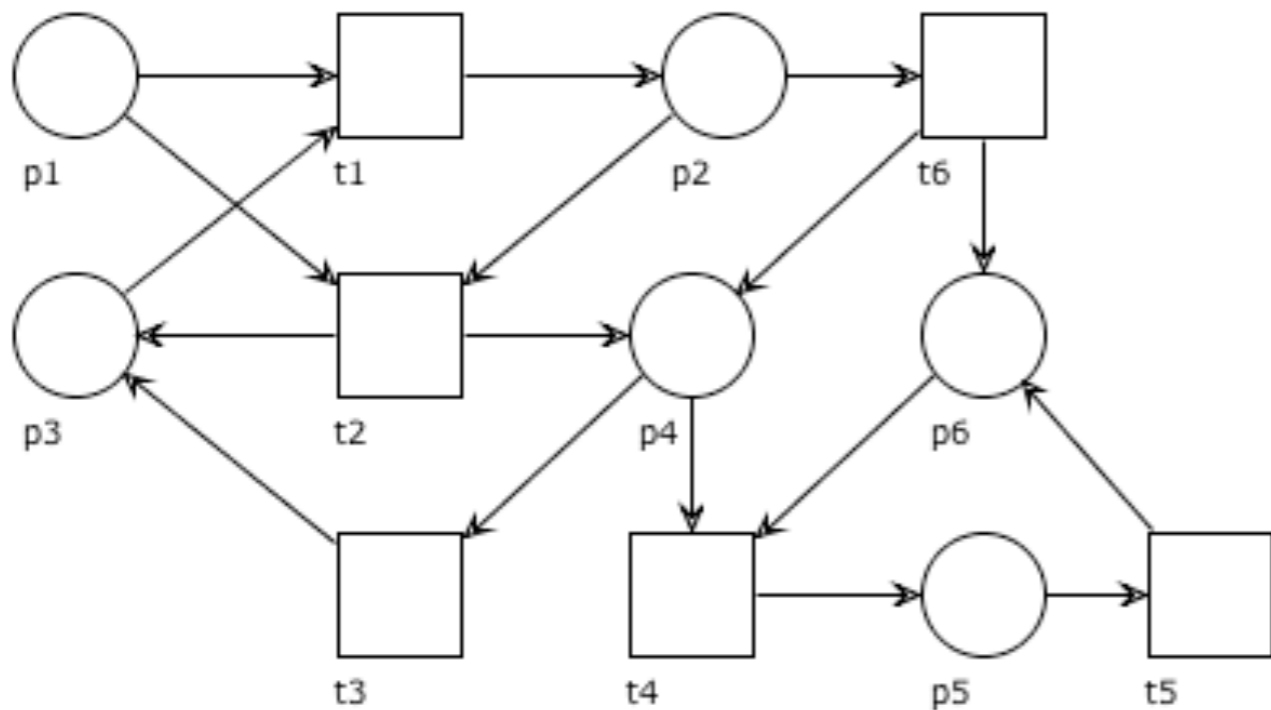
If $M \xrightarrow{t}$, then for any $t' \in [t]$ we have $M \xrightarrow{t'}$

Immediate consequence of the fact that, for free-choice nets

$$t, t' \in [x] \quad \text{iff} \quad \bullet t = \bullet t'$$

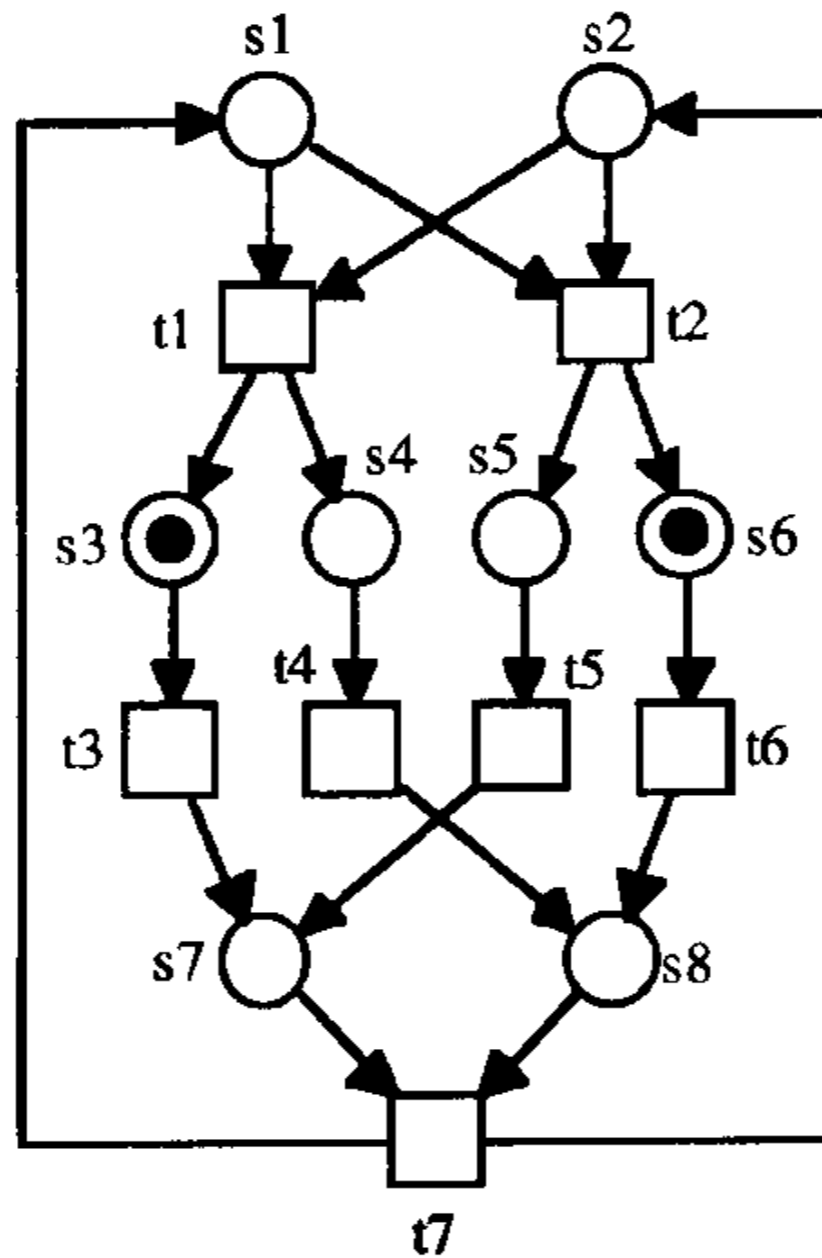
Exercise

Draw all clusters in the nets below



Exercise

Draw all clusters in the free-choice net below



Stable markings

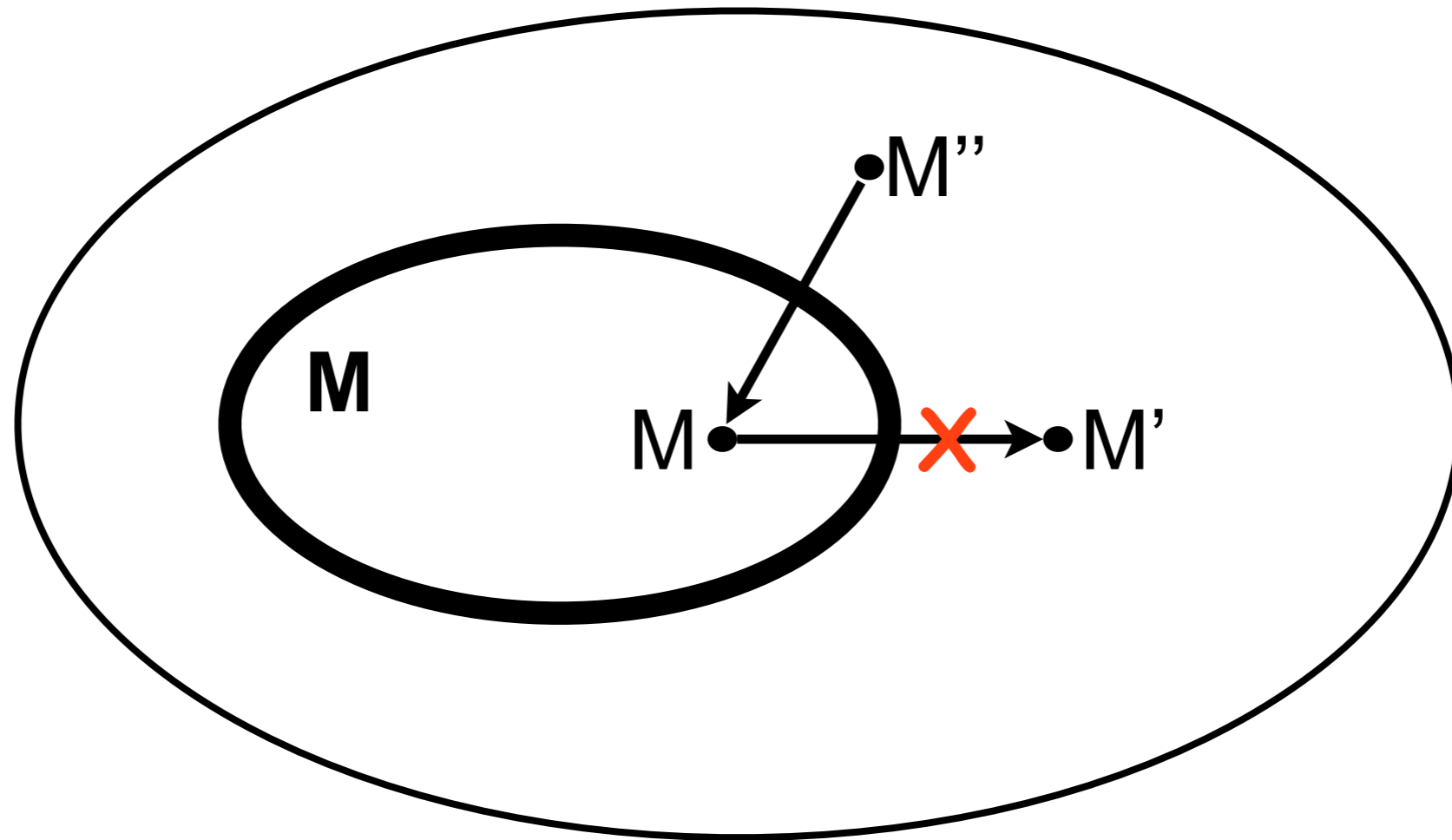
Stable set of markings

Definition: A set of markings \mathbf{M} is called **stable** if

$$M \in \mathbf{M} \quad \text{implies} \quad [M \rangle \subseteq \mathbf{M}$$

(starting from any marking in the stable set \mathbf{M} ,
no marking outside \mathbf{M} is reachable)

Stable set of markings



(starting from any marking **M** in the stable set **M**,
no marking **M'** outside **M** is reachable)

Stability check

\mathbf{M} is stable iff

$$\forall M, t, M'. (M \in \mathbf{M} \wedge M \xrightarrow{t} M' \text{ implies } M' \in \mathbf{M})$$

Question time

Given a net system:

Is the singleton set $\{ 0 \}$ a stable set?

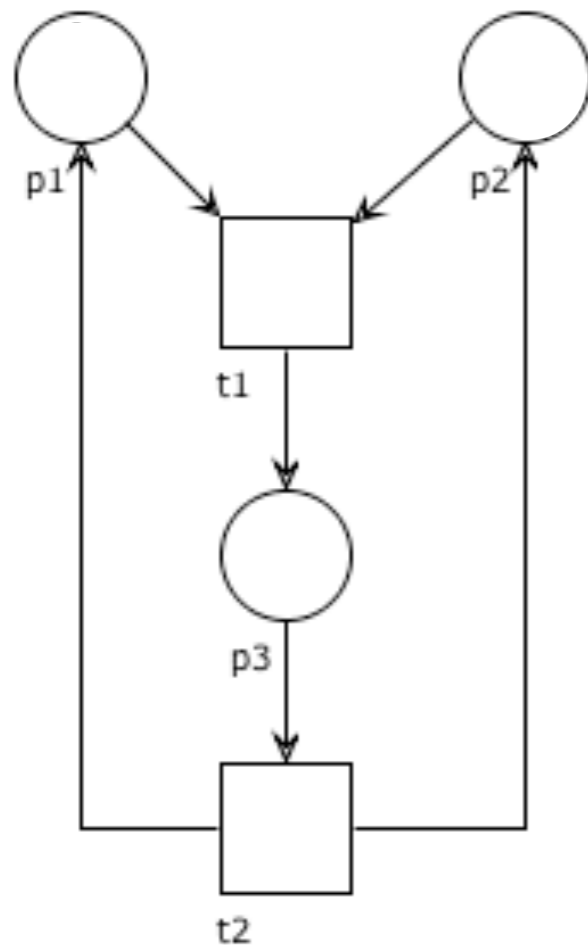
Is the set of all markings a stable set?

Is the set of live markings a stable set?

Is the set of deadlock markings a stable set?

Example

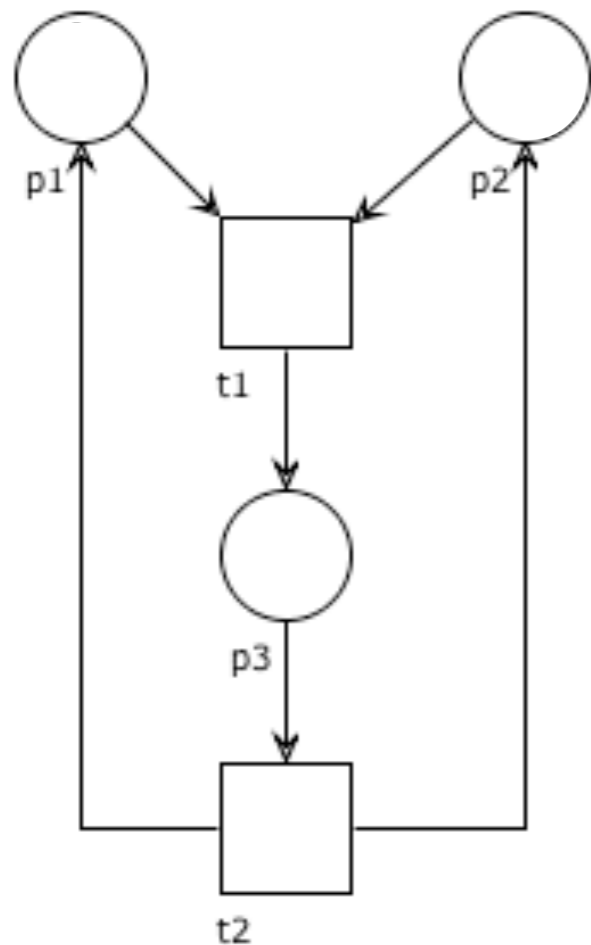
Which of the following is a stable set of markings?



- $\{ 2p_1 + p_2 \}$
- $\{ 2p_1 + p_2, p_1 + 2p_3 \}$
- $\{ p_1, p_2 \}$
- $\{ p_1 + p_2, p_3 \}$

Exercises

Which of the following is a stable set of markings?



$$\{ p_1, p_3 \}$$

$$\{ 2p_1+2p_2, 2p_3 \}$$

$$\{ 2p_1+2p_2, p_1+p_2+p_3, 2p_3 \}$$

$$\{ p_1, 2p_1+2p_2, p_1+p_2+p_3, 2p_3 \}$$

Exercises

Given a net system (P, T, F, M_0) :

Is the set $\{ M \mid M(P)=1 \}$ a stable set?

Is the set of markings reachable from M_0 a stable set?

Is the set $\{ M \mid M(P) < k \}$ a stable set?

Exercises

Let I be an S -invariant

Is the set $\{ M \mid I \cdot M = I \cdot M_0 \}$ a stable set?

Is the set $\{ M \mid I \cdot M \neq I \cdot M_0 \}$ a stable set?

Is the set $\{ M \mid I \cdot M = 1 \}$ a stable set?

Is the set $\{ M \mid I \cdot M = 0 \}$ a stable set?

Exercises

Let \mathbf{M} and \mathbf{M}' be stable sets

Prove that their union is a stable set

Prove that their intersection is a stable set

Is their difference a stable set?

What is the least stable set that includes a marking M_0 ?

What is the largest stable set of a net?

Siphons

Proper siphon

Definition:

A set of places R is a **siphon** if $\bullet R \subseteq R\bullet$

It is a **proper siphon** if $R \neq \emptyset$

Siphons, intuitively

A set of places R is a siphon if
all transitions that can produce tokens in the places of R
require some place in R to be marked

Therefore:
if no token is present in R ,
then no token will ever be produced in R

Siphon check

Let R be a set of places of a net

mark with \checkmark all transitions that consume tokens from R

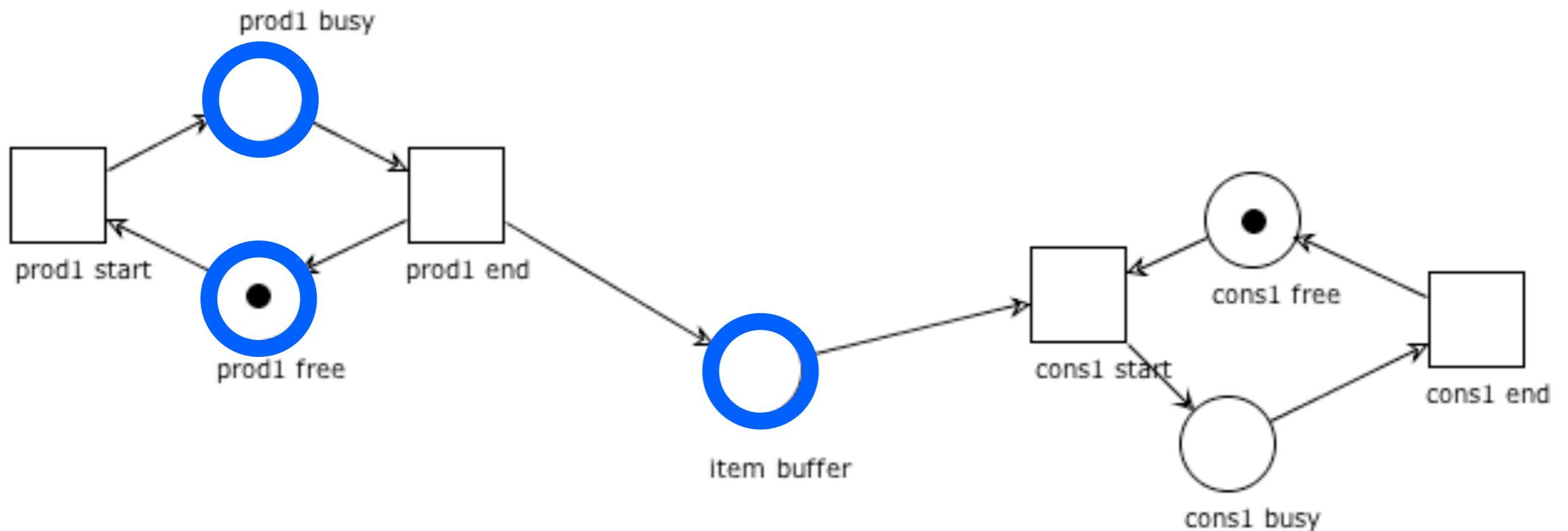
if there is a transition producing tokens in some place of R that is not marked by \checkmark , then R is not a siphon

Otherwise R is a siphon

• $R \subseteq R$ •

Siphon check: example

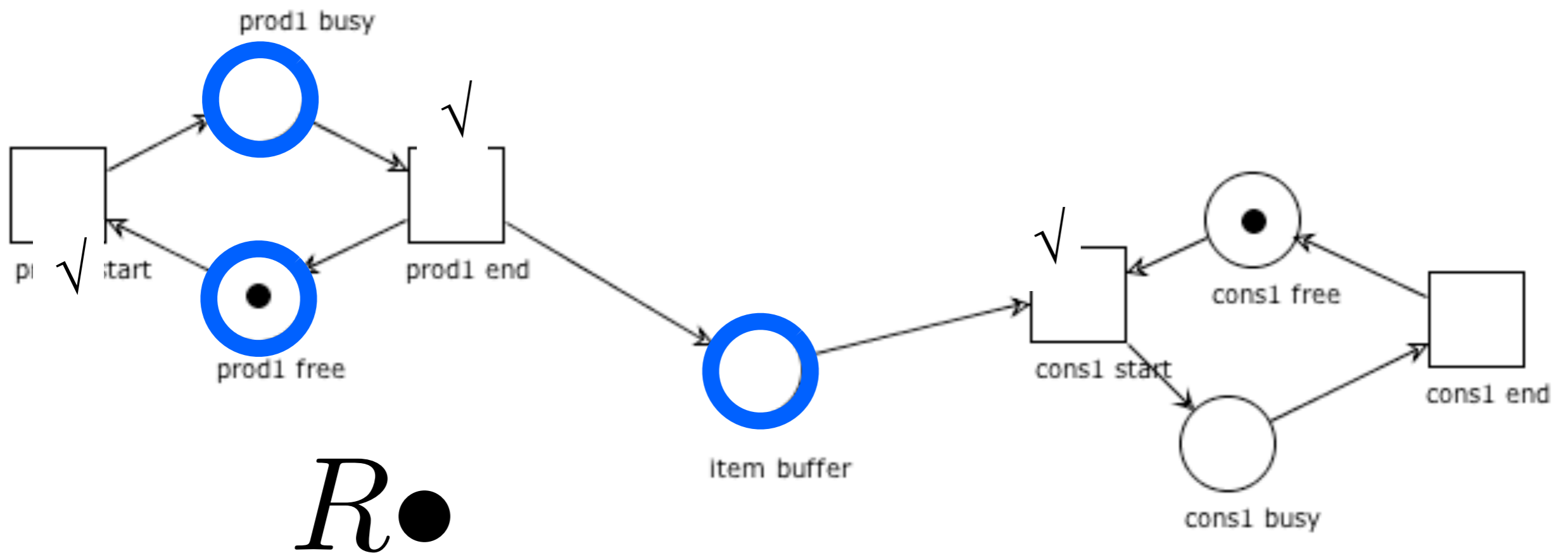
Is $R = \{ \text{prod1 busy}, \text{prod1 free}, \text{itembuffer} \}$ a siphon?



• $R \subseteq R$ •

Siphon check: example

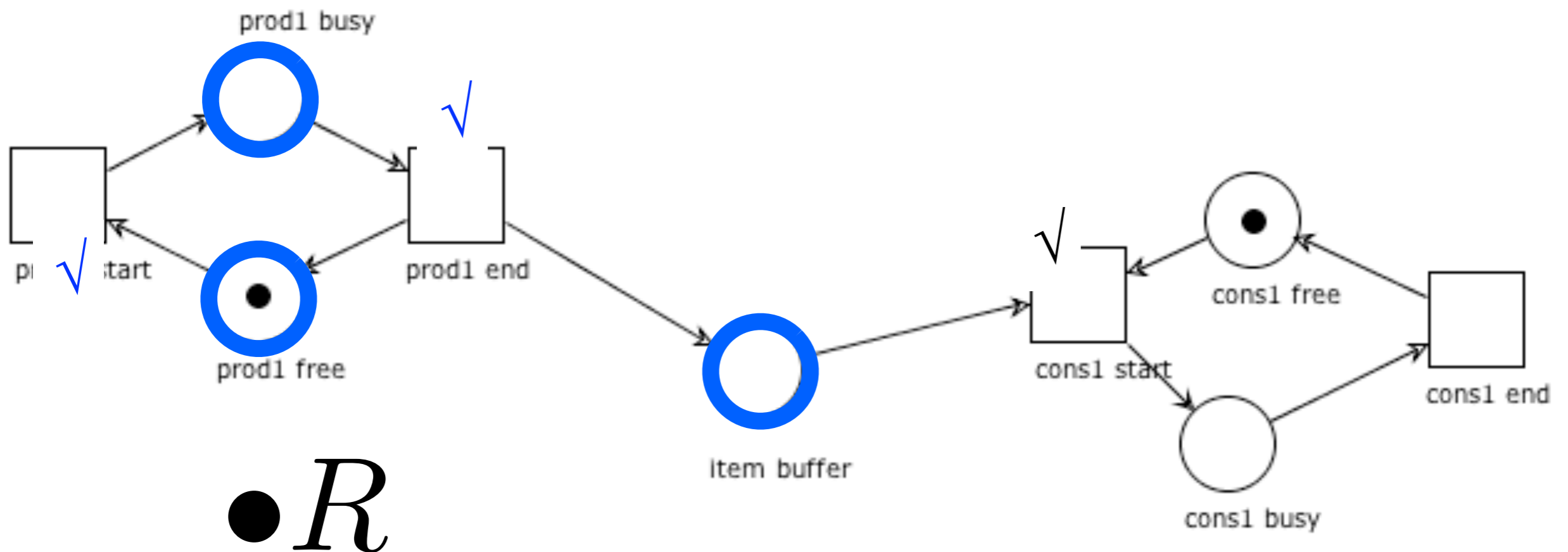
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Siphon check: example

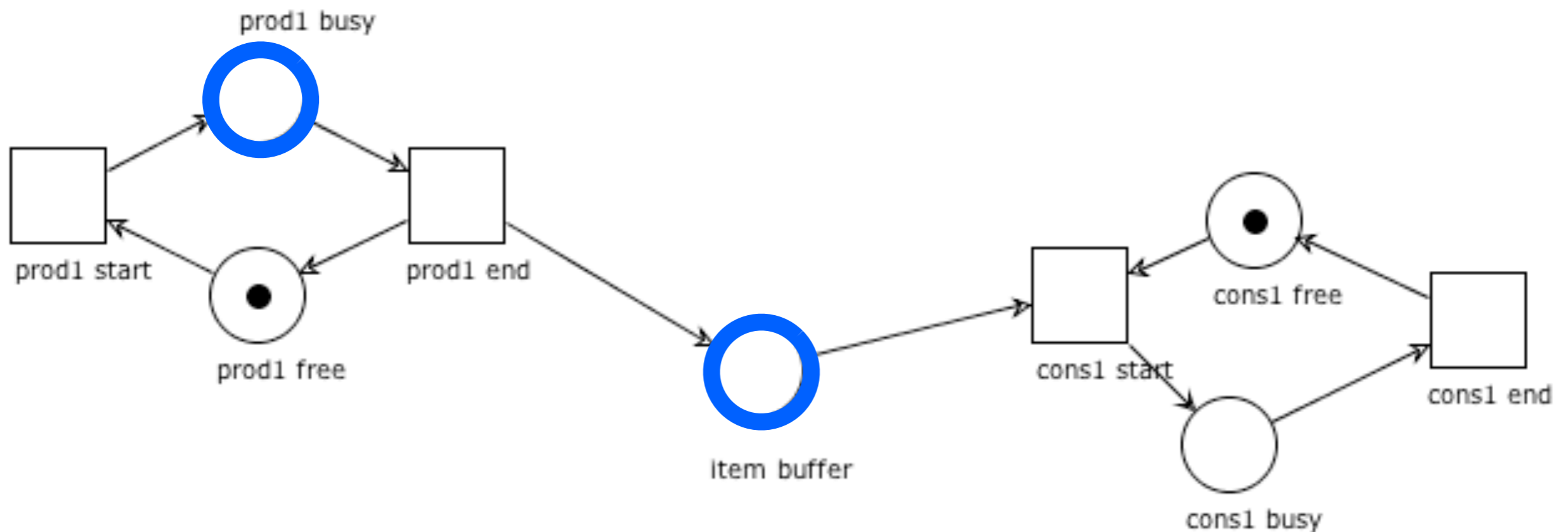
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Siphon check: example

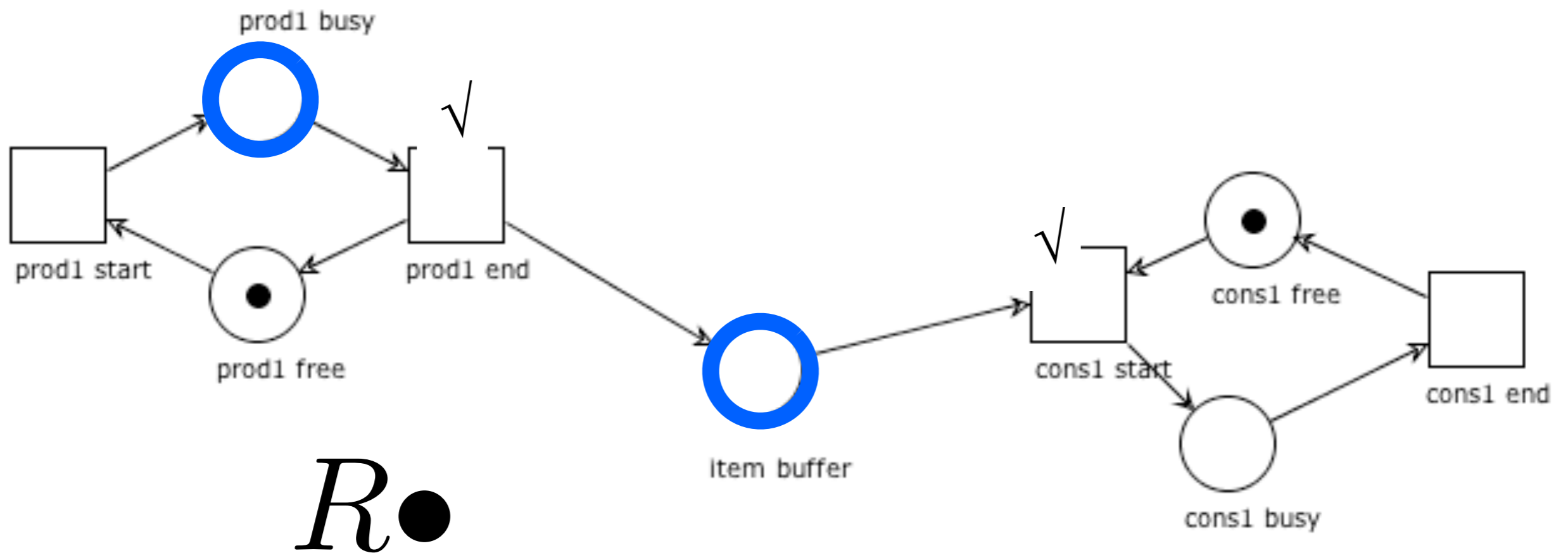
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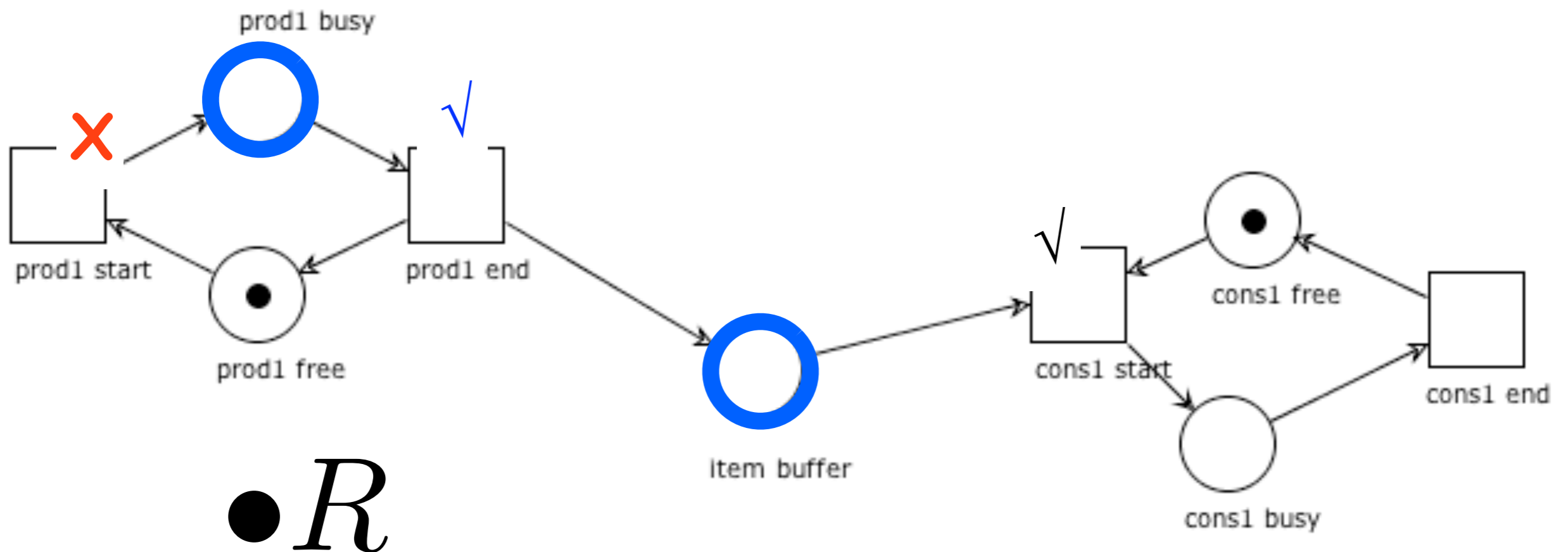
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• $R \subseteq R$ •

Siphon check: example

Is $R = \{ \text{prod1 busy, itembuffer} \}$ a siphon?



Fundamental property of siphons

Proposition: Unmarked siphons remain unmarked

Take a siphon R .

We just need to prove that the set of markings

$$\mathbf{M} = \{ M \mid M(R)=0 \}$$

is stable, which is immediate by definition of siphon

Corollary:

If a siphon R is marked at some reachable marking M ,
then it was initially marked at M_0

Siphons and liveness

Prop.: Live systems have no unmarked proper siphons

(We prove: $M_0(R) > 0$ for every proper siphon R of a live system)

Take $p \in R$ and let $t \in \bullet p \cup p \bullet$

Since the system is live, then there are $M, M' \in [M_0 \rangle$ such that

$$M \xrightarrow{t} M'$$

Therefore p is marked at either M or M'

Therefore R is marked at either M or M'

Therefore R was initially marked (at M_0)

Siphons and liveness

Corollary: If a system has an unmarked proper siphon then it is not live

Siphons and deadlock

Prop.: Deadlocked systems have an unmarked proper siphon

Let M be a deadlocked marking

Let $R = \{ p \mid M(p) = 0 \}$

Since M is deadlock: $R \bullet = T$

Therefore $\bullet R \subseteq T = R \bullet$ and R is a siphon.

Since T cannot be empty, R is proper

A key observation

If we can guarantee that

all proper siphons are marked
at **every** reachable marking,

then the system is deadlock free

Exercise

Prove that the union of siphons is a siphon

Traps

Proper trap

Definition:

A set of places R is a **trap** if $\bullet R \supseteq R\bullet$

It is a **proper trap** if $R \neq \emptyset$

Traps, intuitively

A set of places R is a trap if

all transitions that can consume tokens from R

produce some token in some place of R

Therefore:

if some token is present in R ,
then it is never possible for R to become empty

Trap check

Let R be a set of places of a net

mark with \checkmark all transitions that produce tokens in R

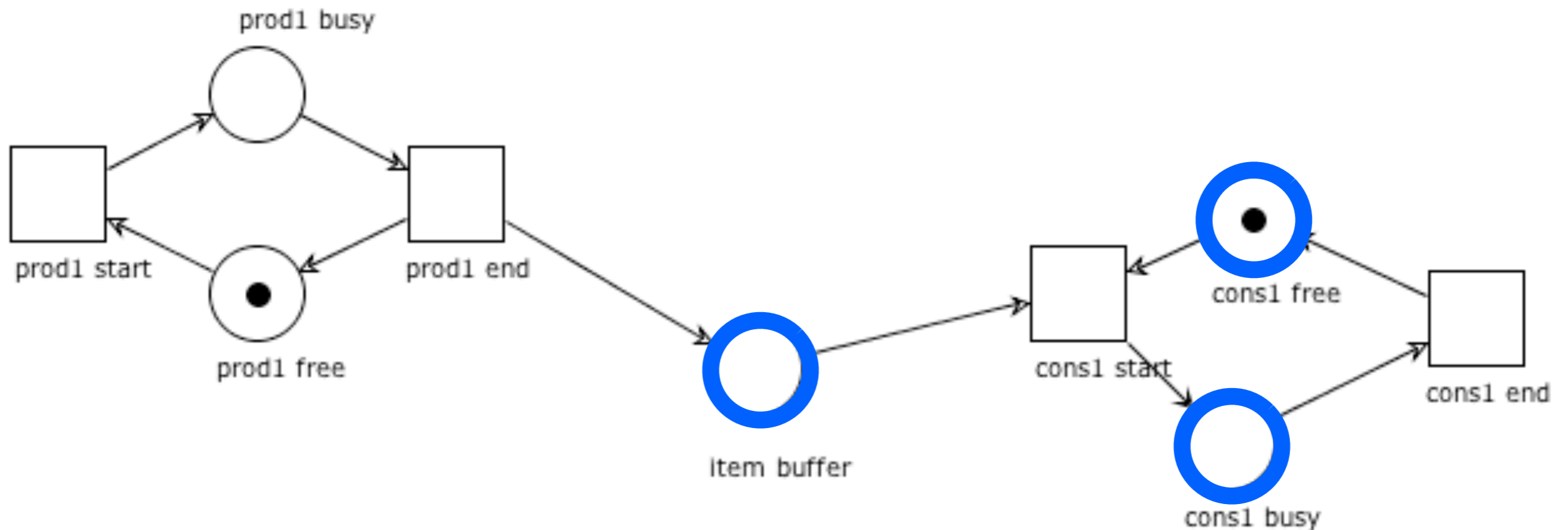
if there is a transition consuming tokens from some place in R that is not marked by \checkmark , then R is not a trap

Otherwise R is a trap

$$\bullet R \subseteq R \bullet$$

Trap check: example

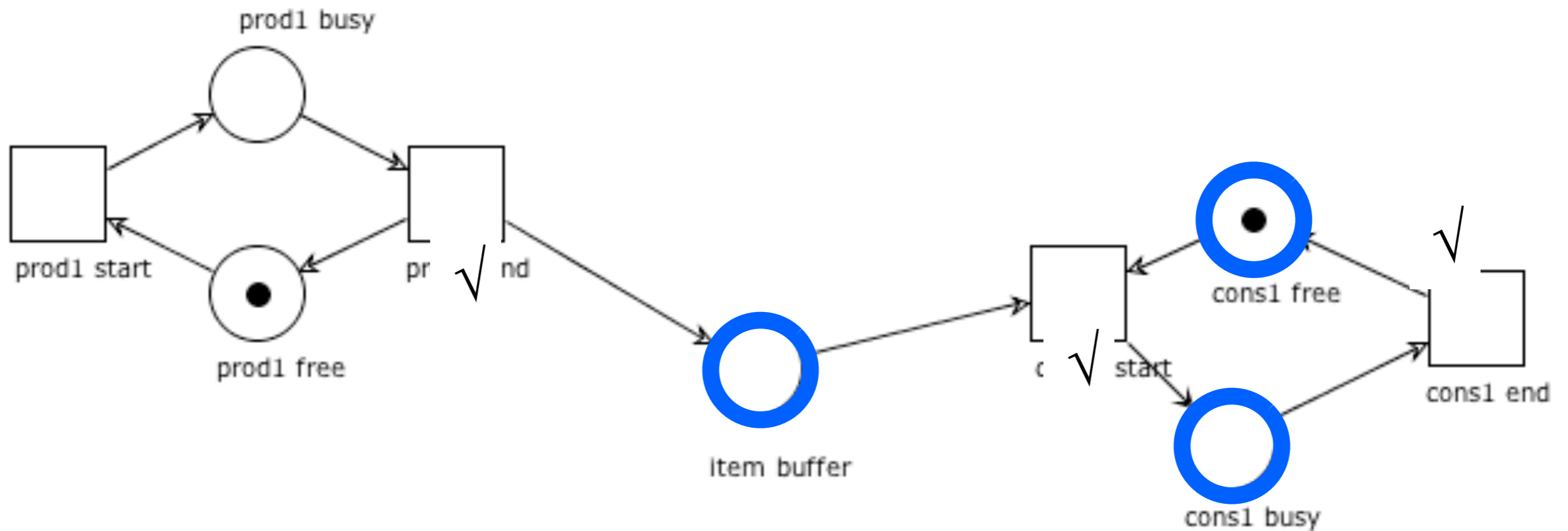
Is $R = \{ \text{itembuffer}, \text{cons1busy}, \text{cons1free} \}$ a trap?



• $R \supseteq R$ •

Trap check: example

Is $R = \{ \text{itembuffer}, \text{cons1busy}, \text{cons1free} \}$ a trap?

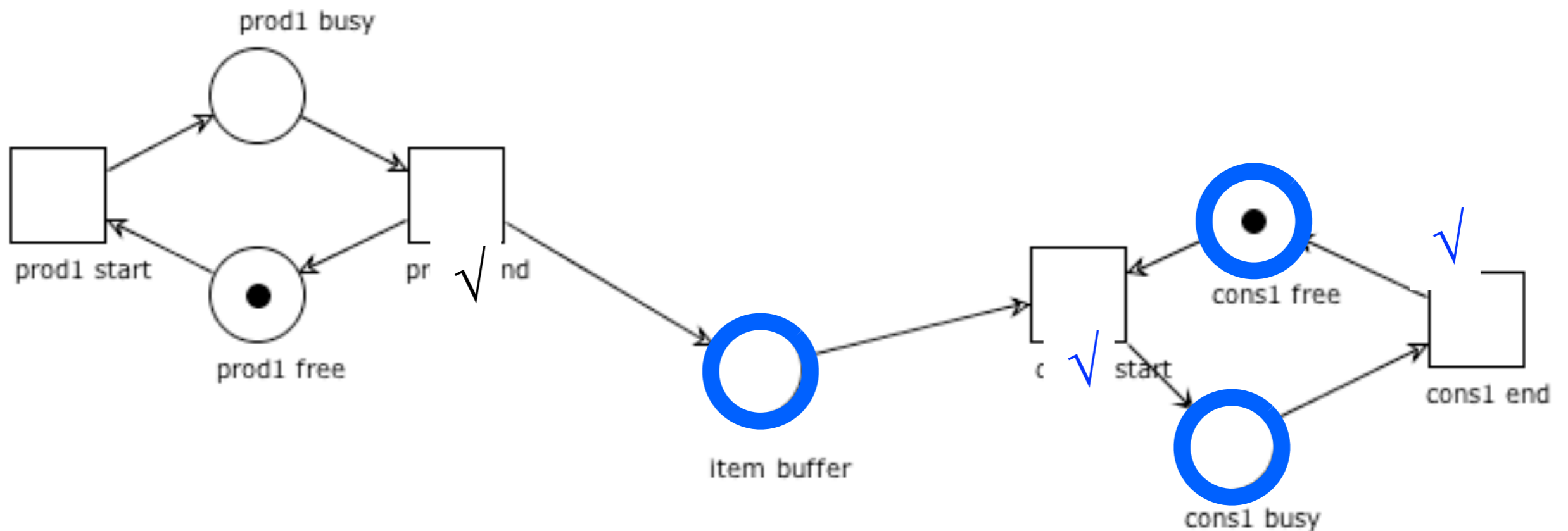


• R

• $R \supseteq R$ •

Trap check: example

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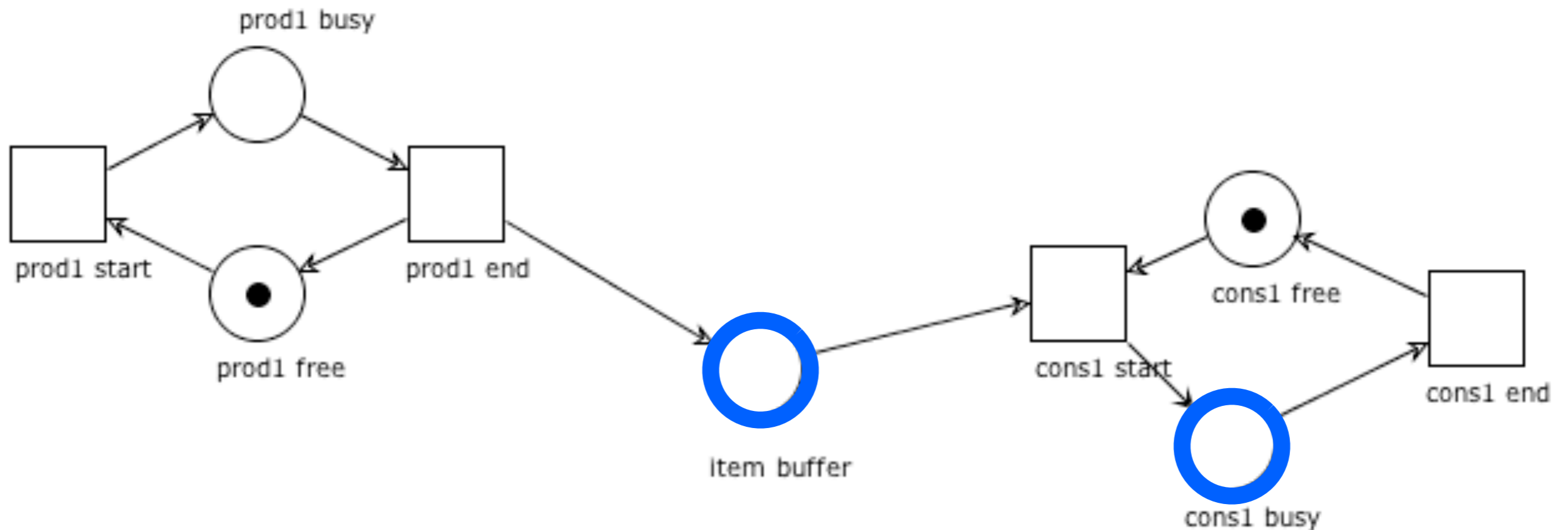


R •

• $R \subseteq R$ •

Trap check: example

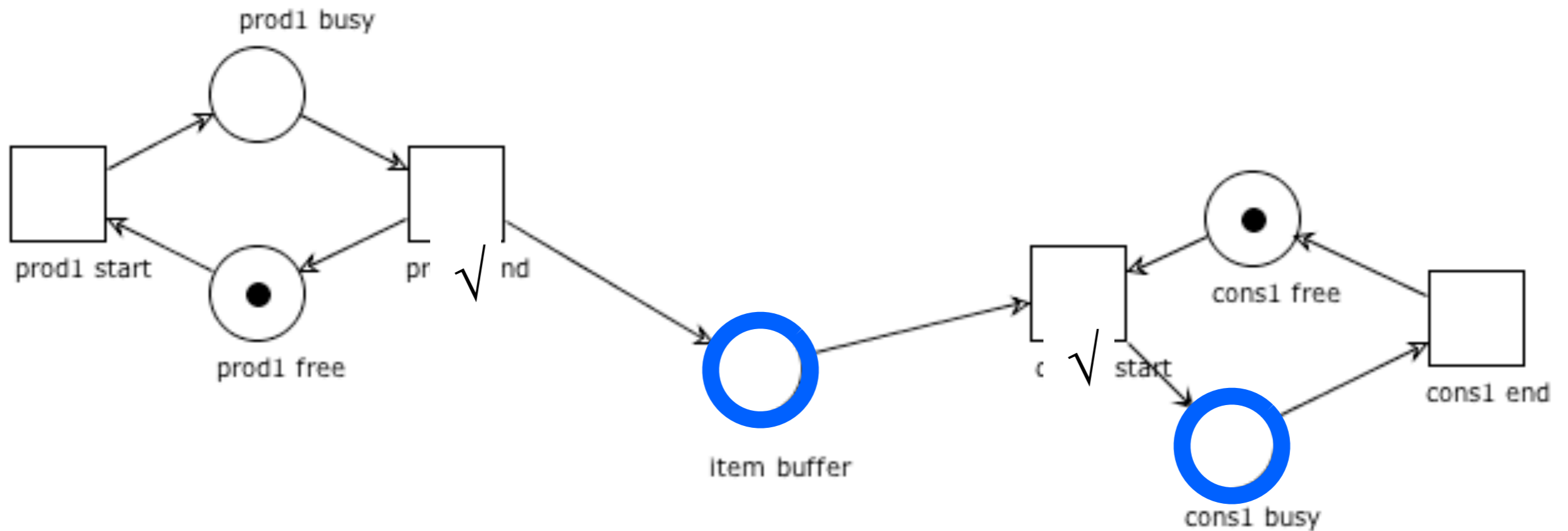
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• $R \supseteq R$ •

Trap check: example

Is $R = \{ \text{itembuffer}, \text{cons1busy} \}$ a trap?

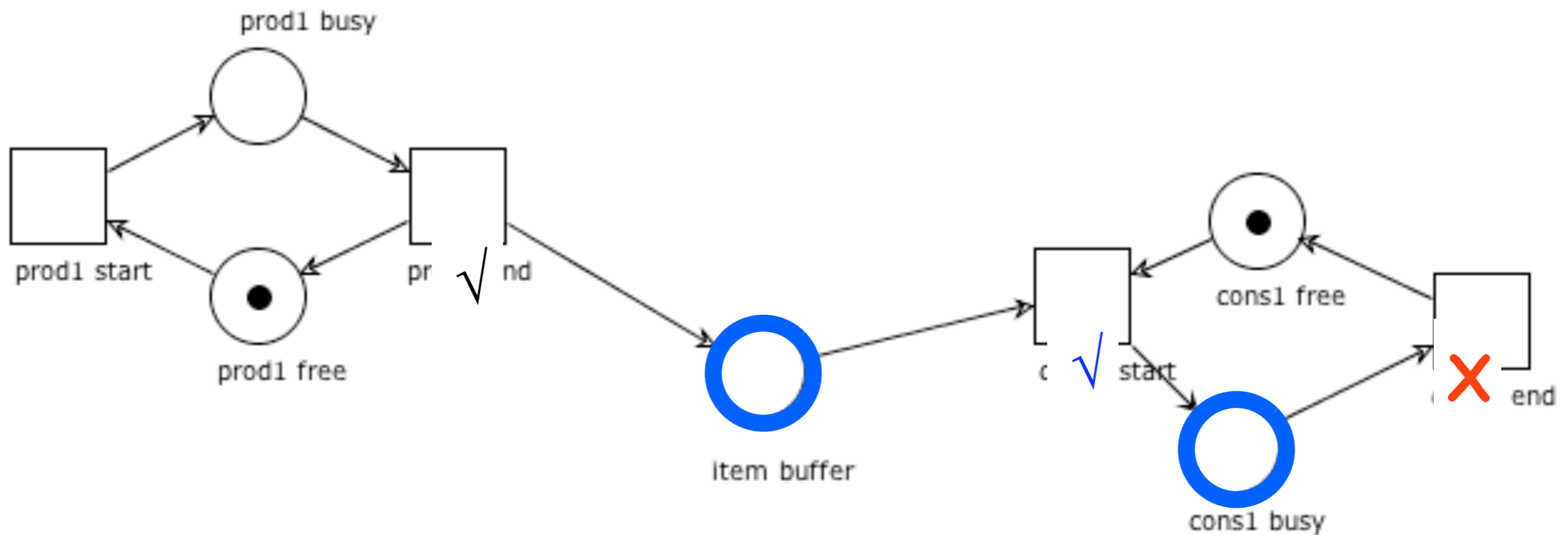


• R

• $R \supseteq R$ •

Trap check: example

Is $R = \{ \text{itembuffer}, \text{cons1busy} \}$ a trap?



R •

Fundamental property of traps

Proposition: Marked traps remain marked

Take a trap R .

We just need to prove that the set of markings

$$\mathbf{M} = \{ M \mid M(R) > 0 \}$$

is stable, which is immediate by definition of trap

Corollary:

If a trap R is unmarked at some reachable marking M ,
then it was initially unmarked at M_0

Exercise

Prove that the union of traps is a trap

Putting pieces together

unmarked siphons stay unmarked
(marked siphons can become unmarked)

if a siphon is marked at M , it was marked at M_0

if all proper siphons always stay marked \Rightarrow deadlock-free

Putting pieces together

if all proper siphons always stay marked \Rightarrow deadlock-free

marked traps stay marked
(unmarked traps can become marked)

if a siphon contains a marked trap, it stays marked

if all siphons contain marked traps, they stay marked
 \Rightarrow deadlock-free

A sufficient condition for deadlock-freedom

Proposition:

If every proper siphon of a system contains a marked trap,
then the system is deadlock-free

We show that if the system is not deadlock free,
then there is a siphon that does not include any marked trap.

Assume some reachable M is dead.

Let R be the set of unmarked places at M .

Then, we have seen that R is a proper siphon.

Since $M(R)=0$, then R includes no trap marked at M .

Therefore, R includes no trap marked at M_0

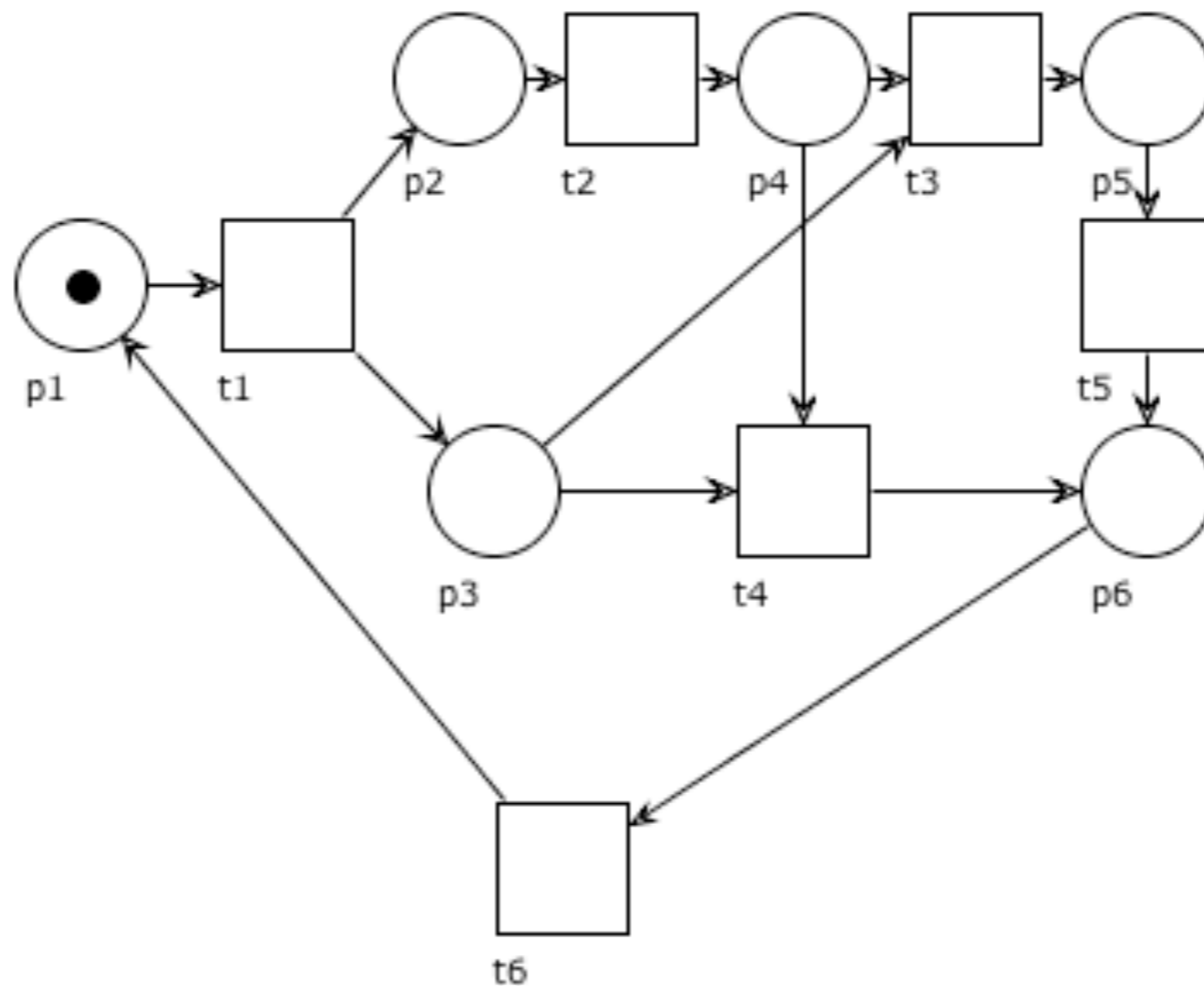
Note

It is easy to observe that every siphon includes a (possibly empty) unique maximal trap with respect to set inclusion

Moreover, a siphon includes a marked trap iff its maximal trap is marked

Exercise

Find all proper siphons and traps in the net below
(at most 2^6 sets to consider)



Liveness = Place-liveness
(in free-choice systems)

Place liveness (recall)

Definition: Let (P, T, F, M_0) be a net system.

A place $p \in P$ is **live** if $\forall M \in [M_0 \rangle. \exists M' \in [M \rangle. M'(p) > 0$

A place p is live

if every time it becomes unmarked

there is still the possibility to be marked in the future

(or if it is always marked)

Definition:

A net system (P, T, F, M_0) is **place-live** if every place $p \in P$ is live

liveness implies place-liveness

Dead nodes (recall)

Definition: Let (P, T, F) be a net system.

A transition $t \in T$ is **dead** at M if $\forall M' \in [M \rangle. M' \not\xrightarrow{t}$

A place $p \in P$ is **dead** at M if $\forall M' \in [M \rangle. M'(p) = 0$

Some obvious facts

If a system is not live,
it has a transition dead at some reachable marking M

If a system is not place-live,
it has a place dead at some reachable marking M

If a place / transition is dead at M , then it remains dead
at any marking M' reachable from M
(the set of dead nodes can only increase during a run)

Every transition in the pre- or post-set of a dead place
is also dead

An obvious facts in free-choice systems

In a free-choice system:

if an output transition t of a place p is dead at M

then any output transition t' of p is dead at M

(because t and t' must have the same pre-set)

Dead t , dead p

Lemma: If the transition t is dead at M in a free-choice system, then there is a non-live place p in the pre-set of t

By contraposition, we prove: if all input places of t are live then t is not dead

Let $\bullet t = [t] \cap P = \{p_1, \dots, p_n\}$

Since all places p_1, \dots, p_n are live at M , there exists

$$M \xrightarrow{\sigma_1} M_1 \xrightarrow{\sigma_2} \dots \xrightarrow{\sigma_n} M_n$$

such that $M_i(p_i) > 0$ for all i

If the sequence contains $u \in [t]$ then t is not dead at M

If no transition in $[t]$ appears in the sequence, then no token in $\bullet t$ is consumed

Hence $M_n(p_i) > 0$ for all i , and $M_n \xrightarrow{t}$ and t is not dead at M

Place-liveness implies liveness in f.c. systems

Proposition: If a free-choice system is place-live,
then it is live

By contraposition, we prove:
non-liveness implies non-place-liveness

If a free-choice system is not live then there is a
transition t dead at some reachable marking M

But then some input place of t must be non-live at M ,
so the system is not place-live

Consequence in f.c. nets: place-liveness = liveness

If a free-choice system is place-live, then it is live

In any system, liveness implies place-liveness

Corollary:

A free-choice system is live **iff** it is place-live

Commoner's theorem

Theorem:

A free-choice system is live
iff

every proper siphon includes an initially marked trap

We show just the “if” direction, which is simpler

We need a technical lemma

Non-liveness and unmarked siphons

Lemma: Every non-live free-choice system has a proper siphon R and a reachable marking M such that $M(R)=0$

By non-liveness: the system is not place-live,
i.e., some p is dead at some $L \in [M_0 \rangle$

Take $M \in [L \rangle$ such that every place not dead at M
is not dead at any marking of $[M \rangle$
i.e. all markings in $[M \rangle$ have the same set R of dead places
(dead places remain dead)

Next we prove that R is a proper siphon and $M(R) = 0$

Non-liveness and unmarked siphons

Lemma: Every non-live free-choice system has a proper siphon R and a reachable marking M such that $M(R)=0$

1. R is a siphon $\bullet R \subseteq R \bullet$
 - any $t \in \bullet R$ is dead at M
 - every t dead at M has an input place in R
(t has some input place dead at some marking reachable from M)
2. R is proper
 p is dead at L , hence it is dead at M , hence $p \in R$, hence $R \neq \emptyset$
3. $M(R) = 0$ because it contains dead places

Commoner's theorem: "if" direction

A free-choice system is live
if
every proper siphon includes an initially marked trap

By contraposition, we prove:
if a free-choice is non-live, then
a proper siphon exists whose traps are all unmarked

Commoner's theorem: "if" direction

We know that a non-live free-choice system contains a proper siphon R such that $M(R)=0$ at some reachable M
(see previous lemma)

So every trap included in R is unmarked at M

(since marked traps remain marked)
every trap included in R must be unmarked initially



Complexity of the
non-liveness problem
in free-choice systems

A non-deterministic algorithm for non-liveness

1. guess a set of places R
(polynomial time)
2. check if R is a siphon ($\bullet R \subseteq R \bullet$)
(polynomial time)
3. if R is a siphon, compute the maximal trap $Q \subseteq R$
4. if $M_0(Q)=0$, then answer "non-live", otherwise "live"
(polynomial time)

A polynomial algorithm for maximal trap in a siphon

$$\bullet R \subseteq R \bullet$$

$$\bullet Q \supseteq Q \bullet$$

3. if R is a siphon, compute the maximal trap $Q \subseteq R$

Input: A net $N = (P, T, F)$ and $R \subseteq P$

Output: $Q \subseteq R$

$Q := R$

while $(\exists p \in Q, \exists t \in p\bullet, t \notin \bullet Q)$

$Q := Q \setminus \{p\}$

return Q

Non-liveness for f.c. nets is in NP

The non-liveness problem for free-choice systems is in NP

Is the same problem in P?

The corresponding deterministic algorithm cannot make
the guess in step 1

It has to explore all possible subsets of places
 $2^{|P|}$ cases!

NP-completeness

We next sketch the proof of the reduction to non-liveness
in a free-choice net of the CNF-SAT problem

(SATisfiability problem
for propositional formulas in Conjunctive Normal Form)

CNF-SAT is an NP-complete problem

CNF-SAT decision problem

Variables: x_1, x_2, \dots, x_n

Literals: $x_1, \bar{x}_1, x_2, \bar{x}_2, \dots, x_n, \bar{x}_n$

Clause: disjunction of literals

Formula: conjunction of clauses

Example: $\phi = (x_1 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_2 \vee \bar{x}_3)$

Is there an assignment of boolean values to the variables such that $\phi = \text{true}$?

The free-choice net of a formula

Given a formula ϕ , the idea is to construct a free-choice system (P, T, F, M_0) and show that

the formula ϕ is satisfiable
iff
 (P, T, F, M_0) is not live

The free-choice net of a formula

Given a formula ϕ , the idea is to construct a free-choice system (P, T, F, M_0) and show that

the formula ϕ is not satisfiable
iff
 (P, T, F, M_0) is live

CNF-SAT formulas

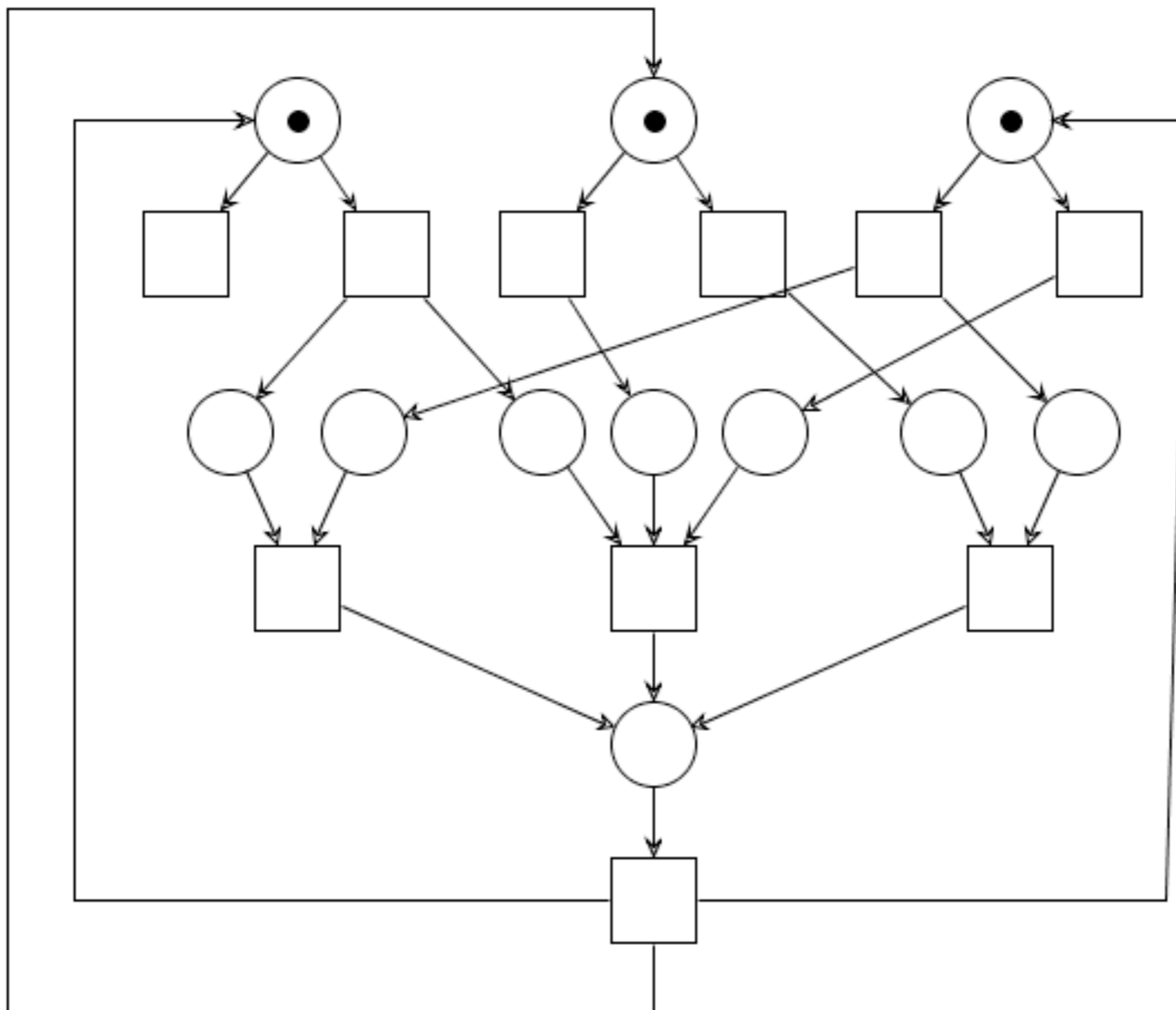
Is there an assignment of boolean values to the variables such that $\phi = \text{true}$?

Is there an assignment of boolean values to the variables such that $\neg\phi = \text{false}$?

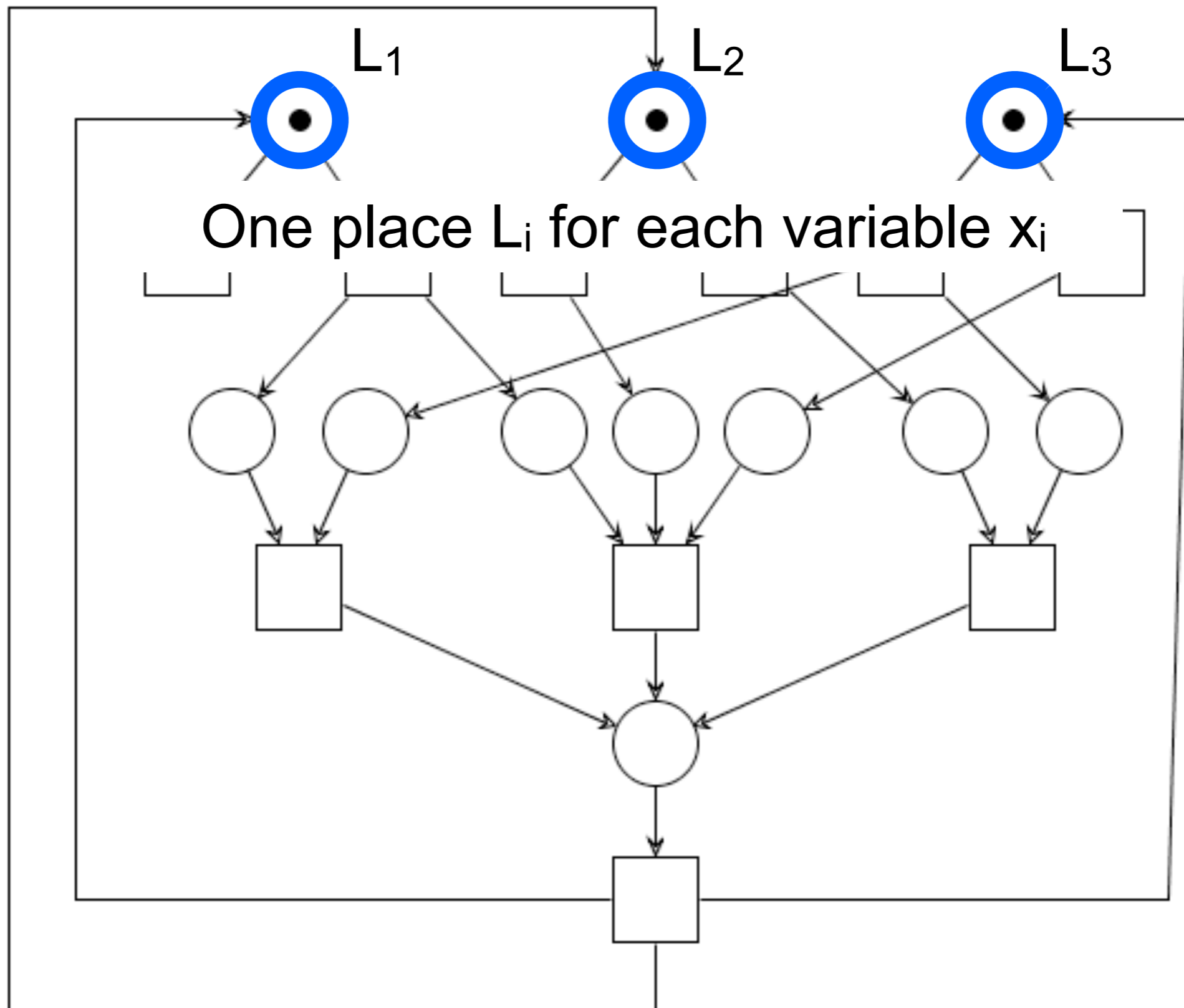
$$\phi = (x_1 \vee \bar{x}_3) \wedge (x_1 \vee \bar{x}_2 \vee x_3) \wedge (x_2 \vee \bar{x}_3)$$

$$\neg\phi = (\bar{x}_1 \wedge x_3) \vee (\bar{x}_1 \wedge x_2 \wedge \bar{x}_3) \vee (\bar{x}_2 \wedge x_3)$$

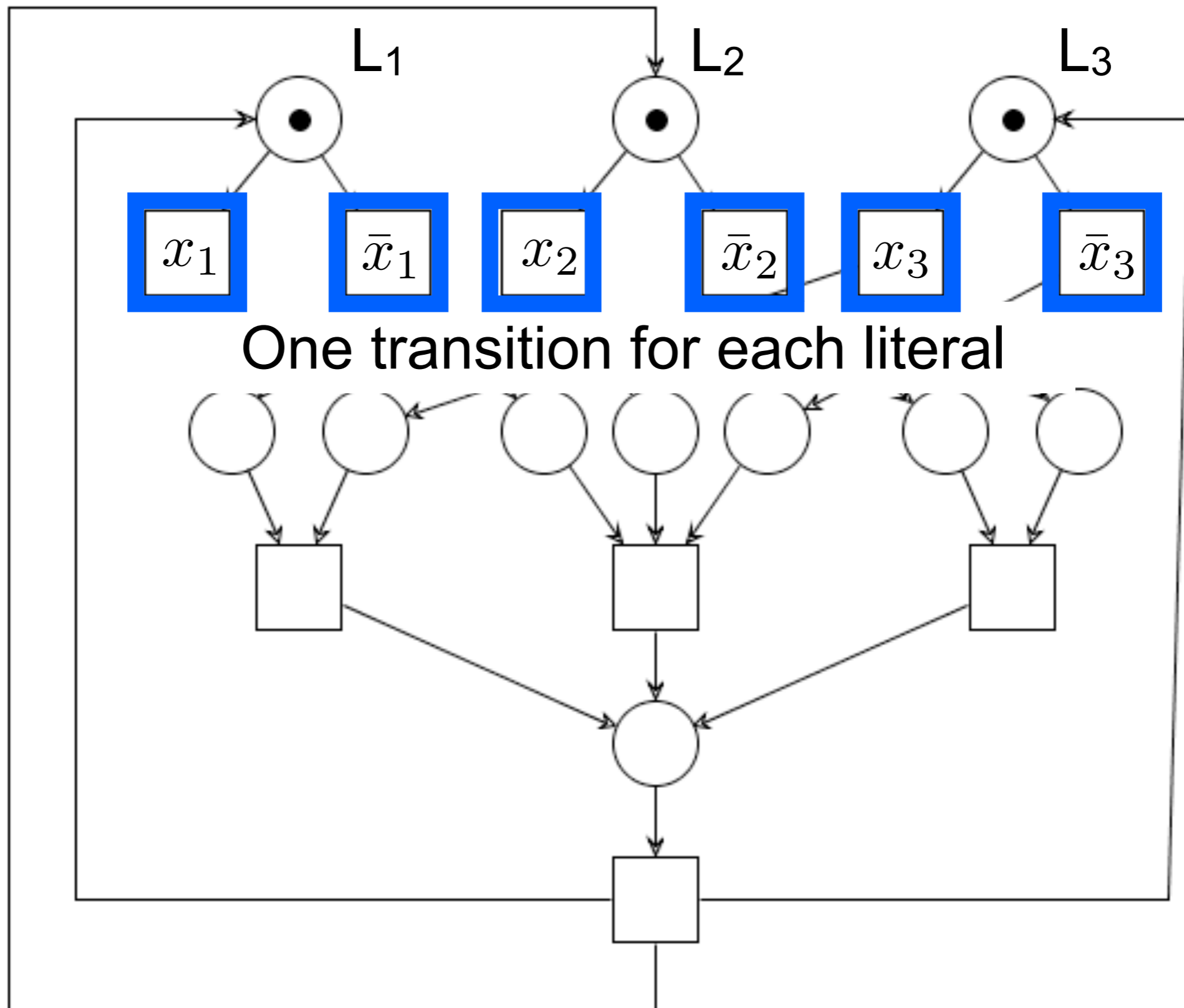
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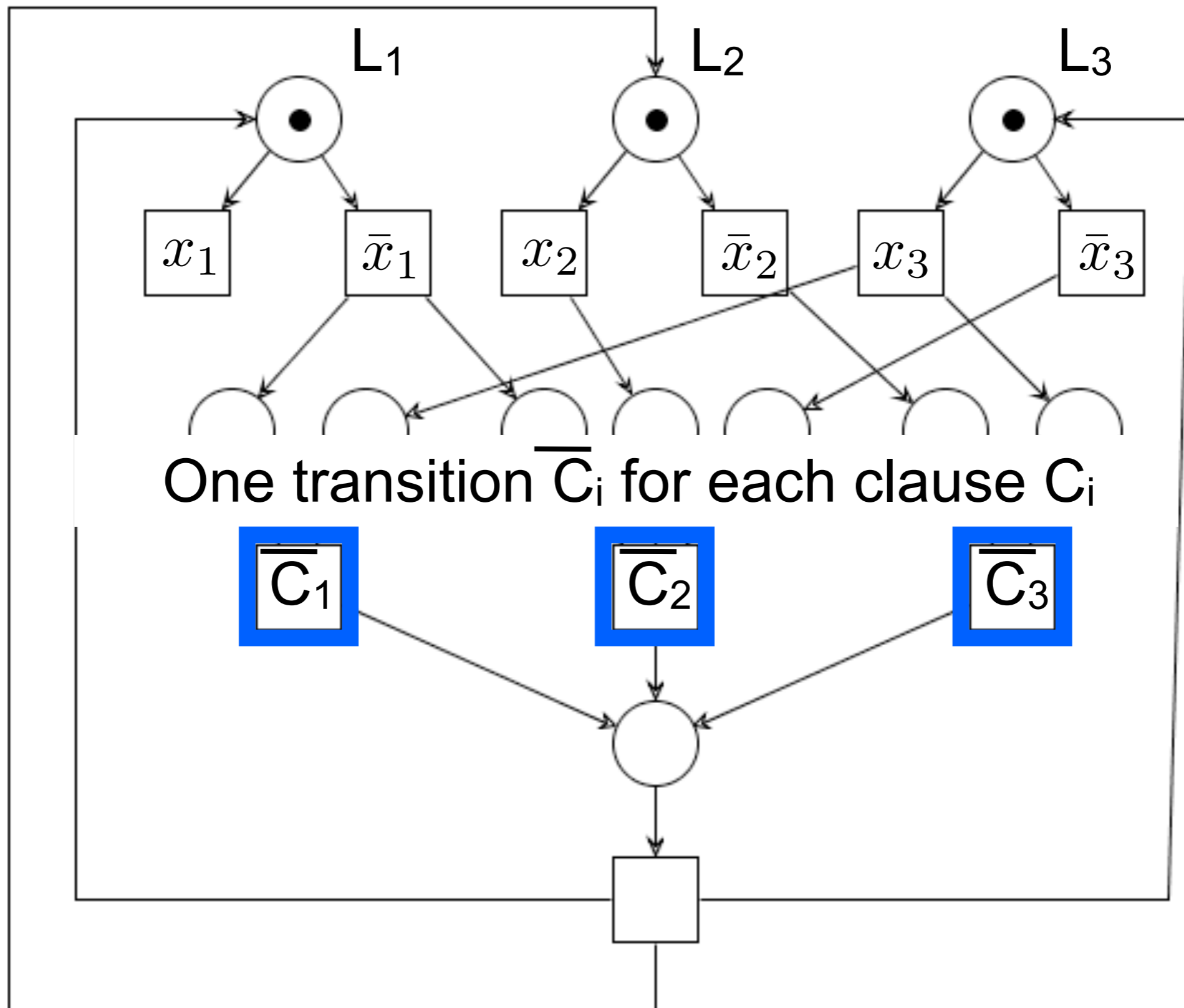
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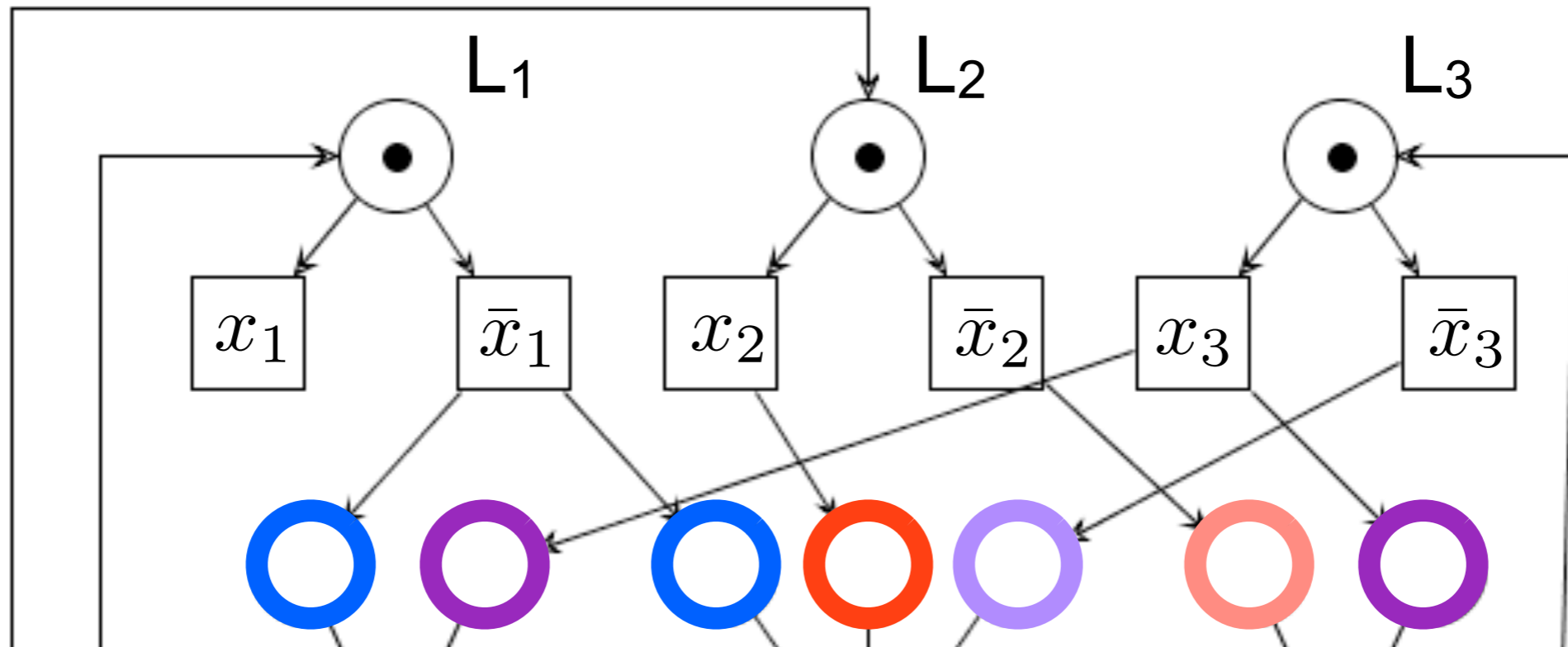
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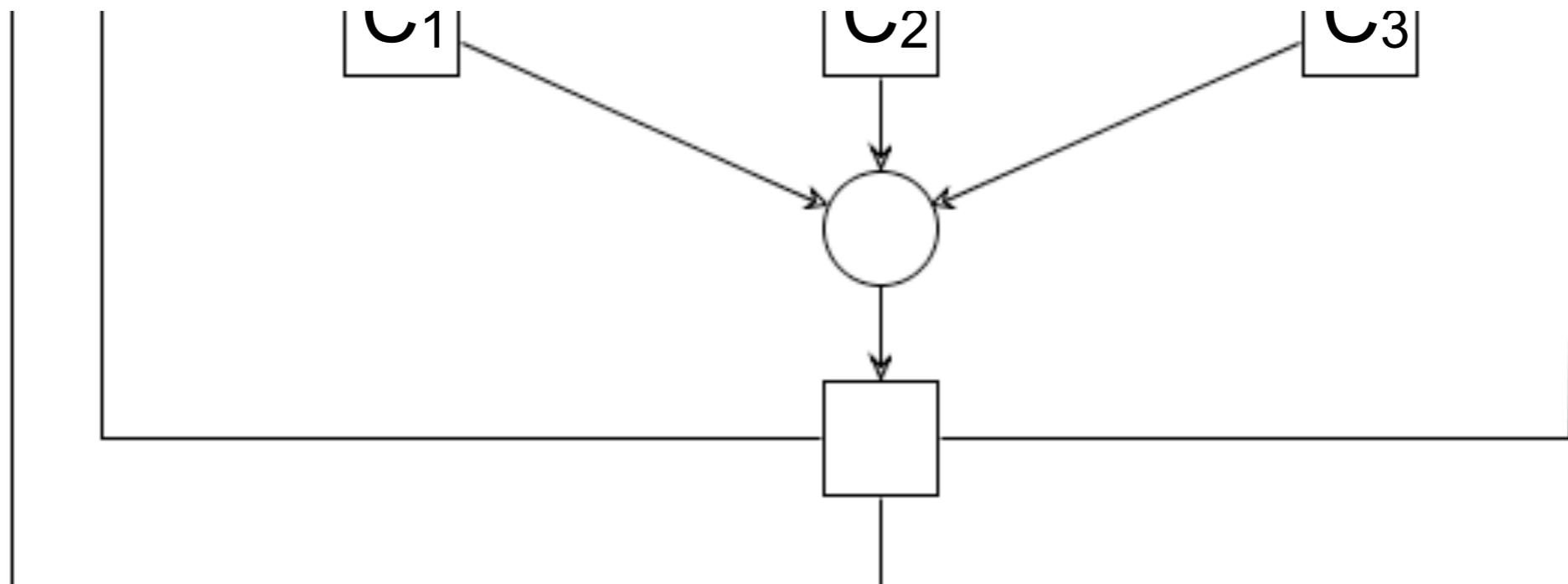
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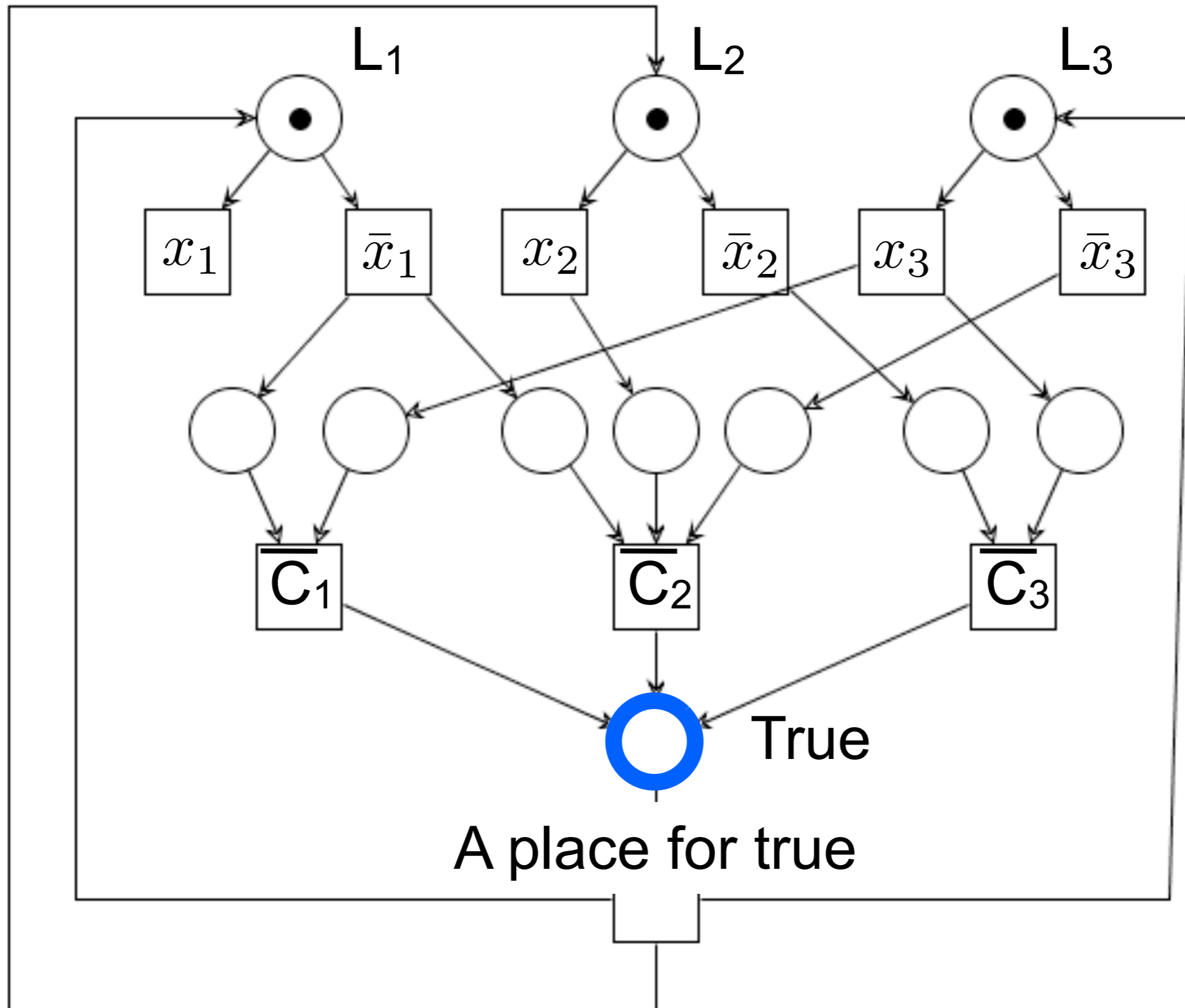
$$\neg\phi = (\bar{x}_1 \wedge x_3) \vee (\bar{x}_1 \wedge x_2 \wedge \bar{x}_3) \vee (\bar{x}_2 \wedge x_3)$$



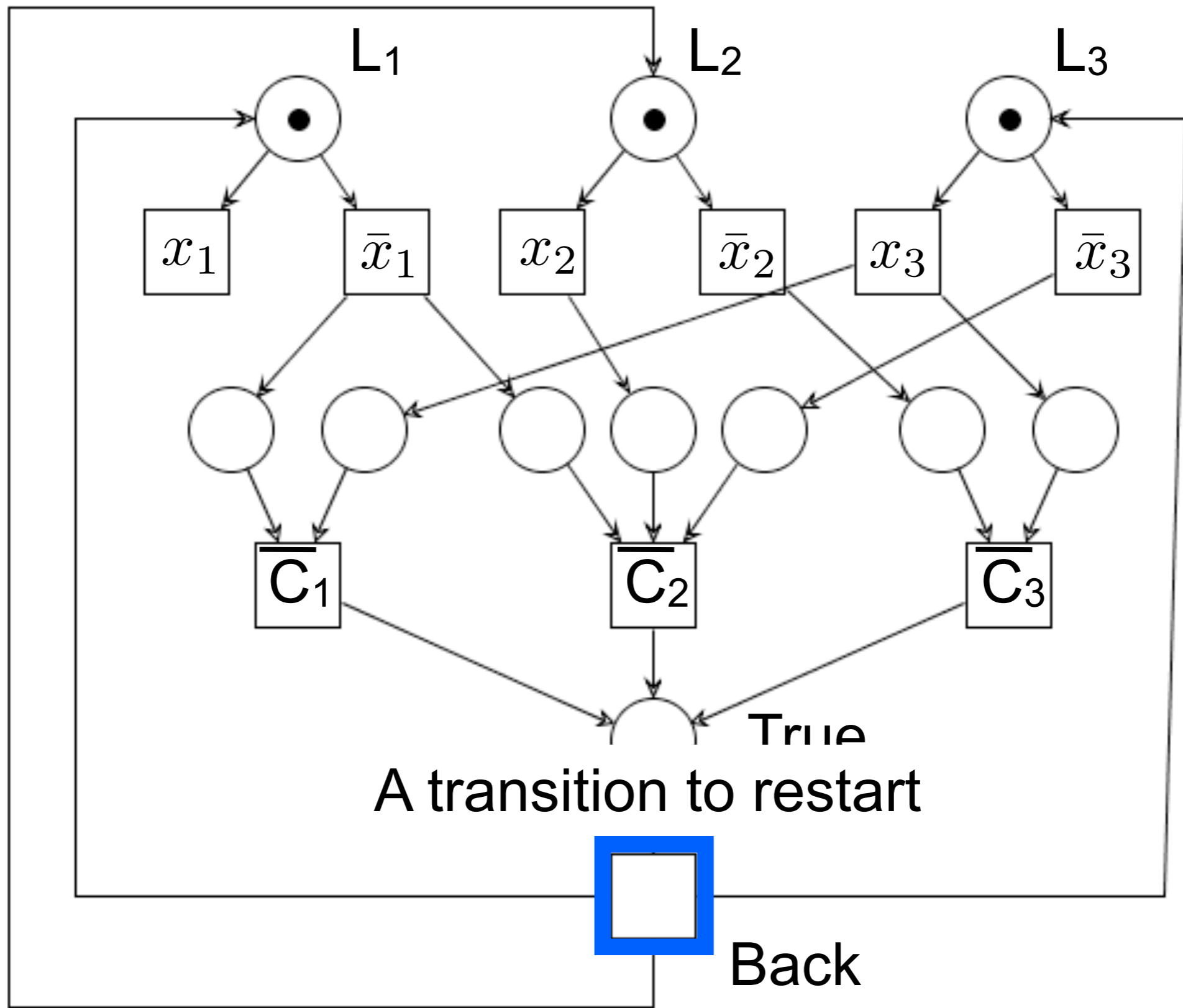
A place for each occurrence of a literal



$$\neg\phi = (\bar{x}_1 \wedge x_3) \vee (\bar{x}_1 \wedge x_2 \wedge \bar{x}_3) \vee (\bar{x}_2 \wedge x_3)$$



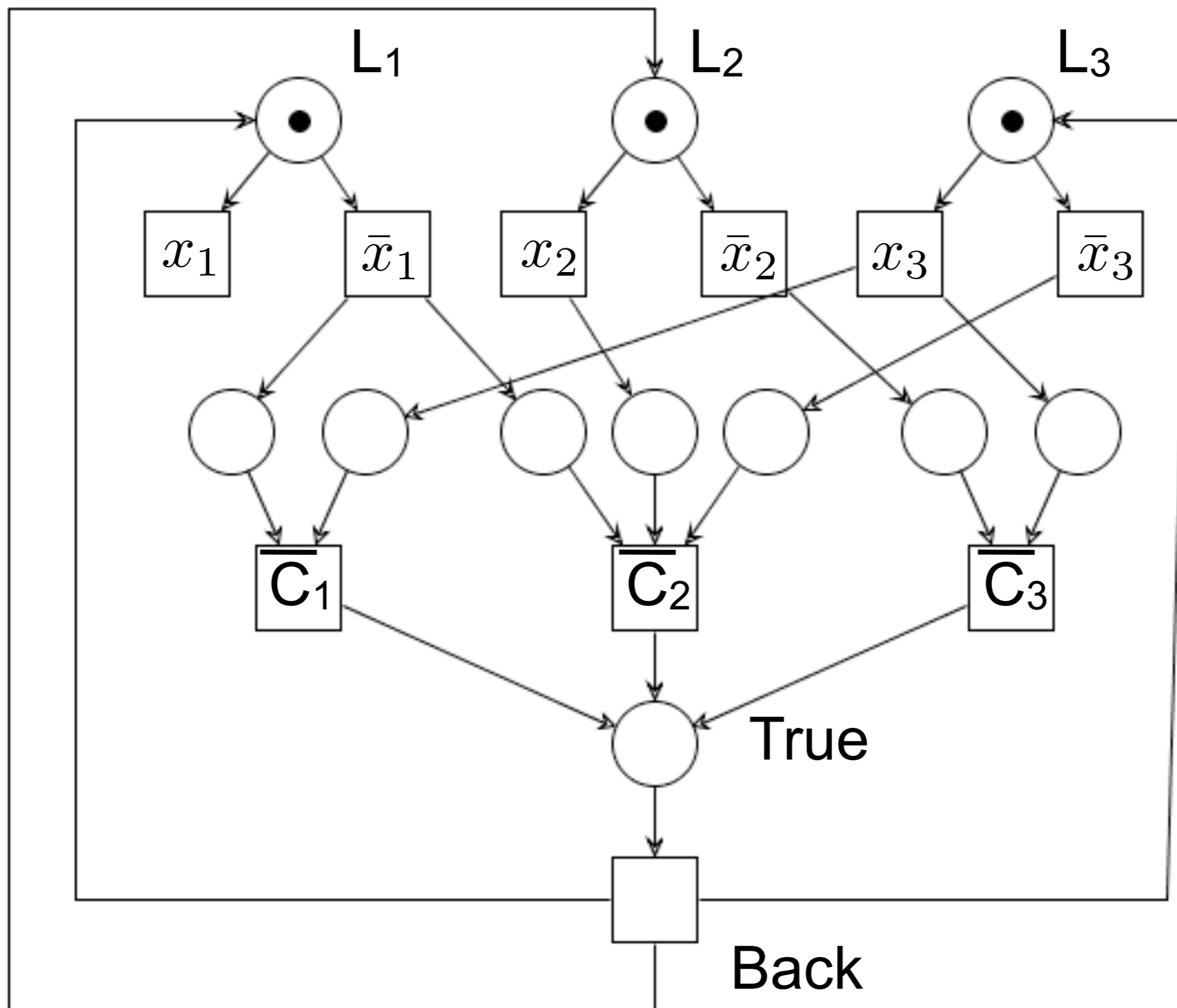
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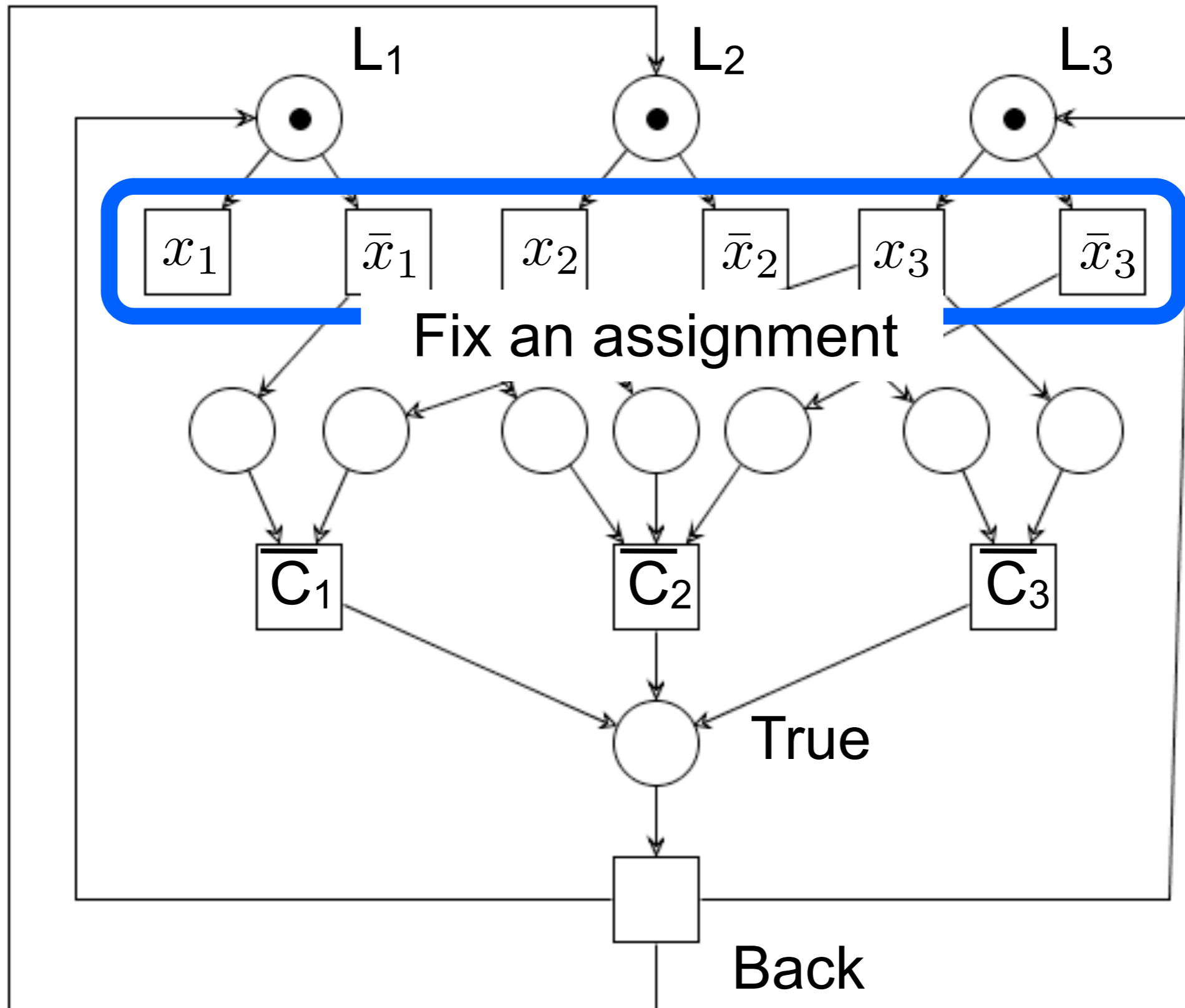
A transition to restart

Back

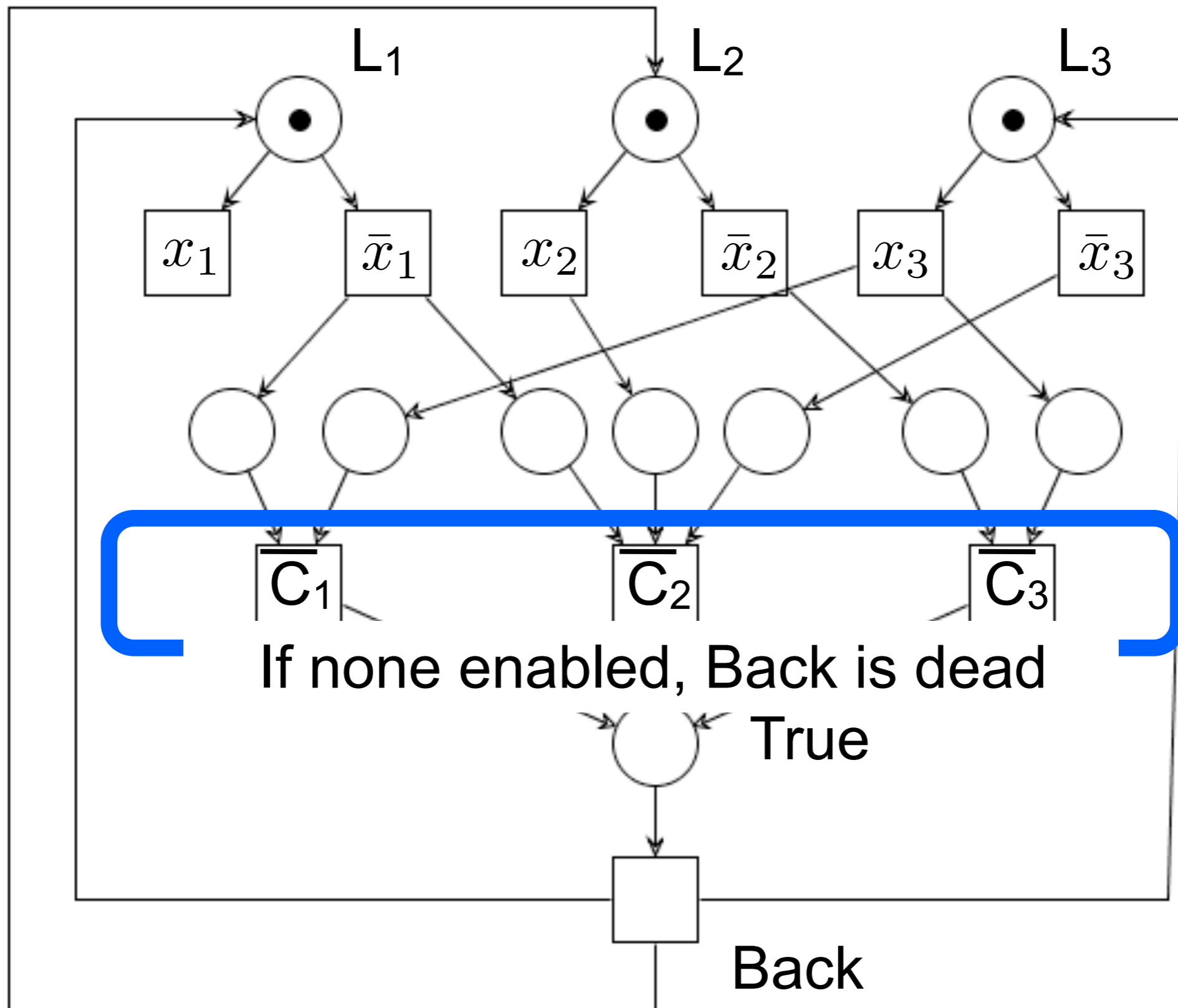
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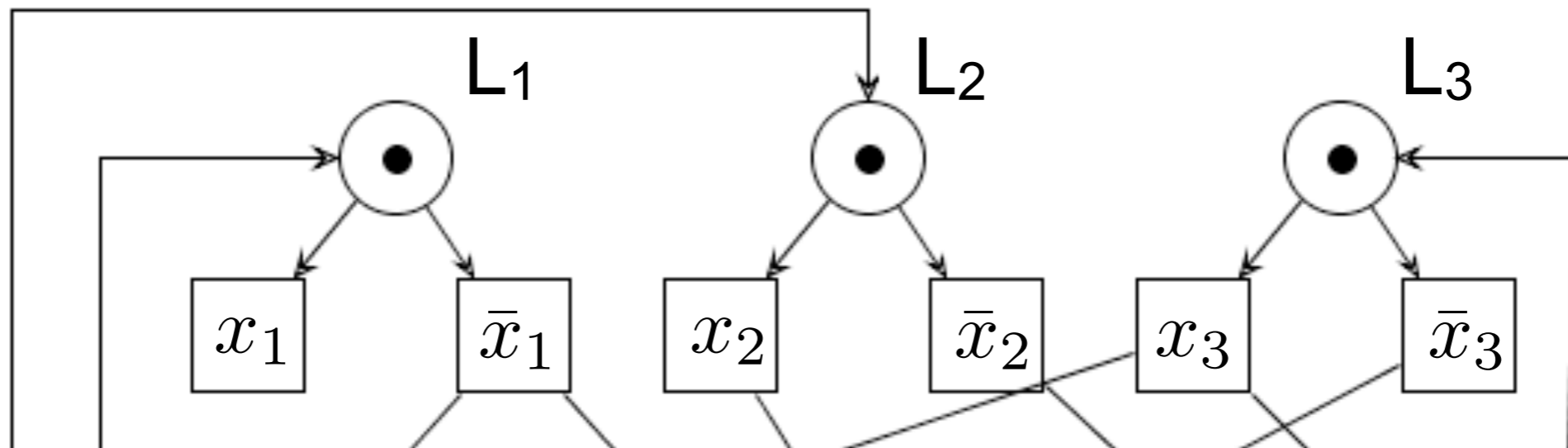
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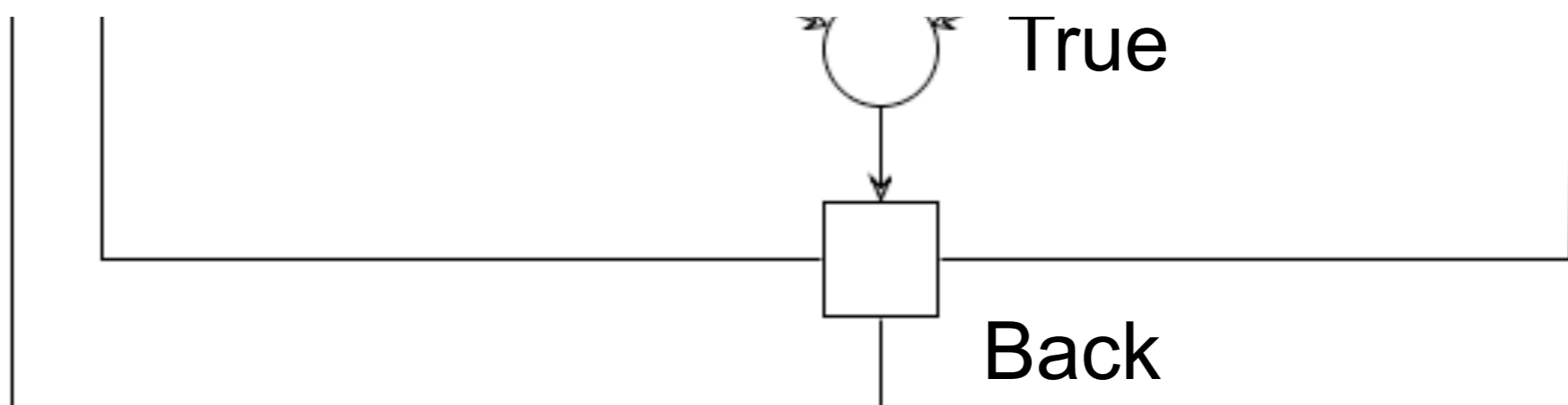


$$\neg\phi = (\bar{x}_1 \wedge x_3) \vee (\bar{x}_1 \wedge x_2 \wedge \bar{x}_3) \vee (\bar{x}_2 \wedge x_3)$$

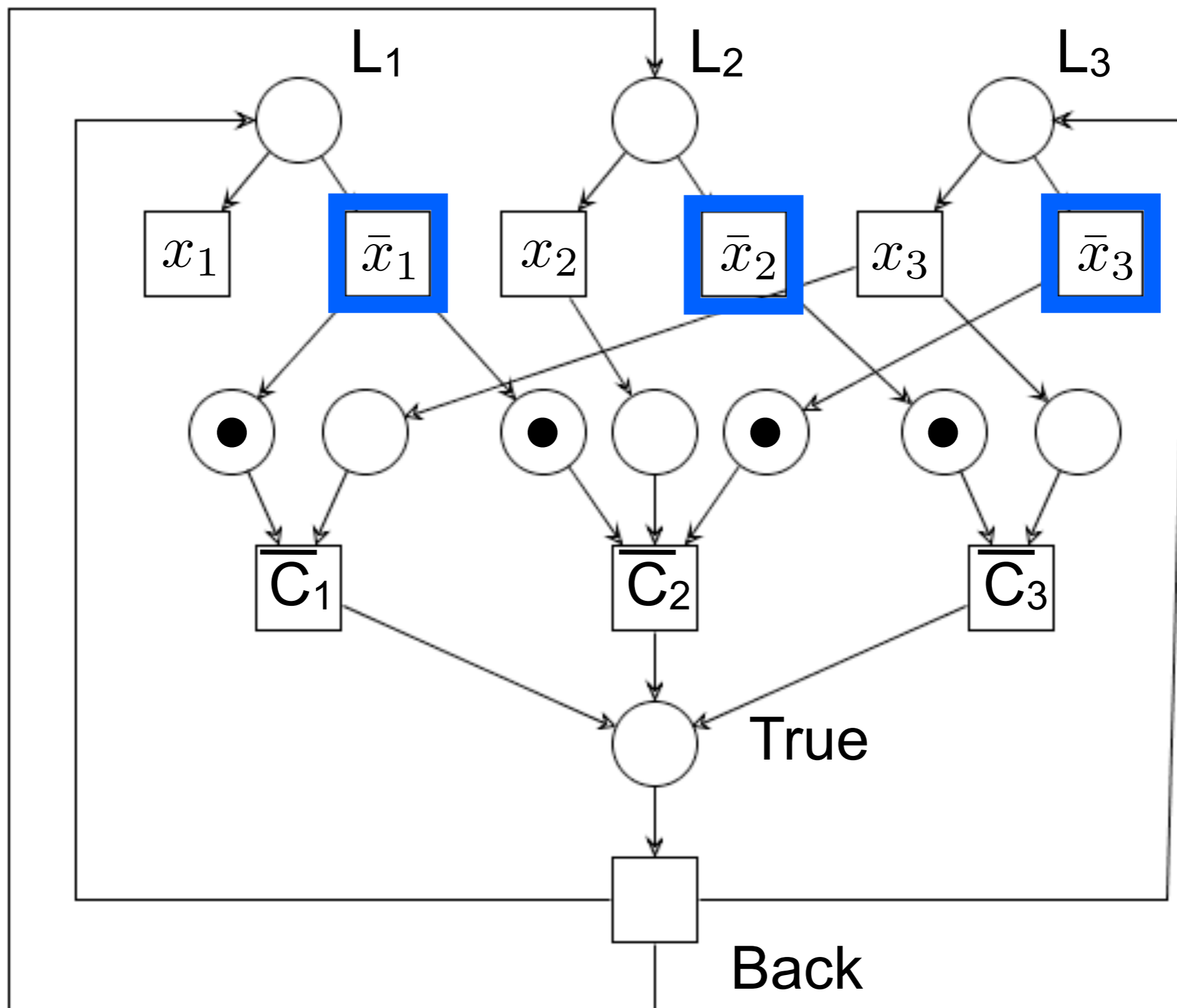


If ϕ is satisfiable, then the net is not live

If the net is not live, then ϕ is satisfiable



$$\neg\phi = (\bar{x}_1 \wedge x_3) \vee (\bar{x}_1 \wedge x_2 \wedge \bar{x}_3) \vee (\bar{x}_2 \wedge x_3)$$



Main consequence



No polynomial algorithm to decide liveness of a free-choice system is available

(unless $P=NP$)

Exercise

Draw the net corresponding to the formula

$$x_2 \wedge (x_1 \vee \bar{x}_3 \vee \bar{x}_4) \wedge (x_1 \vee \bar{x}_2) \wedge (\bar{x}_1 \vee x_4) \wedge (\bar{x}_2 \vee \bar{x}_4)$$

Is it satisfiable?

Live and bounded
free-choice nets

Rank Theorem

(main result, proof omitted)

Theorem:

A free-choice system (P, T, F, M_0) is live and bounded
iff

1. it has at least one place and one transition polynomial
2. it is connected polynomial
3. M_0 marks every proper siphon
4. it has a positive S-invariant polynomial
5. it has a positive T-invariant polynomial
6. $\text{rank}(N) = |C_N| - 1$ polynomial

(where C_N is the set of clusters)

A polynomial algorithm for maximal siphon in R

Input: A net $N = (P, T, F, M_0)$, $R \subseteq P$

Output: $Q \subseteq R$ maximal siphon in R

$Q := R$

while $(\exists p \in Q, \exists t \in \bullet p, t \notin Q \bullet)$

$Q := Q \setminus \{p\}$

return Q

A polynomial algorithm for maximal unmarked siphon

3. M_0 marks every proper siphon **polynomial**

Input: A net $N = (P, T, F, M_0)$, $R = \{p \mid M_0(p) = 0\}$

Output: $Q \subseteq R$ maximal unmarked siphon

$Q := R$

while $(\exists p \in Q, \exists t \in \bullet p, t \notin Q \bullet)$

$Q := Q \setminus \{p\}$

return Q

If Q is empty then M_0 marks every proper siphon

Main consequence

**The problem to decide
if a free-choice system is live and bounded
can be solved in polynomial time
(using the Rank Theorem)**



Coverability

S-Coverability analysis

A case is often composed by parallel threads of control
(each thread imposing some order over its tasks)

The notion of S-coverability allows to reveal such threads

A technique to find a positive S -invariant

Decompose the free-choice net N in suitable S -nets so
that any place of N belongs to an S -net
(the same place can appear in more S -nets)

Each S -net induces a uniform S -invariant

A positive S -invariant is obtained
as the sum of the S -invariants of each subnet

S-component

a set of nodes

Definition: Let $N = (P, T, F)$ and $\emptyset \subset X \subseteq P \cup T$
Let $N' = (P \cap X, T \cap X, F \cap (X \times X))$ be a subnet of N .
 N' is an **S-component** if

1. it is a strongly connected S-net
2. for every place $p \in X \cap P$, we have $\bullet p \cup p \bullet \subseteq X$

if a place is selected
the all the attached transitions must be selected

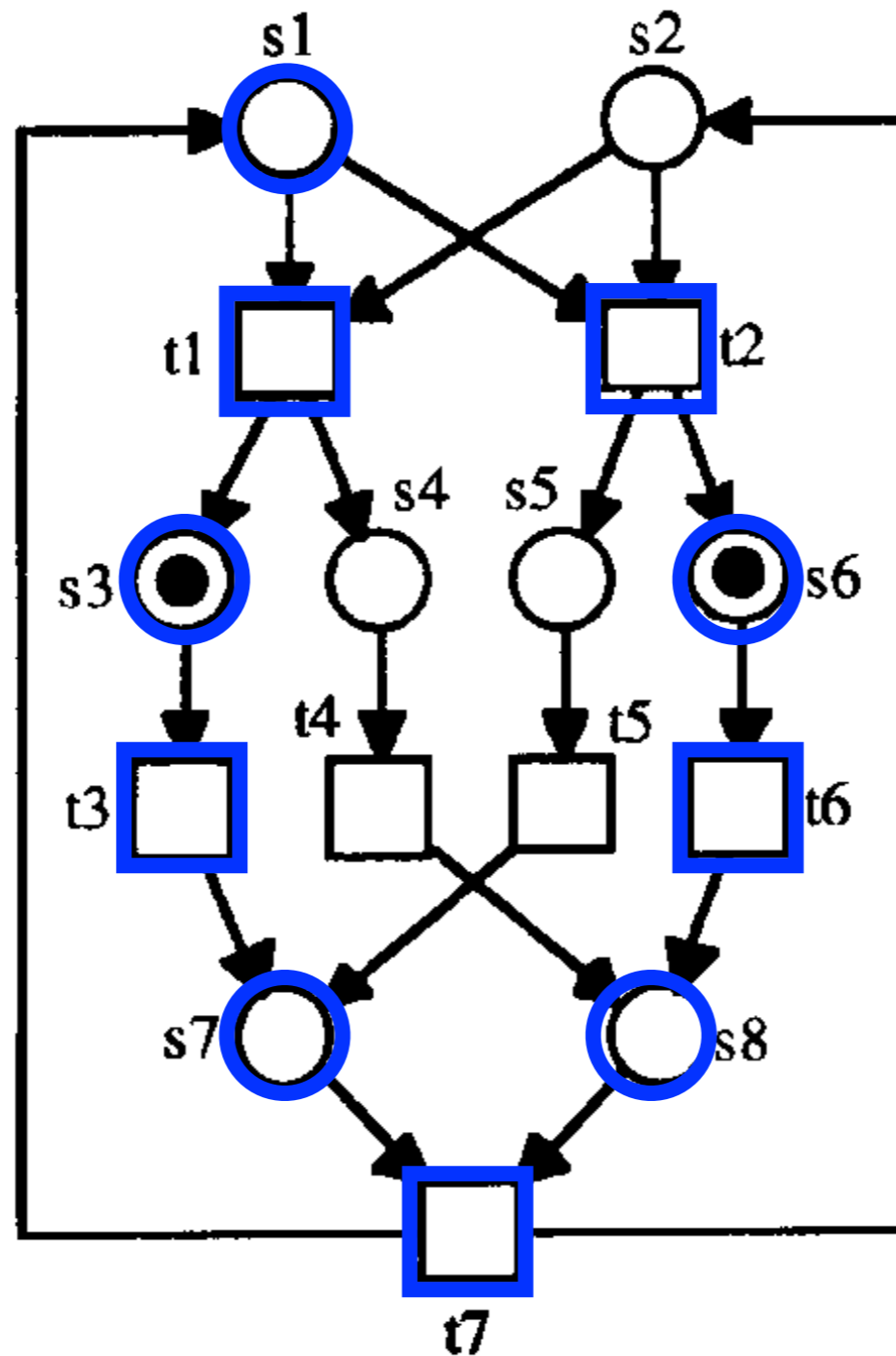
S-cover

Definition: Let **C** be a set of S-components of a net **N**

C is an **S-cover** if every place p of **N** belongs to one or more S-components in **C**

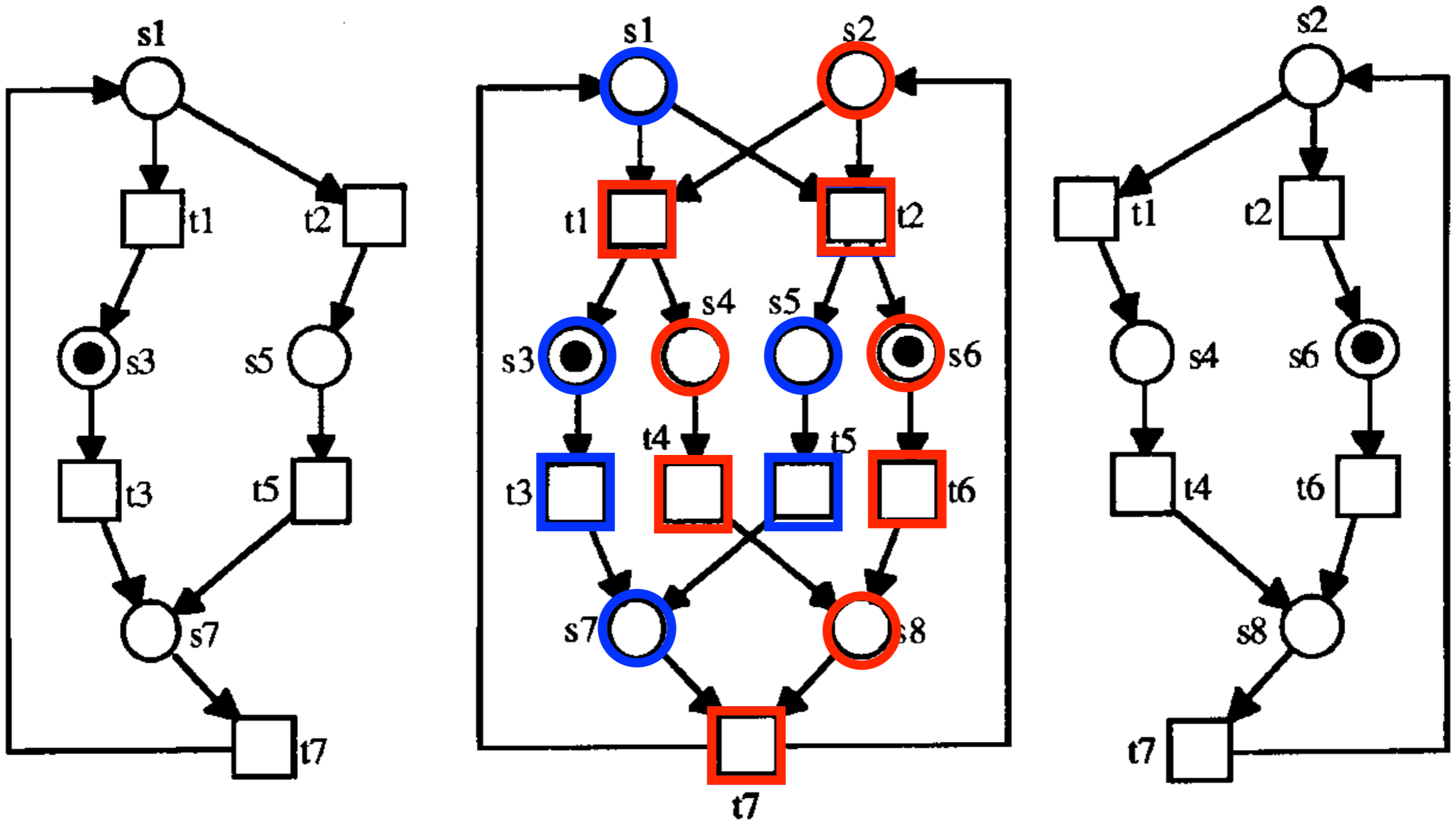
We say that **N** is **covered by S-components** if it has an S-cover

S-cover: example



not an S-net

S-cover: example



S-coverability theorem

Theorem: If a free-choice system is live and bounded then it is S-coverable

(proof omitted)

Consequence:

free-choice + not S-coverable \Rightarrow not (live and bounded)

A technique to find a positive T-invariant

Decompose the free-choice net N in suitable T-nets so
that any transition of N belongs to a T-net
(the same transition can appear in more T-nets)

Each T-net induces a uniform T-invariant

A positive T-invariant is obtained
as the sum of the T-invariants of each subnet

T-component

a set of nodes

Definition: Let $N = (P, T, F)$ and $\emptyset \subset X \subseteq P \cup T$
Let $N' = (P \cap X, T \cap X, F \cap (X \times X))$ be a subnet of N .
 N' is a **T-component** if

1. it is a strongly connected T-net
2. for every transition $t \in X \cap T$, we have $\bullet t \cup t \bullet \subseteq X$

if a transition is selected
the all the attached places must be selected

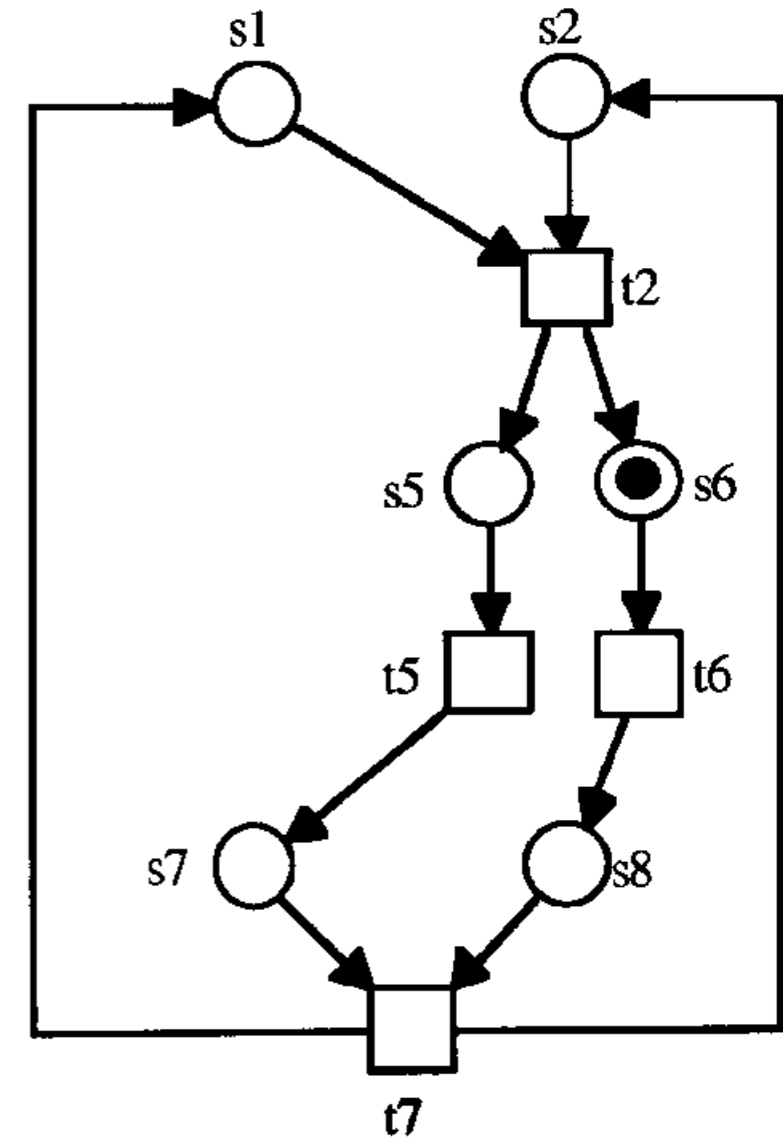
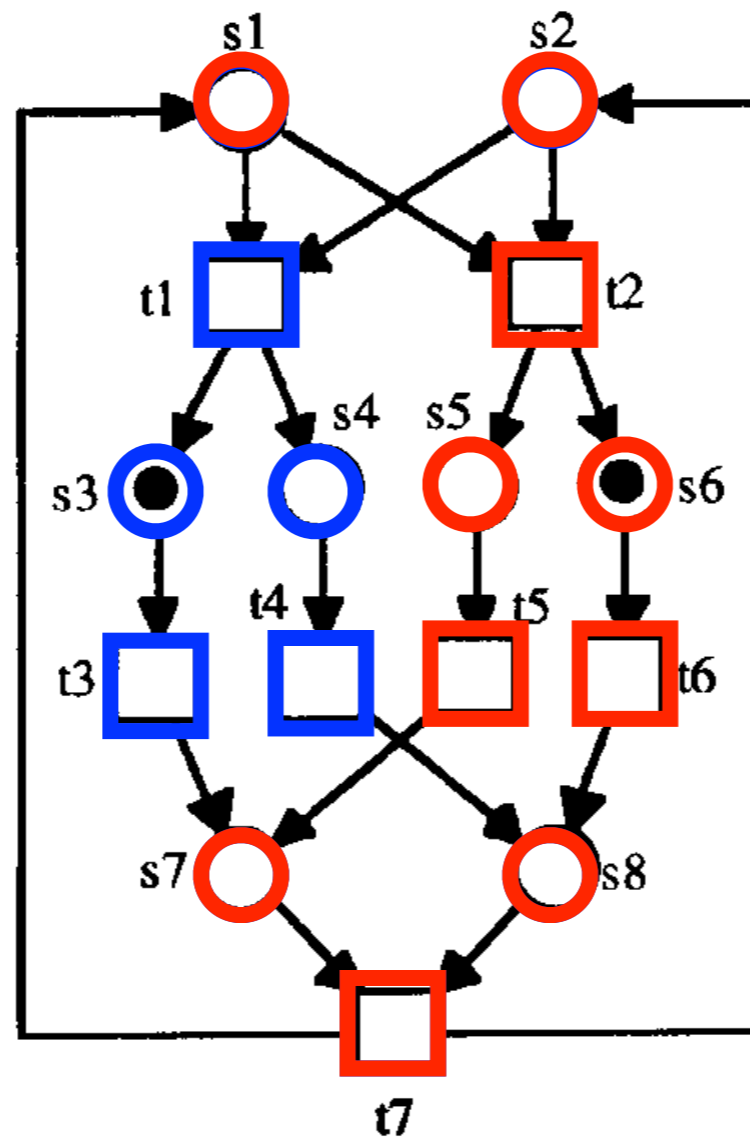
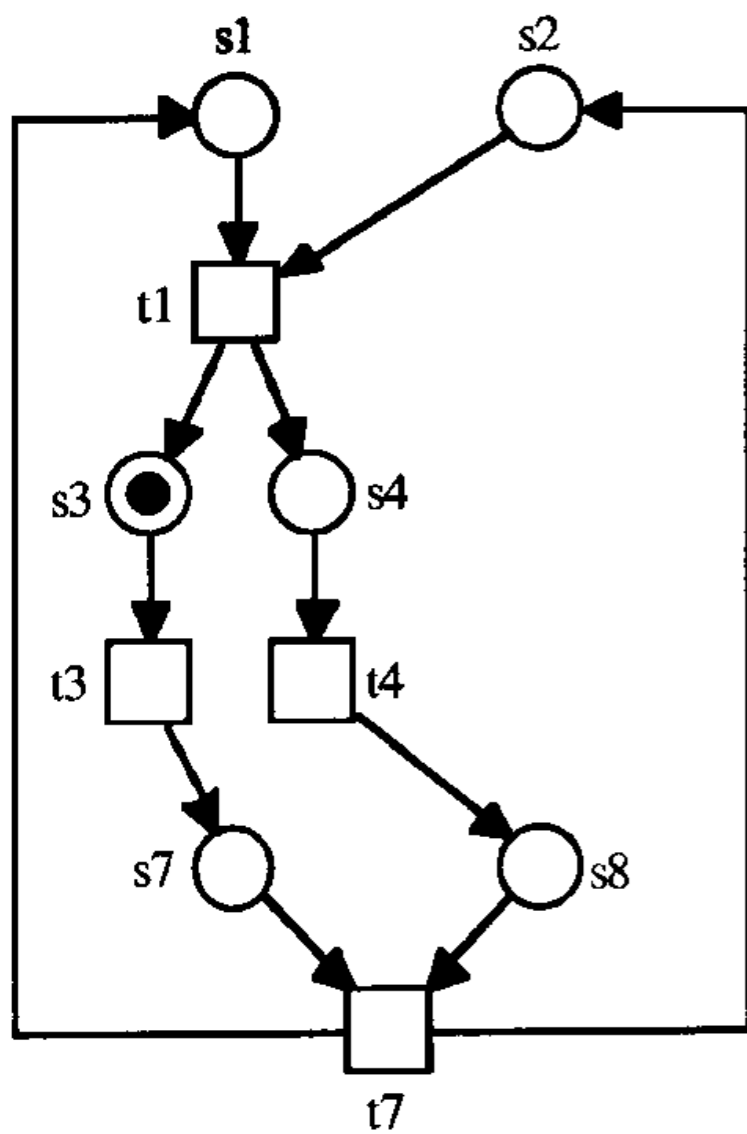
T-cover

Definition: Let \mathbf{C} be a set of T-components of a net N

\mathbf{C} is a **T-cover** if every transition t of N belongs to one or more T-components in \mathbf{C}

We say that N is **covered by T-components** if it has a T-cover

T-cover: example



T-coverability theorem

Theorem: If a free-choice system is live and bounded then it is T-coverable

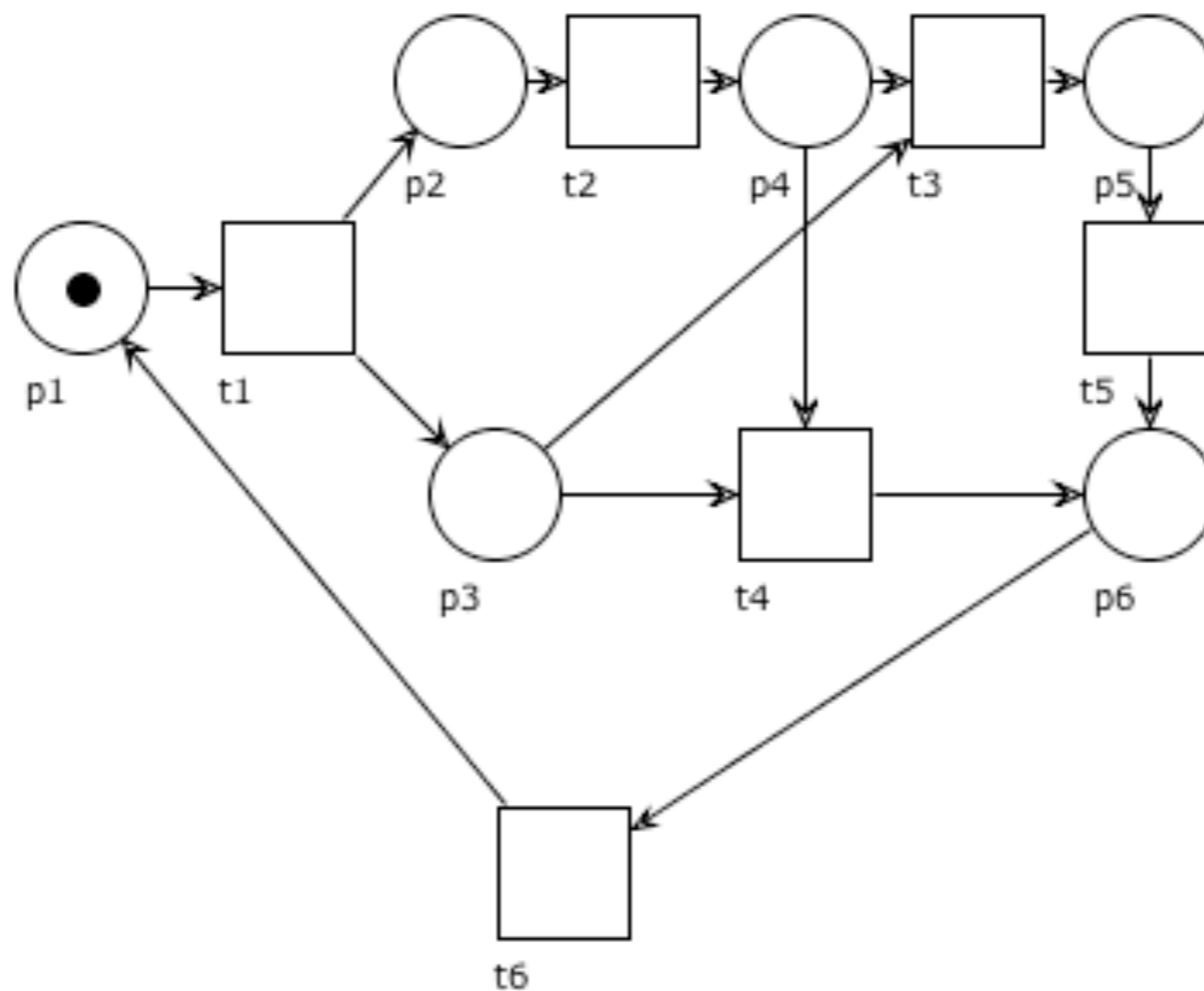
(proof omitted)

Consequence:

free-choice + not T-coverable \Rightarrow not (live and bounded)

Exercise

Find an S-cover and a T-cover for the net below and derive suitable S- and T-invariants



Compositionality

Compositionality of sound free-choice nets

Lemma:

If a free-choice workflow net N is sound
then it is safe

(because N^* is S -coverable and $M_0=i$ has just one token)

Proposition:

If N and N' are sound free-choice workflow nets
then $N[N'/t]$ is a sound free-choice workflow net

(we just need to show that $N[N'/t]$ is free-choice)