Methods for the specification and verification of business processes

MPB (6 cfu, 295AA)

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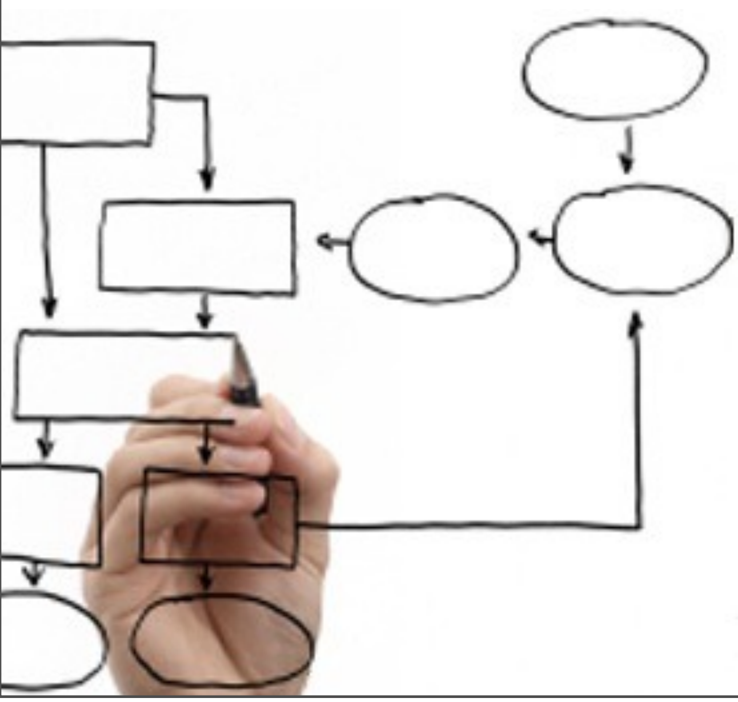


08 - More on nets

Object

To continue the overview of the basic concepts of Petri nets

Enabling and firing

A transition t is **enabled** at marking M iff $\bullet t \subseteq M$ and we write $M \xrightarrow{t}$ (also $M\,[t\rangle$)

A transition t that is enabled at M can **fire**. The **firing** of t at M changes the state to

$$M' = M - \bullet t + t \bullet$$

and we write $M \xrightarrow{t} M'$ (also $M\,[t\rangle\,M'$)

Some remarks

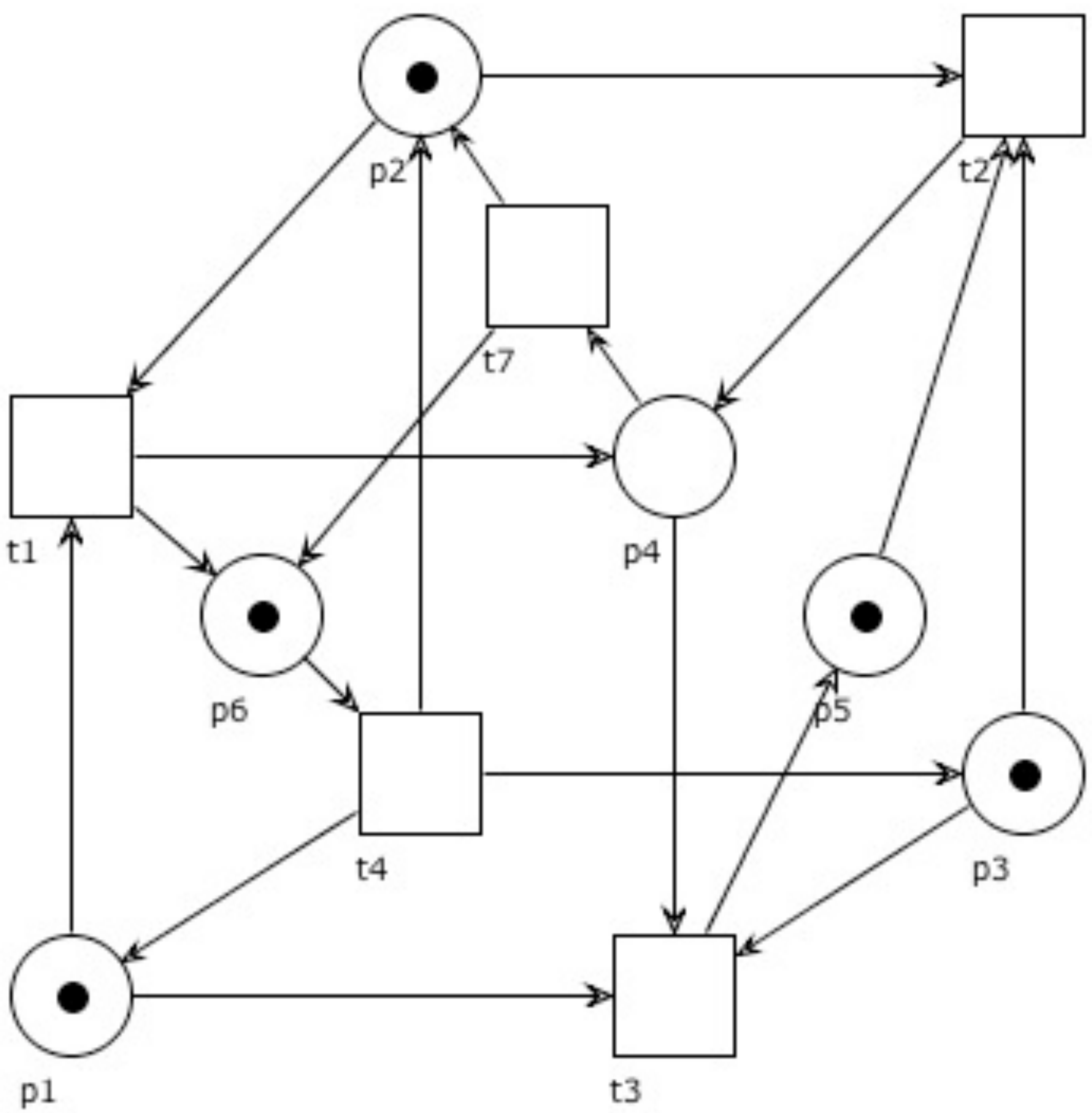
Firing is an atomic action

Our semantics is interleaving:
multiple transitions may be enabled,
but only one fires at a time

The network is static, but
the overal number of tokens may vary over time
(if transitions are fired for which the number of input places is not equal to the number of output places)

Example

$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$

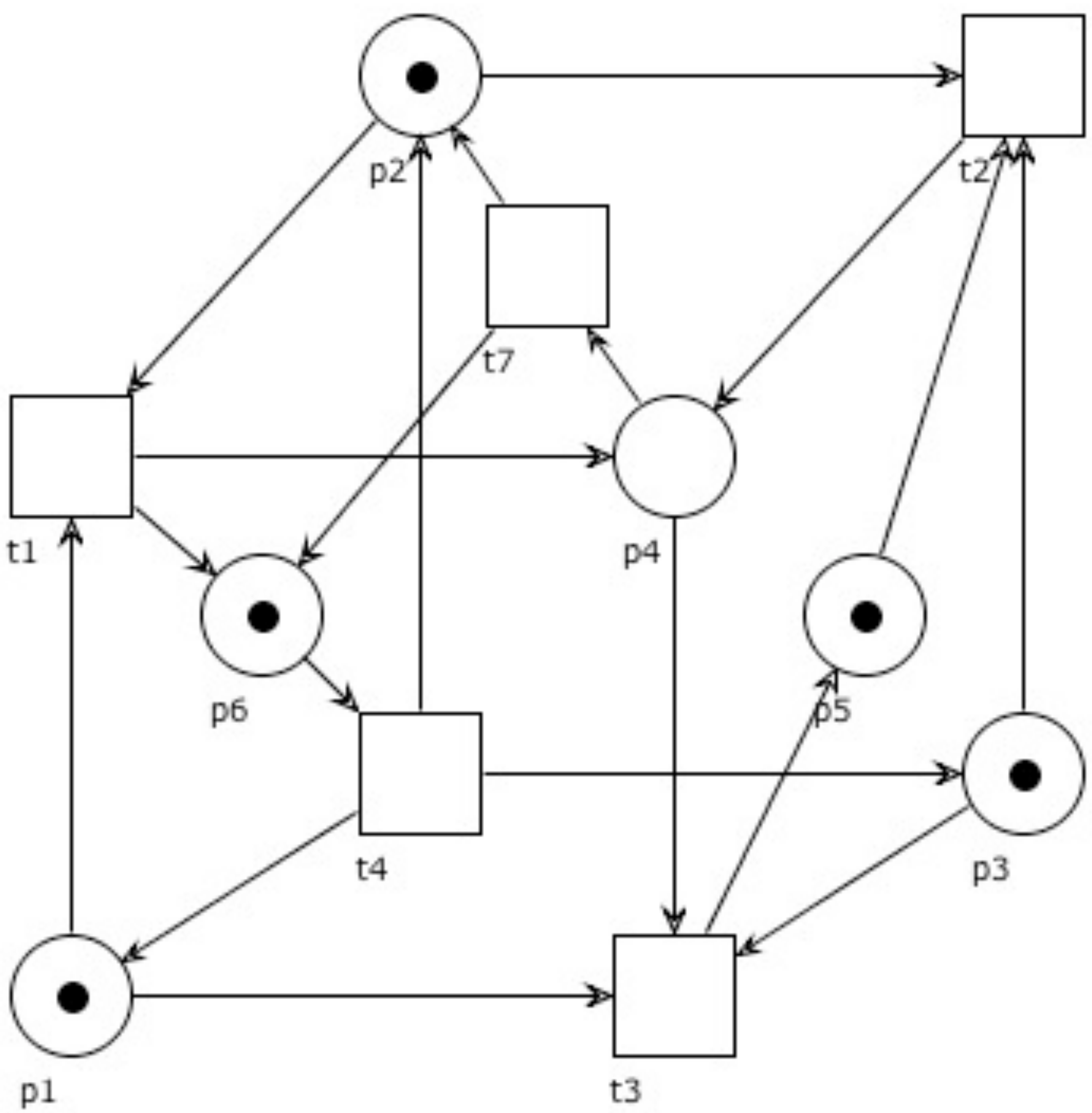


Which of the following holds true?

- $M_0 \xrightarrow{t_1}$
- $M_0 \xrightarrow{t_2}$
- $M_0 \xrightarrow{t_3}$
- $M_0 \xrightarrow{t_7}$

Example

$$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$$



Which of the following holds true?

- $M_0 \xrightarrow{t_1} p_3 + p_4 + p_5 + p_6$
- $M_0 \xrightarrow{t_2} p_1 + p_4 + p_6$
- $M_0 \xrightarrow{t_4} 2p_1 + 2p_2 + 2p_3 + p_5$

Notation

We write $M \to$ if $M \xrightarrow{t}$ for some transition t

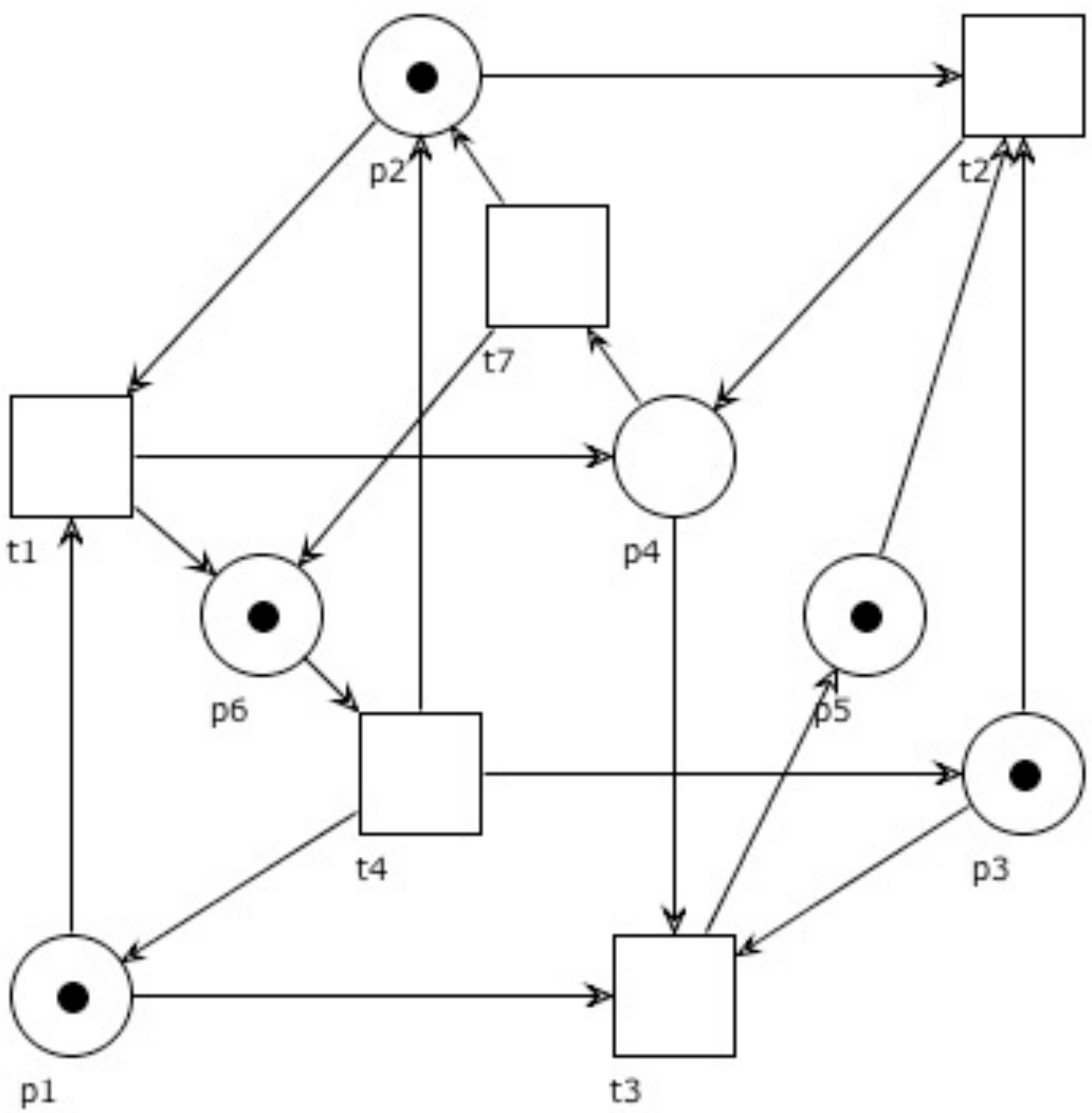
We write $M \to M'$ if $M \xrightarrow{t} M'$ for some transition t

We write $M \stackrel{t}{\not\to}$ if transition t is not enabled at M

We write $M \not\to$ if no transition is enabled at M

Example

$$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$$



We can write that

- $M_0 \longrightarrow$
- $M_0 \longrightarrow p_1 + p_4 + p_6$
- $M_0 \overset{t_7}{\not\longrightarrow}$
- $p_1 + p_5 \not\longrightarrow$

Firing sequence

Let $\sigma = t_1 t_2 ... t_{n-1} \in T^*$ be a sequence of transitions.

We write $M \stackrel{\sigma}{\to} M'$ (and $M \stackrel{\sigma}{\to}$) if:

there is a sequence of markings $M_1, ..., M_n$

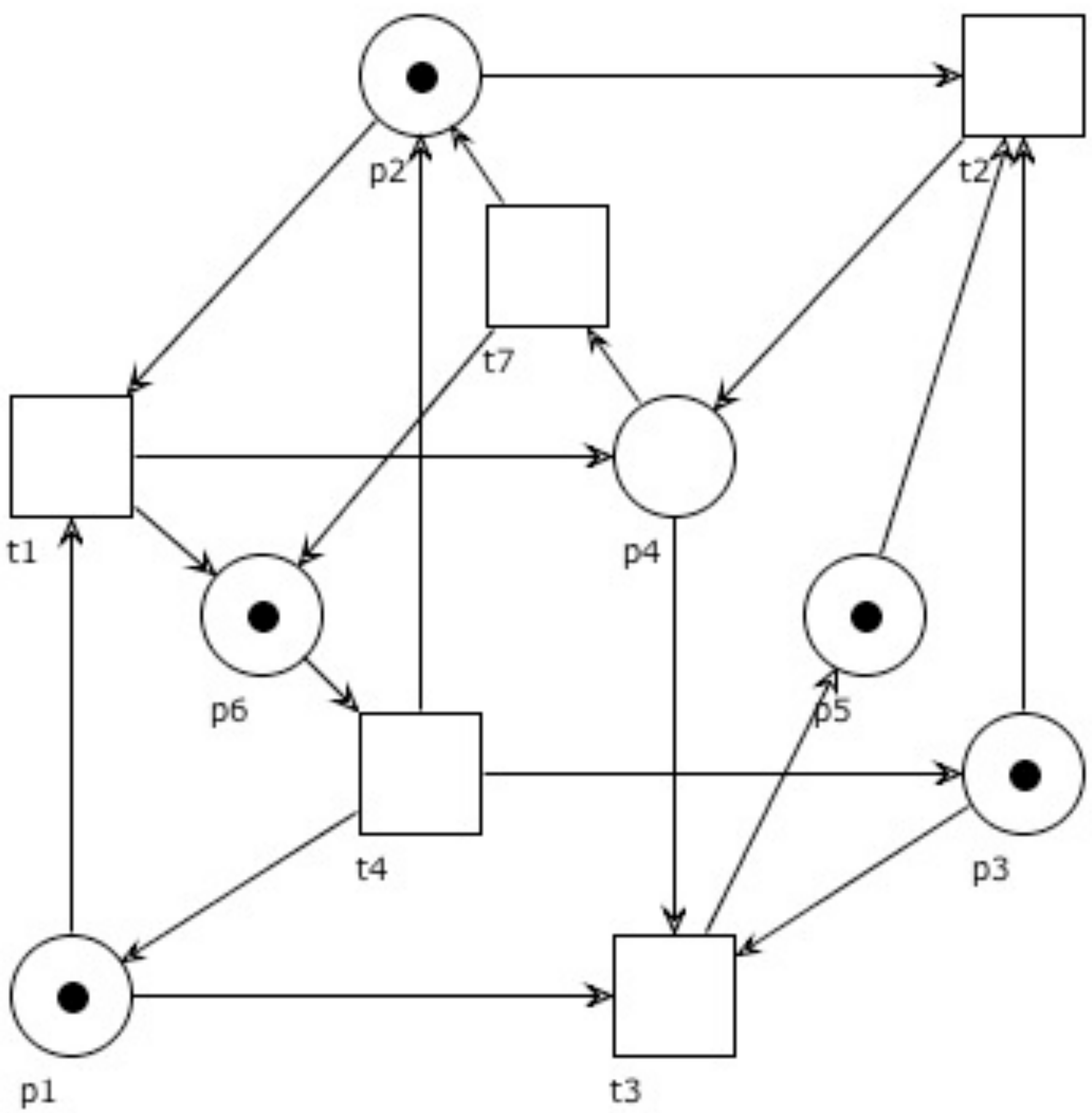
with $M = M_1$ and $M' = M_n$

and $M_i \xrightarrow{t_i} M_{i+1}$ for $1 \leq i < n$

(i.e. $M = M_1 \xrightarrow{t_1} M_2 \xrightarrow{t_2} ... \xrightarrow{t_{n-1}} M_n = M'$)

Exercise

$$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$$

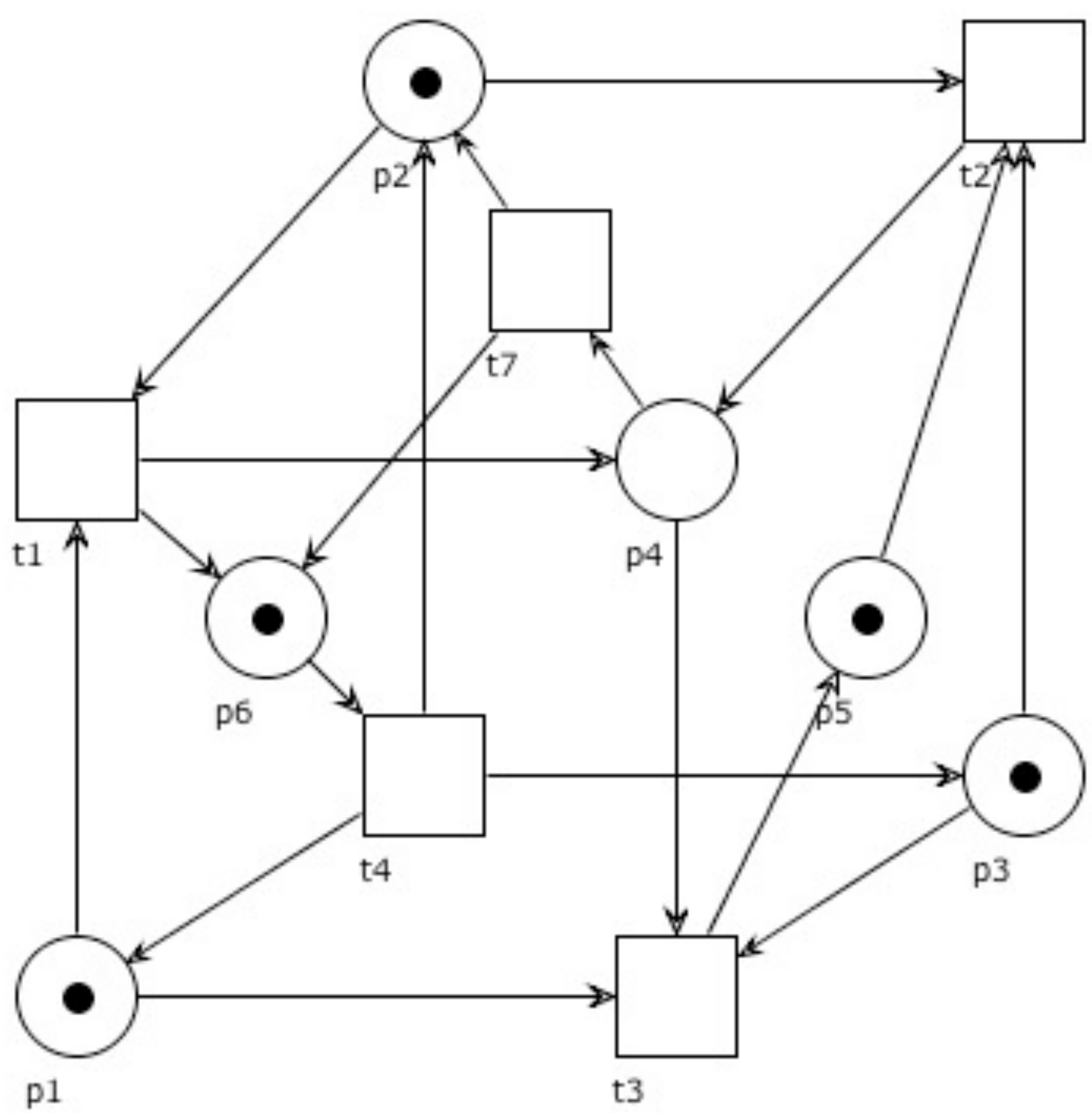


Which of the following holds true?

- $M_0 \xrightarrow{t_1 t_4 t_2 t_3}$
- $M_0 \xrightarrow{t_2 t_7 t_4}$
- $M_0 \xrightarrow{t_1 t_2 t_7}$
- $M_0 \xrightarrow{t_1 t_4 t_2 t_1}$

Example

$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$



We have that

- $M_0 \xrightarrow{t_1 t_4 t_2 t_3} p_4 + p_5 + p_6$
- $M_0 \xrightarrow{t_2 t_7 t_4} 2p_1 + 2p_2 + p_3 + p_6$
- $M_0 \xrightarrow{t_1 t_4 t_3 t_2 t_7} p_2 + p_5 + 2p_6$

Infinite sequence

Let $\sigma = t_1 t_2 ... \in T^\omega$ be an infinite sequence of transitions.

We write $M \xrightarrow{\sigma}$ if:

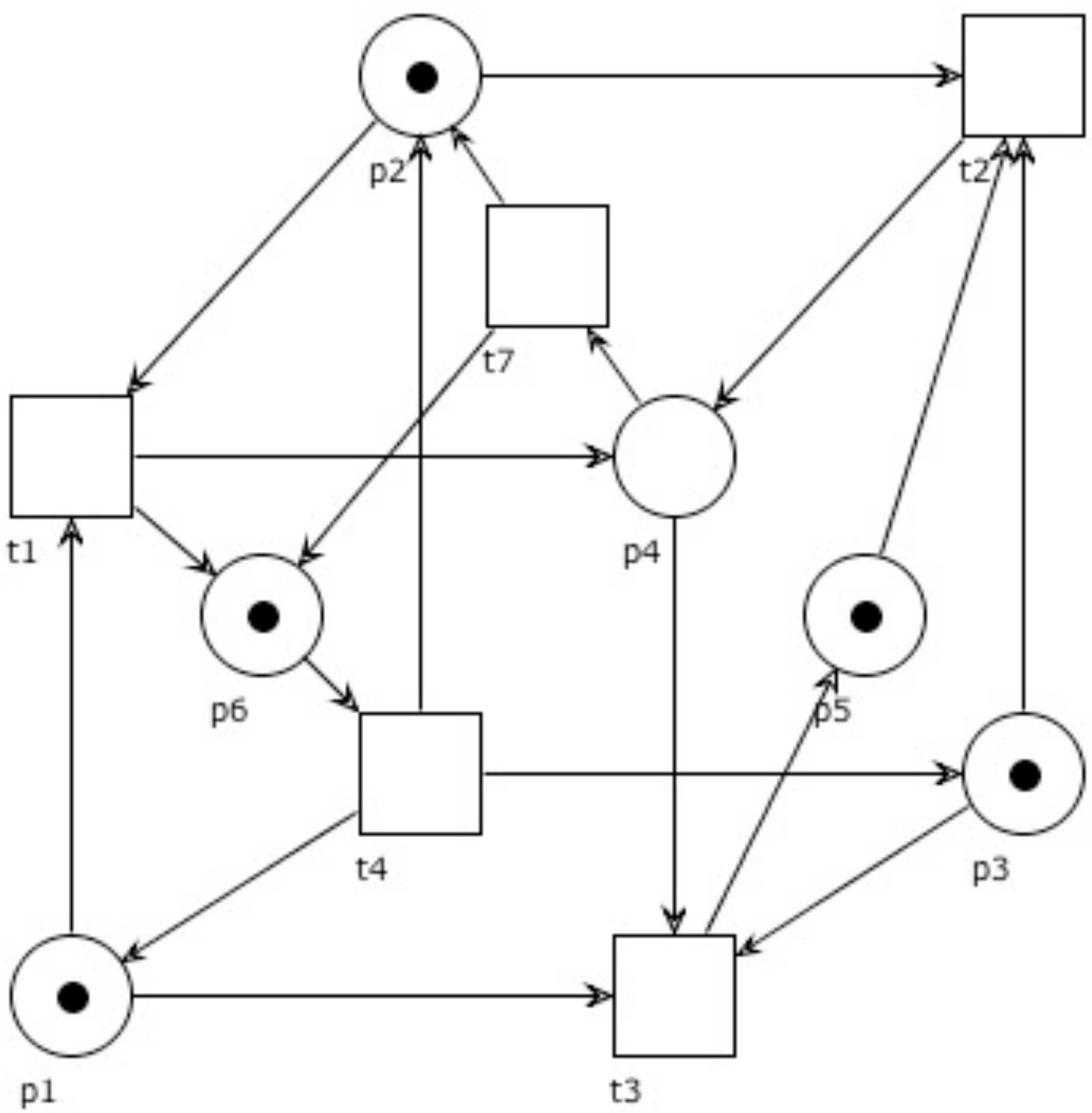
there is an infinite sequence of markings $M_1, M_2, ...$

with $M = M_1$ and $M_i \xrightarrow{t_i} M_{i+1}$ for $1 \leq i$

(i.e. $M = M_1 \xrightarrow{t_1} M_2 \xrightarrow{t_2} ...$)

Example

$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$



We have that

- $M_0 \xrightarrow{t_1 t_4 t_1 t_4 t_1 t_4 \cdots}$
- $M_0 \xrightarrow{t_1 t_4 t_7 t_1 t_4 t_7 t_1 t_4 t_7 \cdots}$

Enabled sequence

We say that an occurrence sequence σ is **enabled** if $M \xrightarrow{\sigma}$

(σ can be finite or infinite)

Note that an infinite sequence can be represented as a map $\sigma : \mathbb{N} \to T$, where $\sigma(i) = t_i$

More on sequences

Concatenation:
for $\sigma_1 = a_1...a_n$ and $\sigma_2 = b_1...b_m$, we let $\sigma_1\sigma_2 = a_1...a_n b_1...b_m$
for $\sigma_1 = a_1...a_n$ and $\sigma_2 = b_1 b_2...$, we let $\sigma_1\sigma_2 = a_1...a_n b_1 b_2...$

σ is a **prefix** of σ' if $\sigma = \sigma'$ or $\sigma\sigma'' = \sigma'$ for some σ''
σ is a **proper prefix** of σ' if $\sigma\sigma'' = \sigma'$ for some σ''

More on sequences

Restriction: (also extraction / projection)
given $T' \subseteq T$ we inductively define $\sigma_{|T'}$ as:

$$\epsilon_{|T'} = \epsilon \qquad (t\sigma)_{|T'} = \begin{cases} t(\sigma_{|T'}) & \text{if } t \in T' \\ \sigma_{|T'} & \text{if } t \notin T' \end{cases}$$

Enabledness

Proposition: $M \xrightarrow{\sigma}$ iff $M \xrightarrow{\sigma'}$ for every prefix σ' of σ

$(\Rightarrow)$ immediate from definition

$(\Leftarrow)$ trivial if σ is finite (σ itself is a prefix of σ)

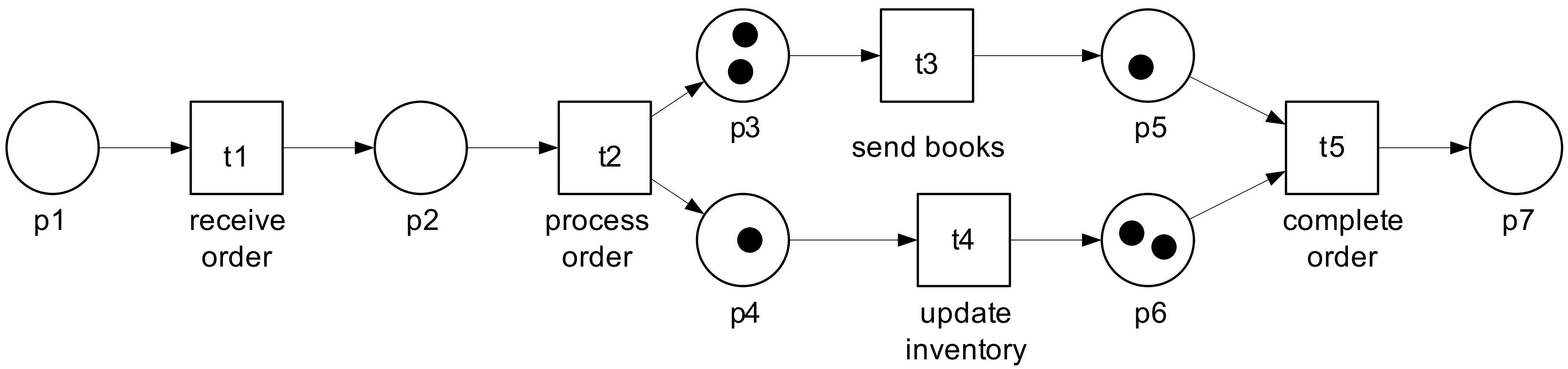
When σ is infinite: taken any $i \in \mathbb{N}$ we need to prove that $t_i = \sigma(i)$ is enabled after the firing of the prefix $\sigma' = t_1 t_2 ... t_{i-1}$ of σ.

But this is obvious, because

$$M \xrightarrow{t_1} M_1 \xrightarrow{t_2} ... \xrightarrow{t_{i-1}} M_{i-1} \xrightarrow{t_i} M_i$$

is also a finite prefix of σ and therefore $M_{i-1} \xrightarrow{t_i}$

Exercises



M. Weske: Business Process Management,
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Which are the currently enabled transitions?

For each of them, which state would their firing lead to?

What are the reachable states?

Occurrence graph (aka Reachability graph)

The reachability graph is a graph that represents all possible occurrence sequences of a net

Nodes of the graphs = reachable markings
Arcs of the graphs = firings

Formally, $OG(N) = ([M_0\rangle, A)$ where $A \subseteq [M_0\rangle \times T \times [M_0\rangle$ s.t.

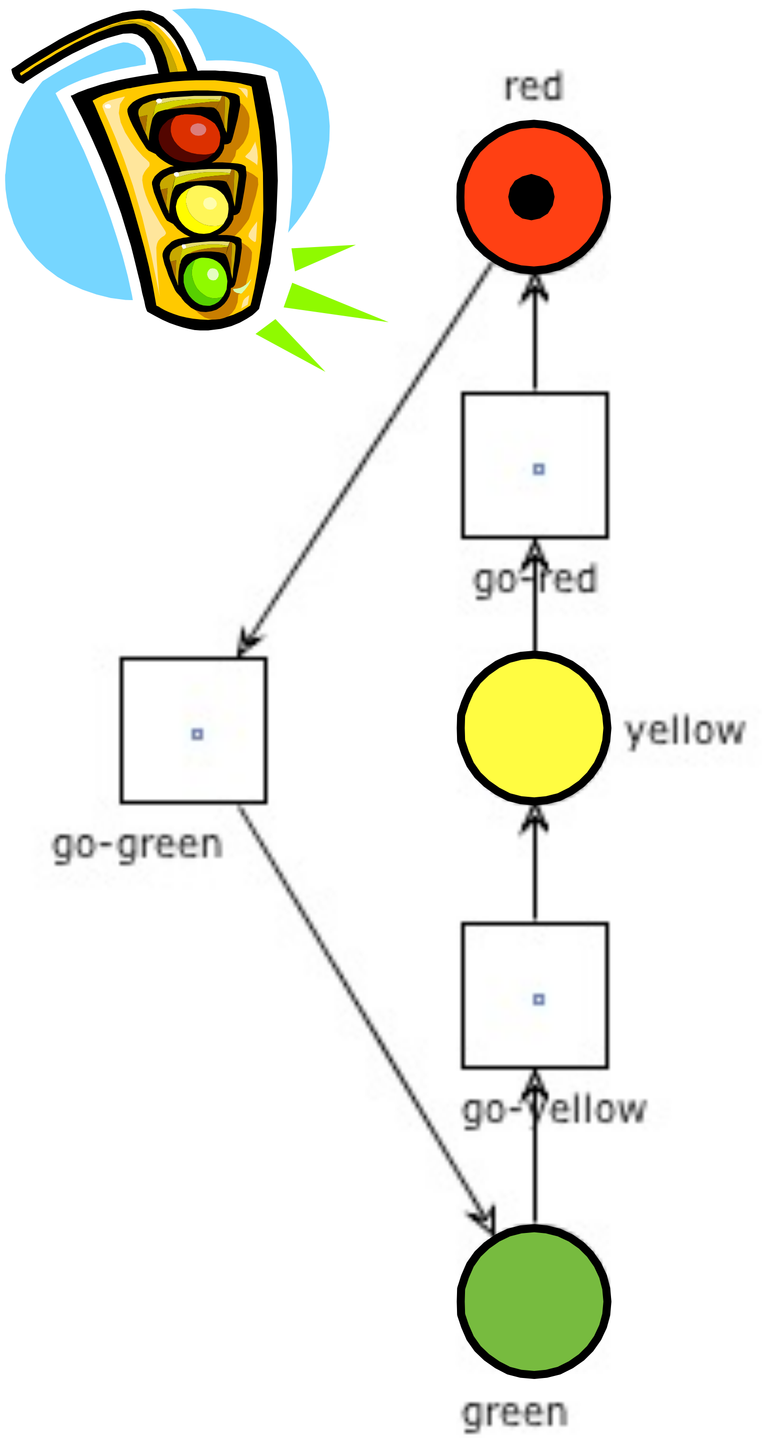
$$(M, t, M') \in A \quad \text{iff} \quad M \xrightarrow{t} M'$$

How to compute OG(N)

The occurrence graph can be constructed as follows:

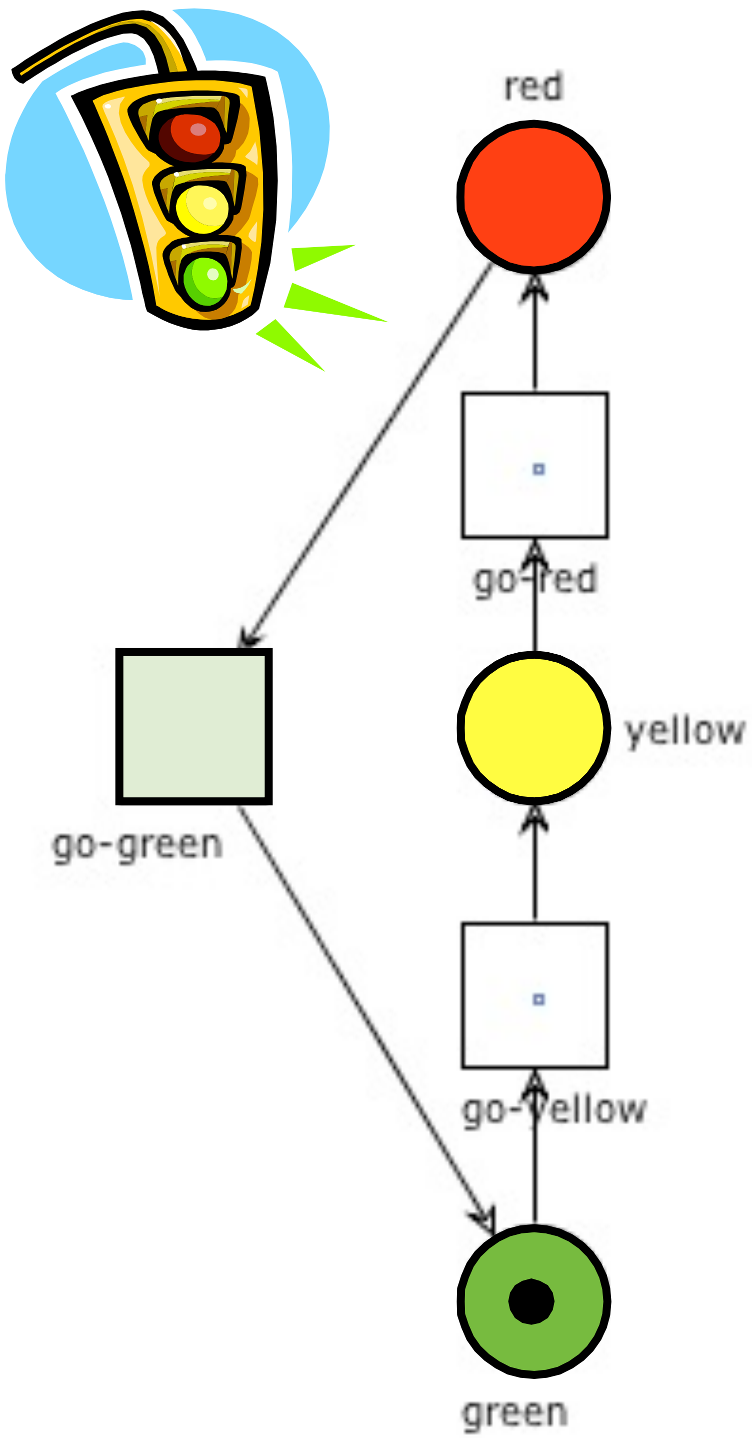
1. $Nodes = \{\},\ Arcs = \{\},\ Todo = \{M_0\}$
2. $M = next(Todo)$
3. $Nodes = Nodes \cup \{M\},\ Todo = Todo \setminus \{M\}$
4. $Firings = \{(M, t, M') \mid \exists t \in T, \exists M' \in \mu(P),\ M \xrightarrow{t} M'\}$
5. $New = \{M' \mid (M, t, M') \in Firings\} \setminus (Nodes \cup Todo)$
6. $Todo = Todo \cup New,\ Arcs = Arcs \cup Firings$
7. $isEmpty(Todo)\ ?\ \mathbf{stop}\ :\ \mathbf{goto}\ 2$

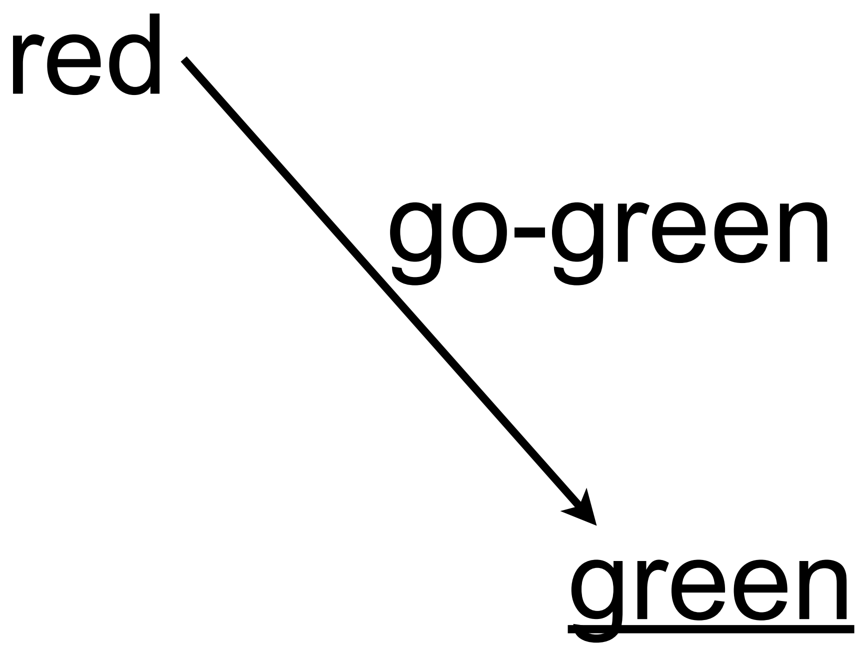
Example: traffic light



red

Example: traffic light





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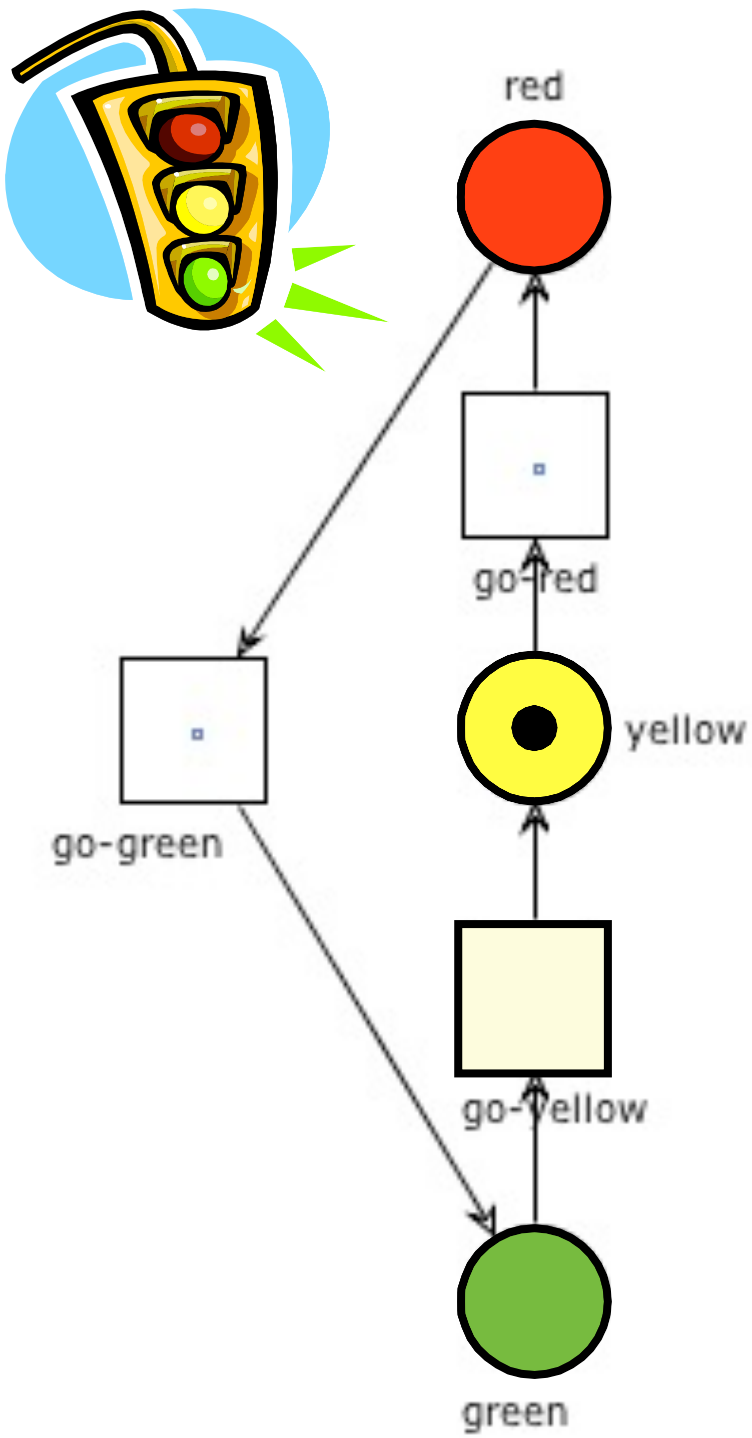

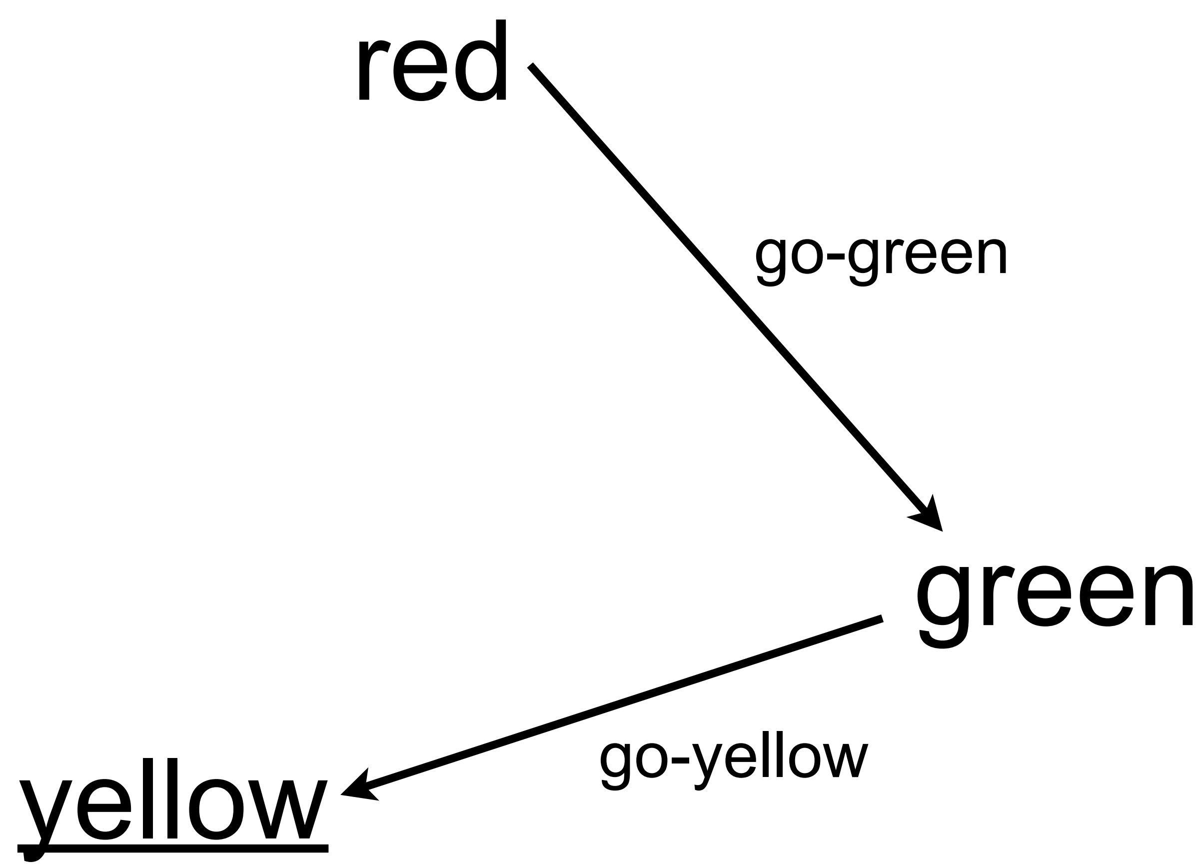

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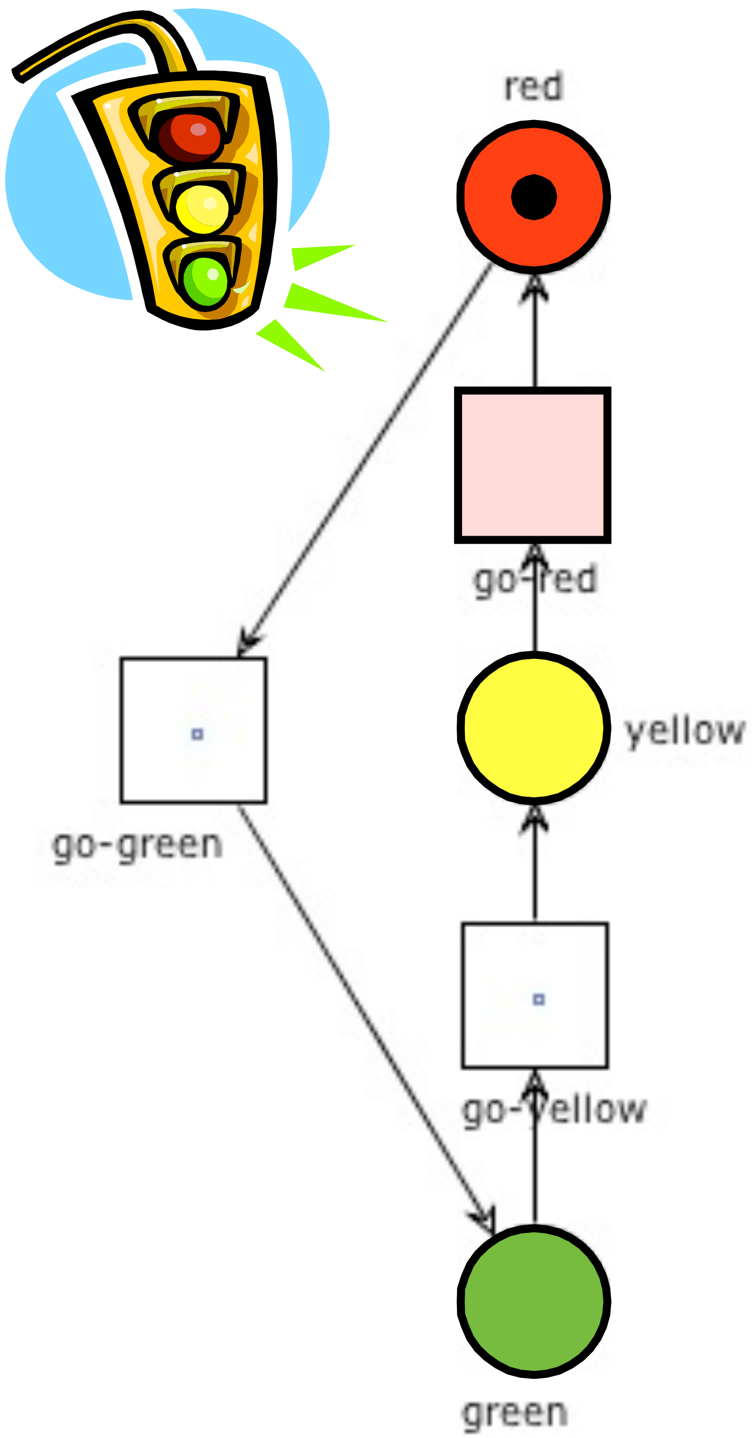

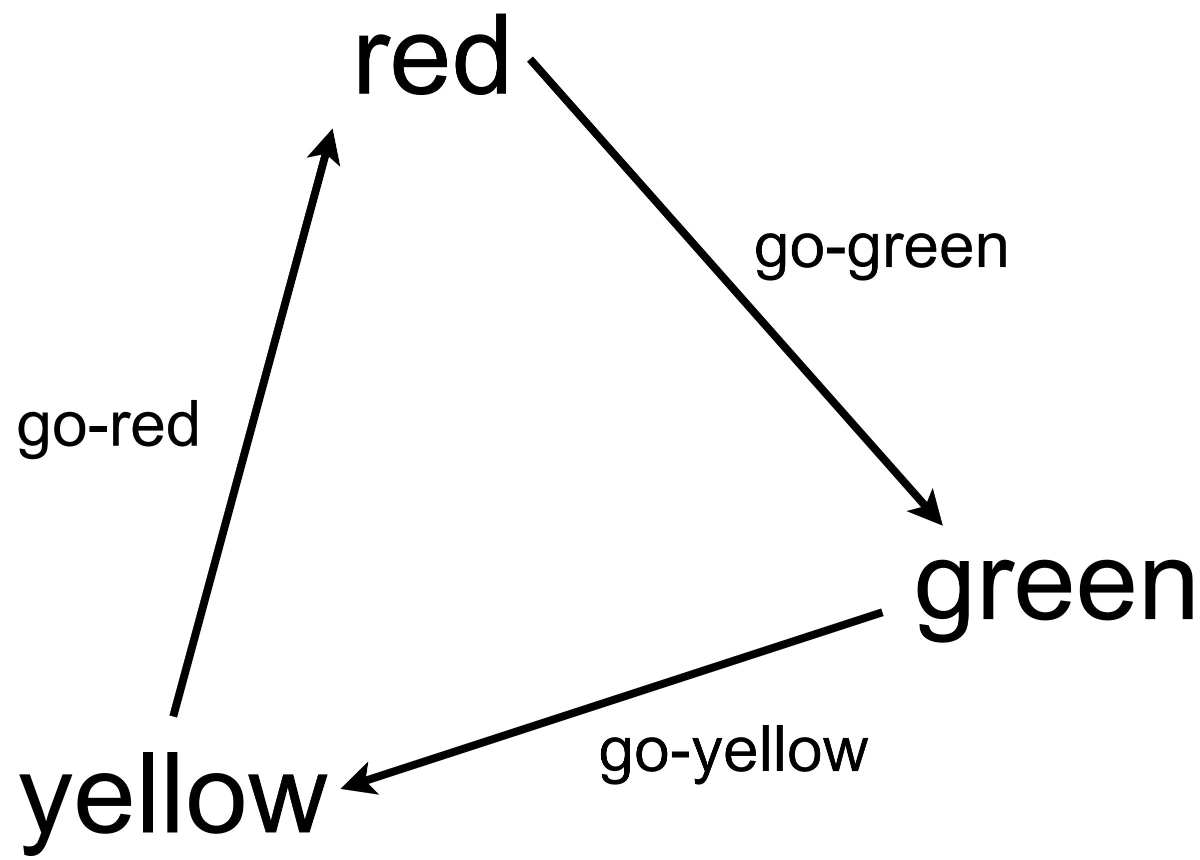

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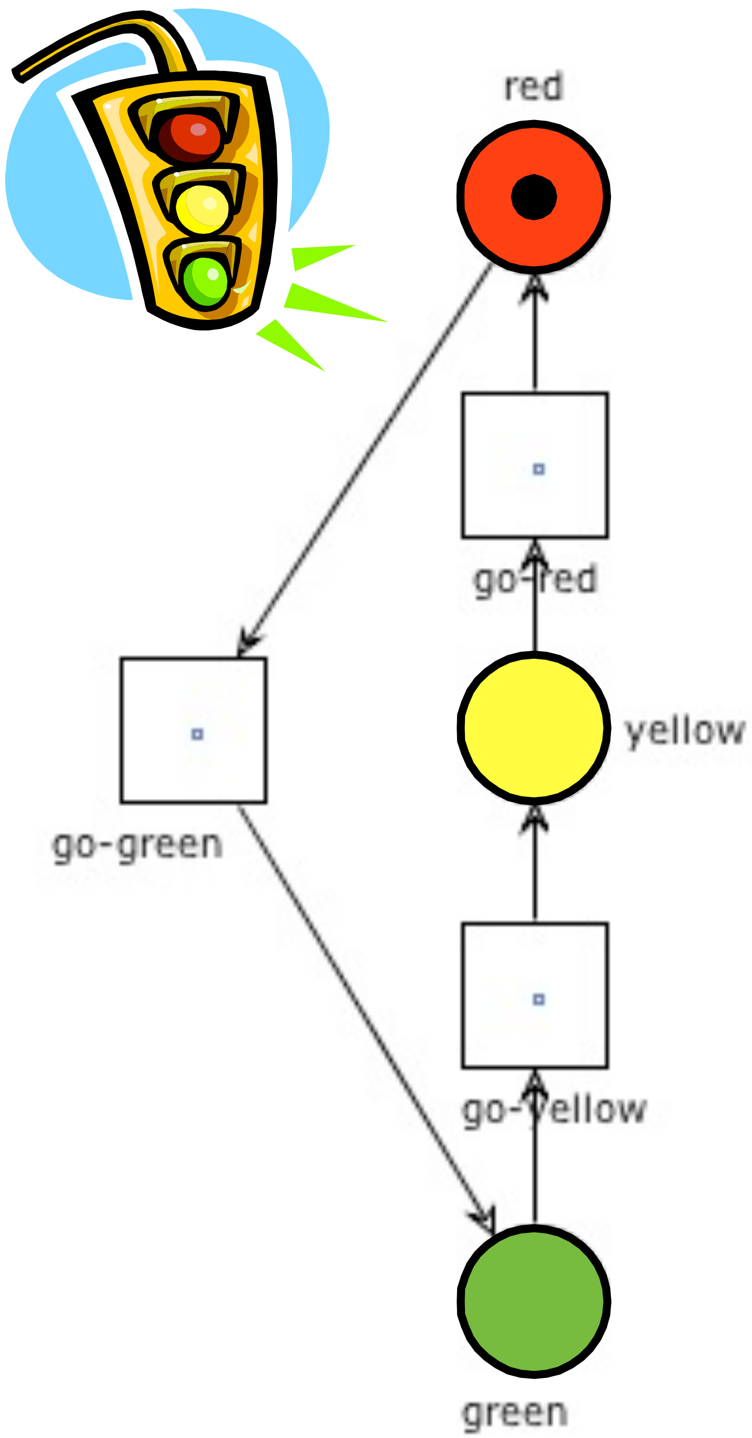
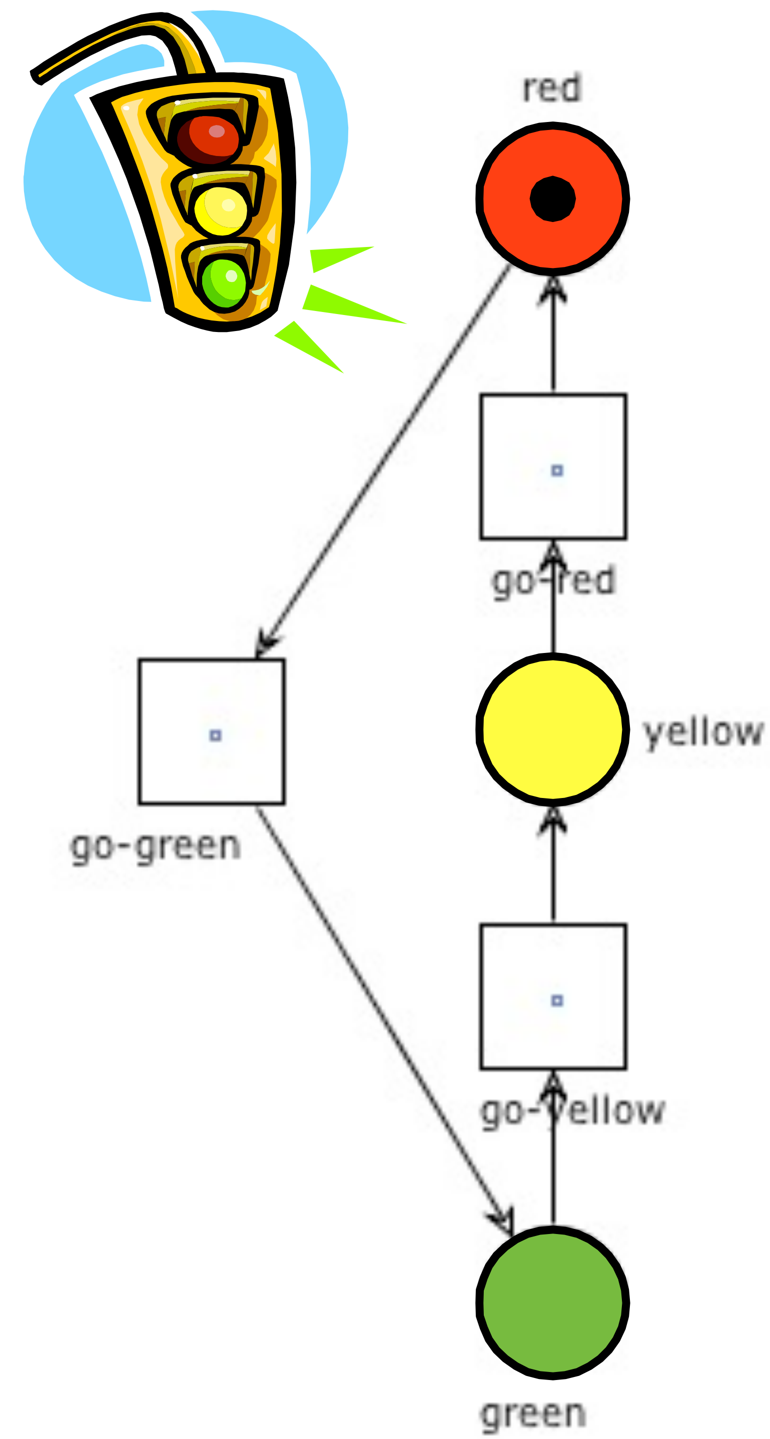


red + red’



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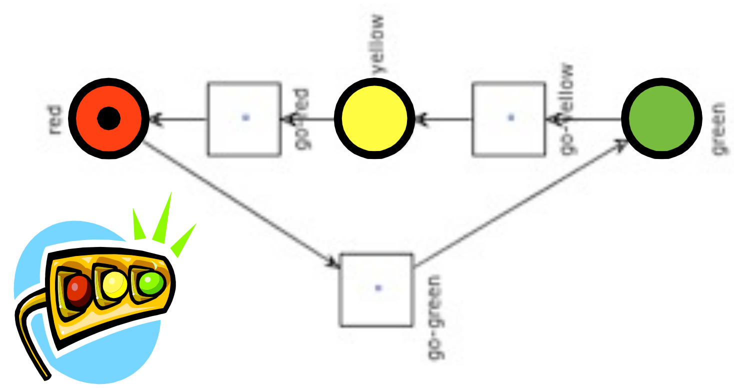

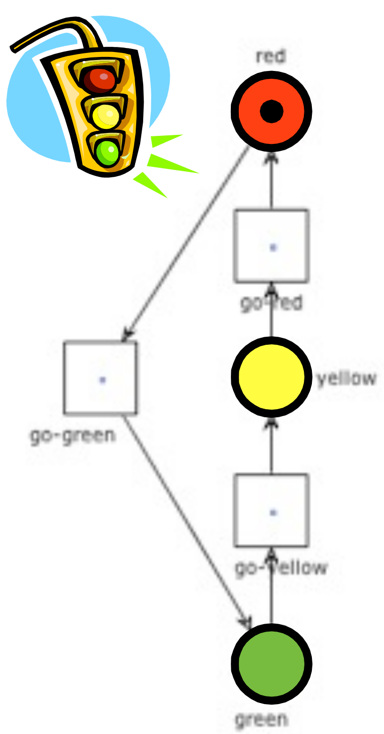

red + red’

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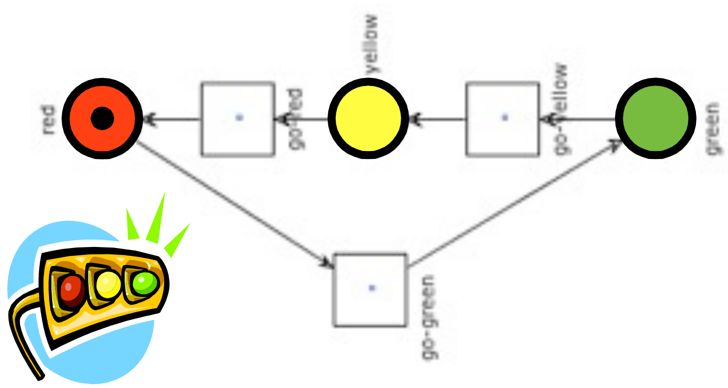

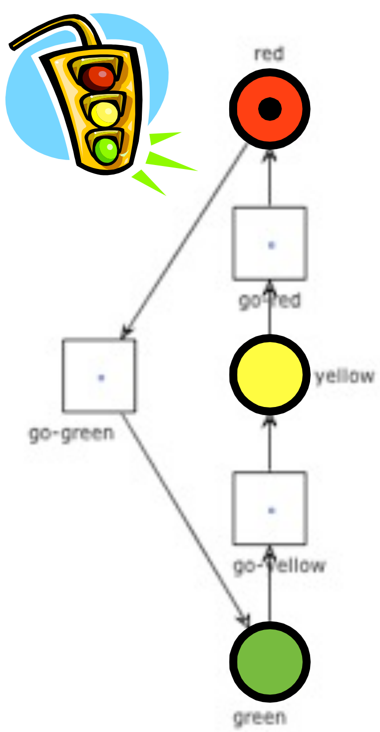

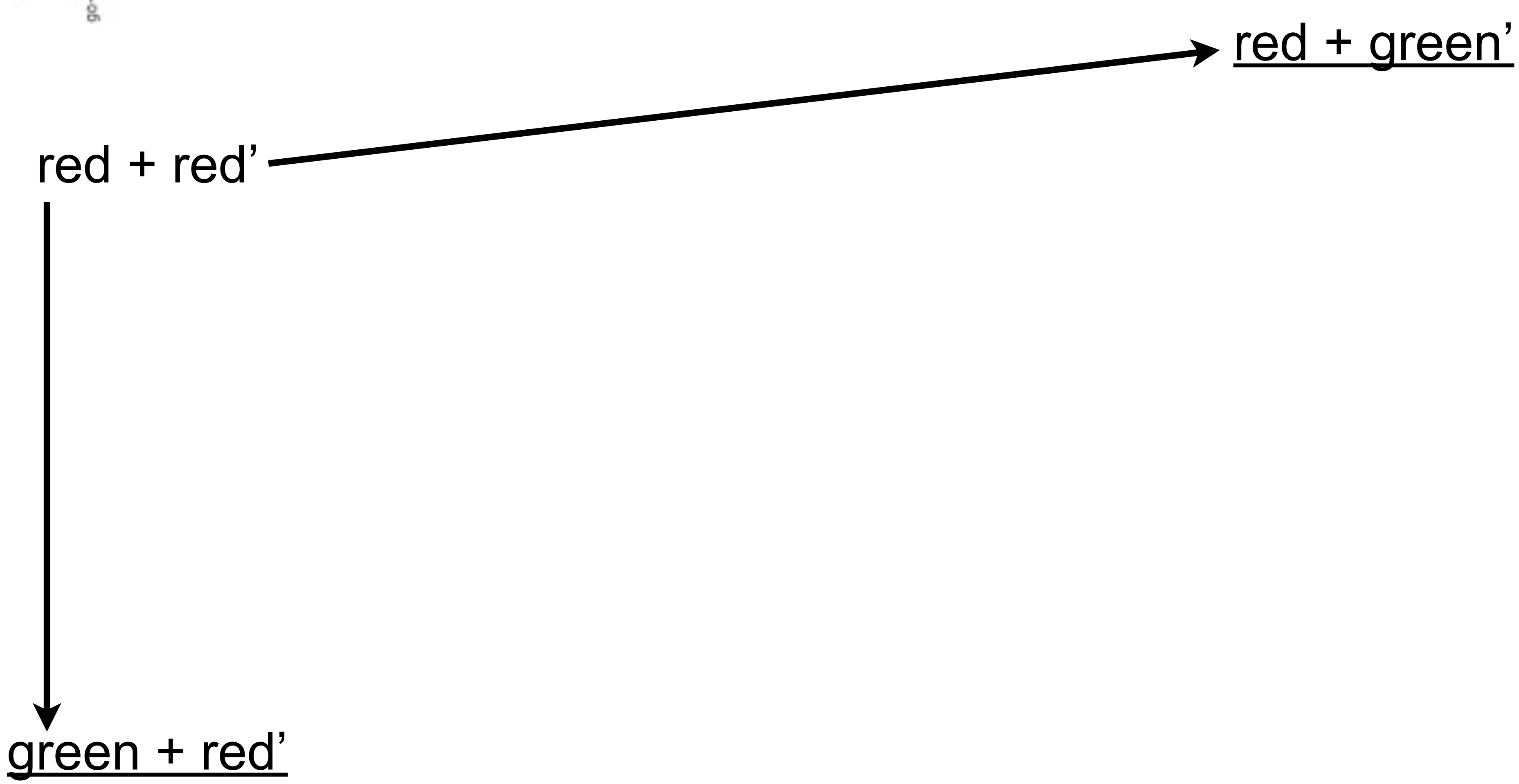

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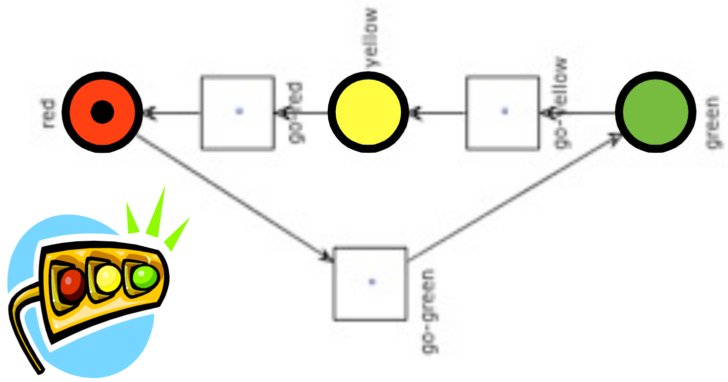

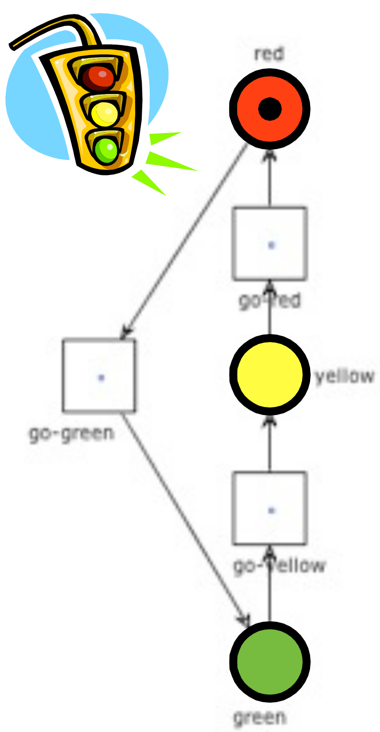

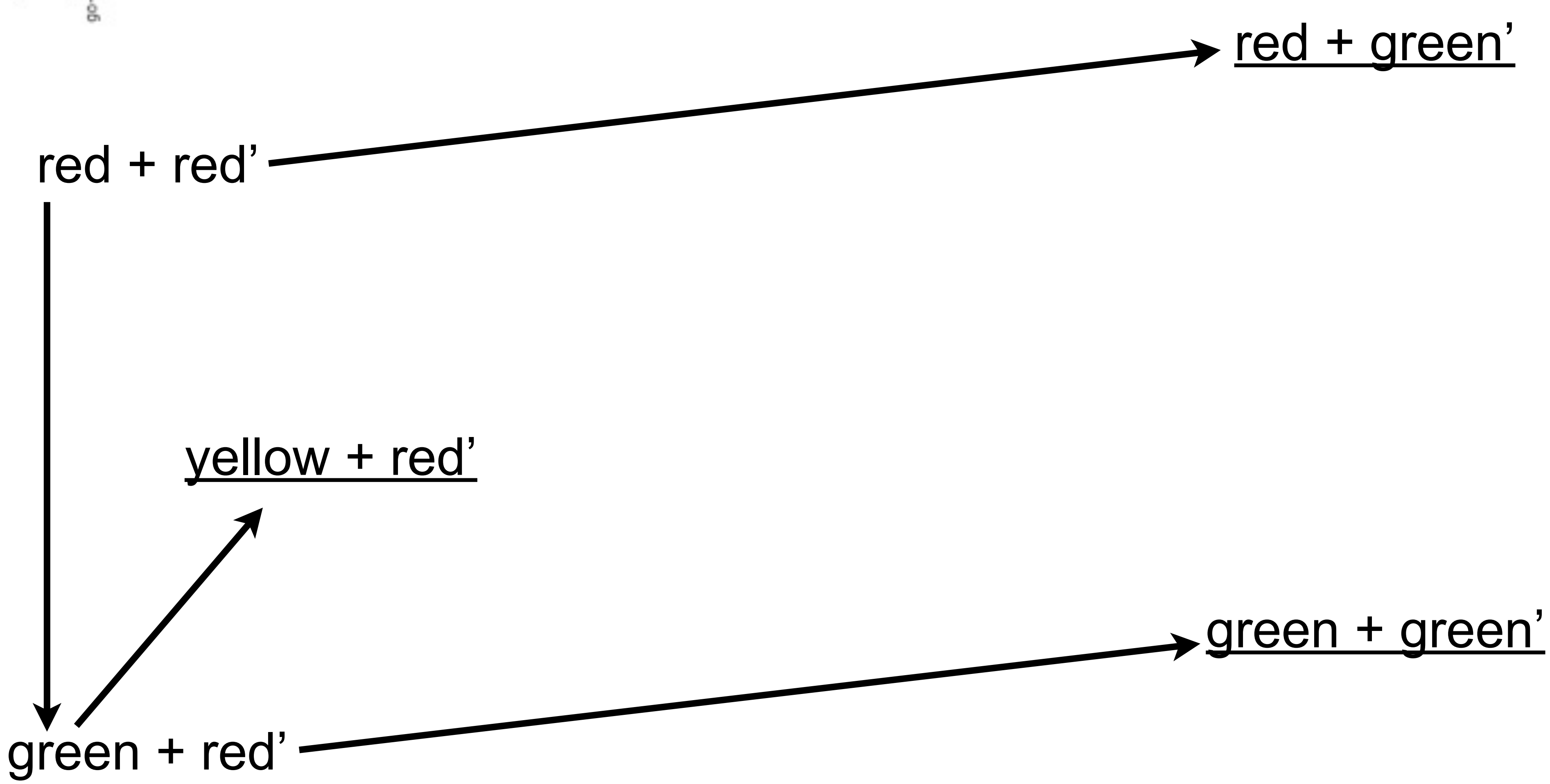

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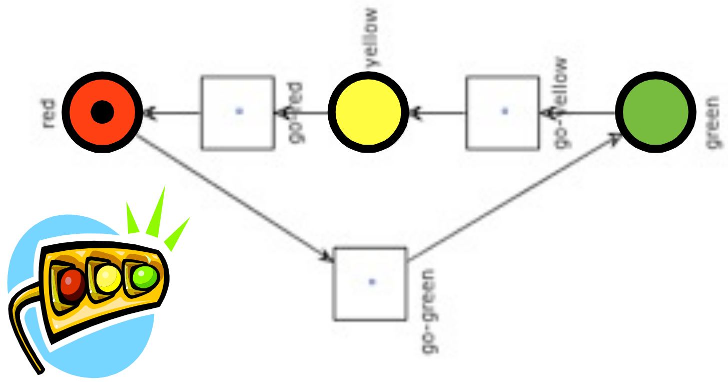

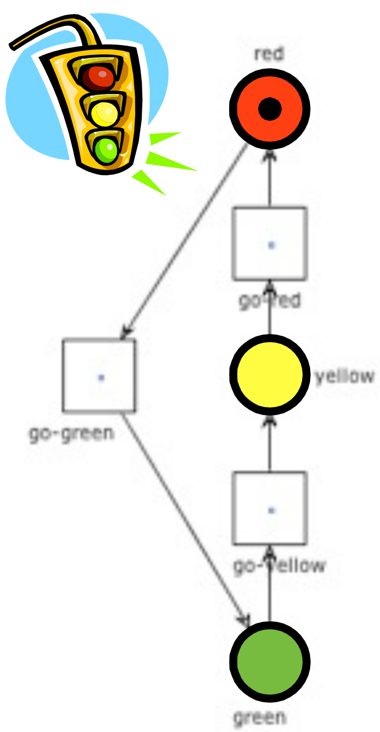

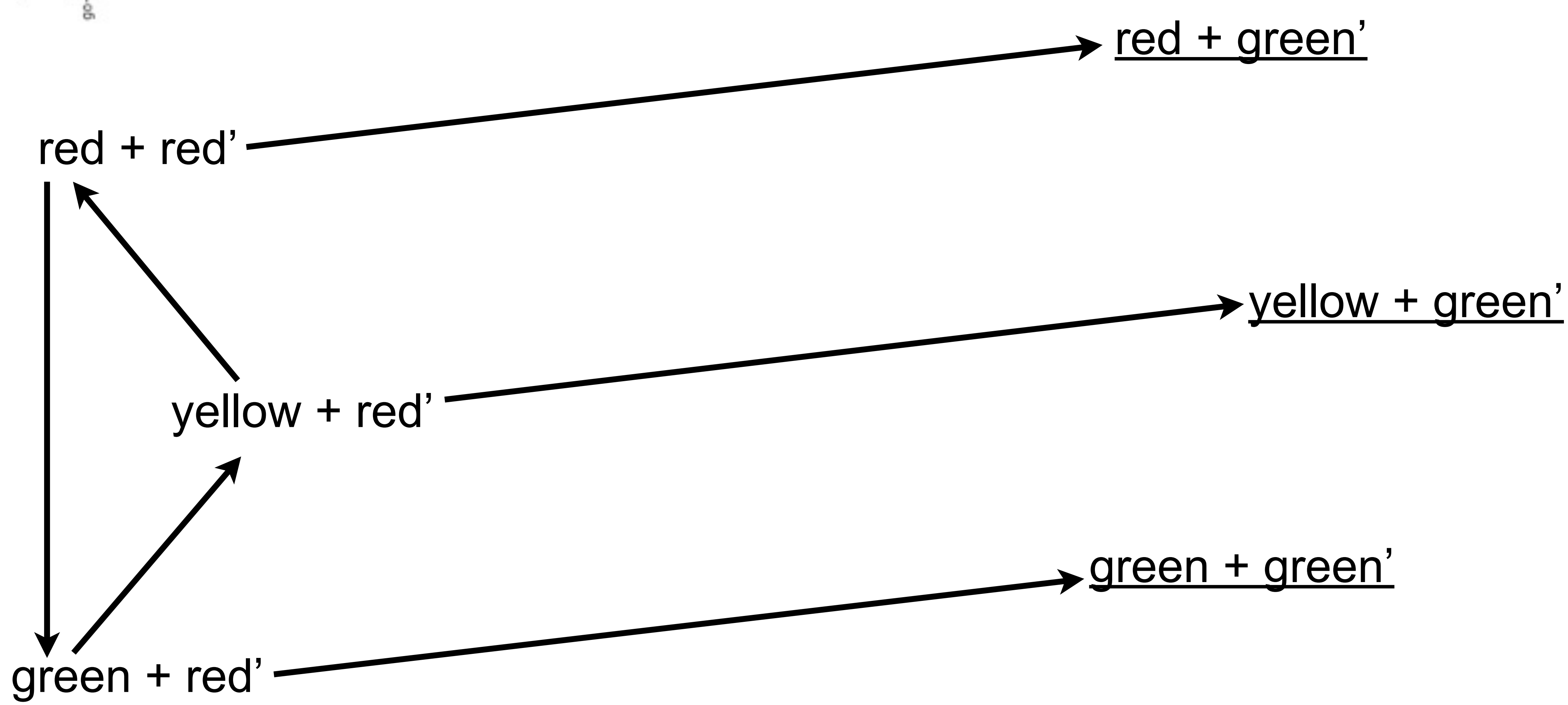

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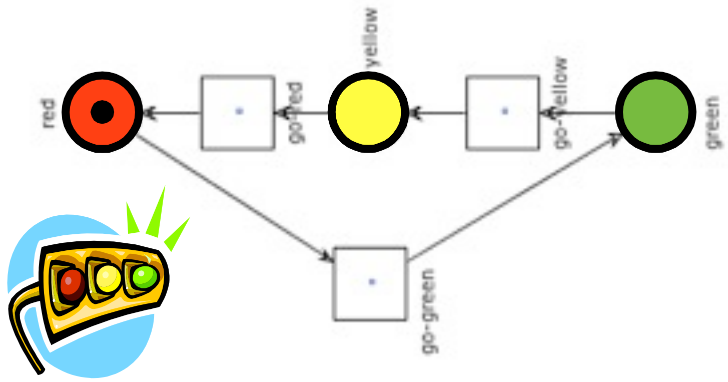

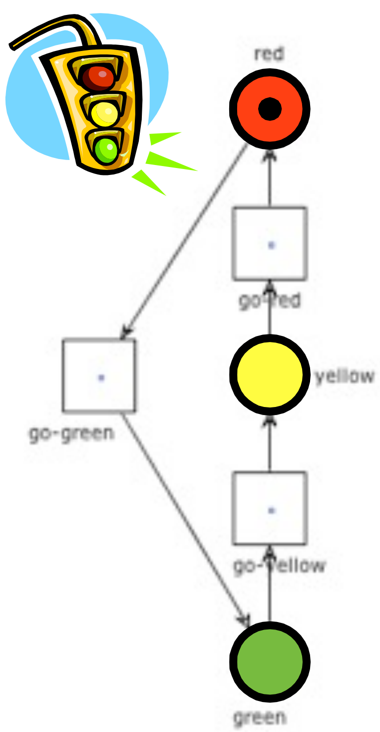

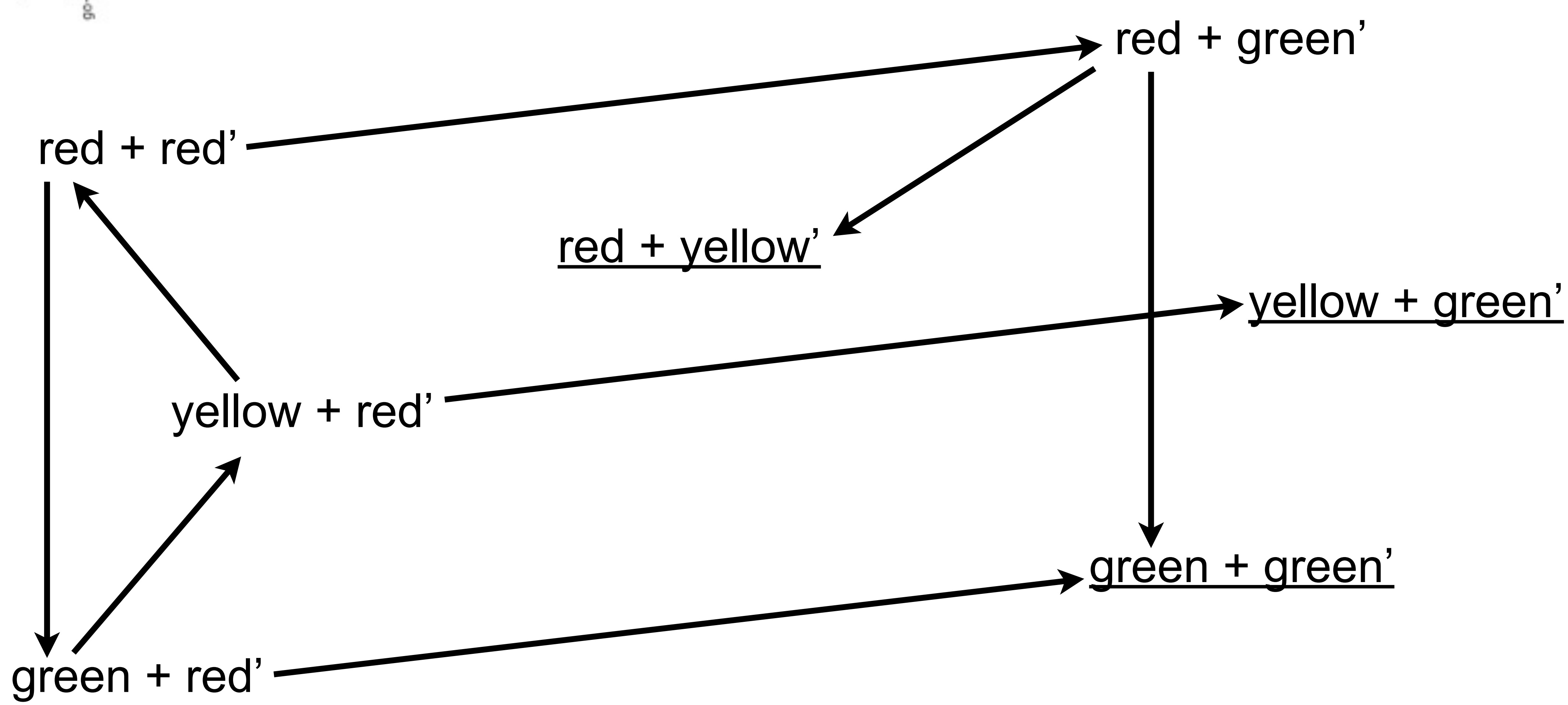

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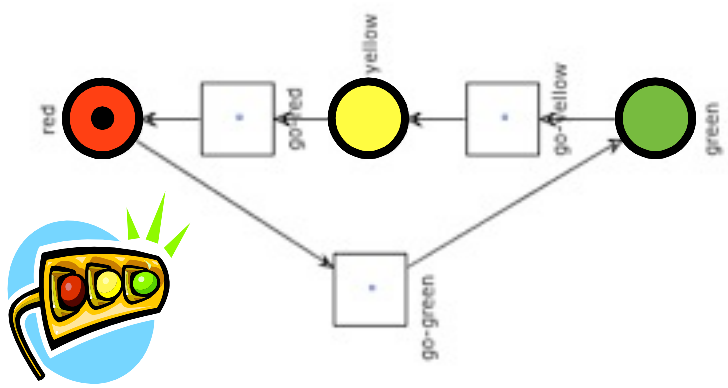

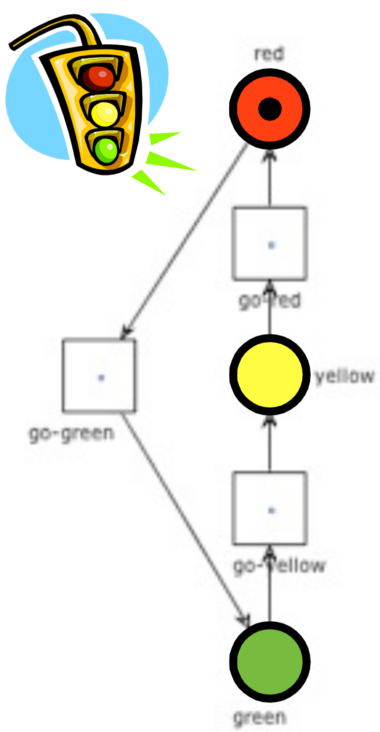

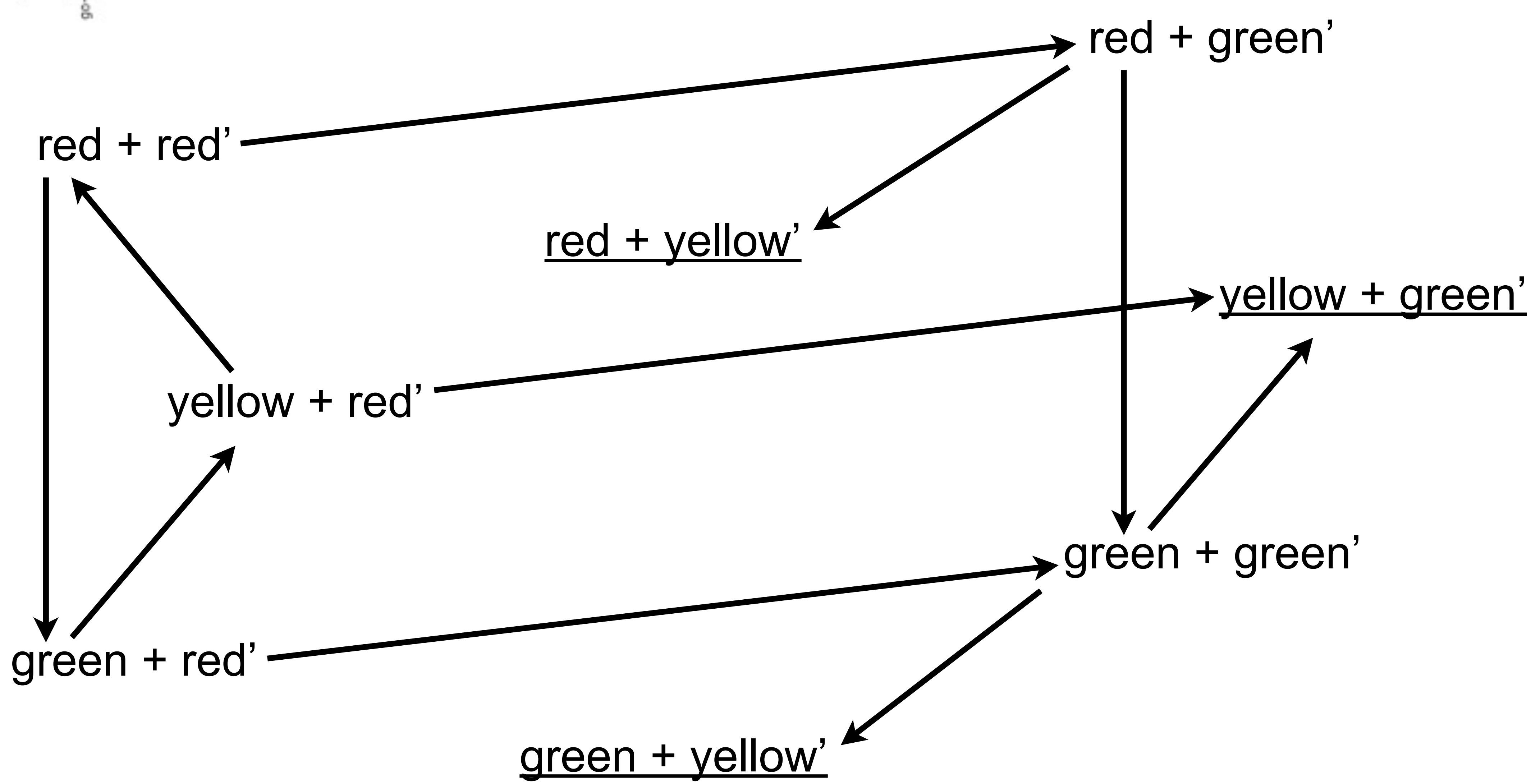

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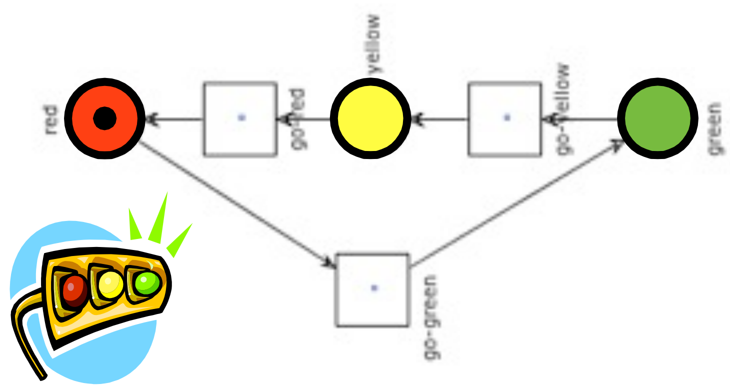

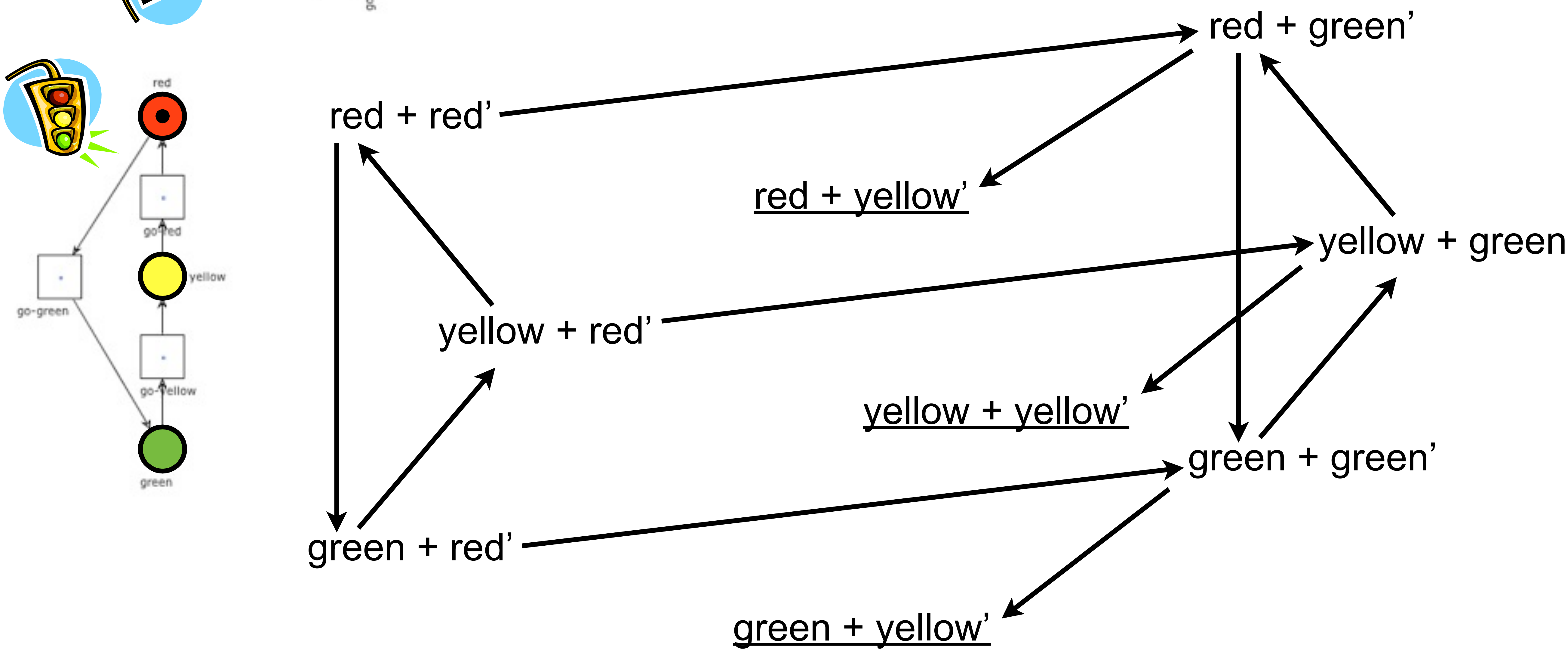

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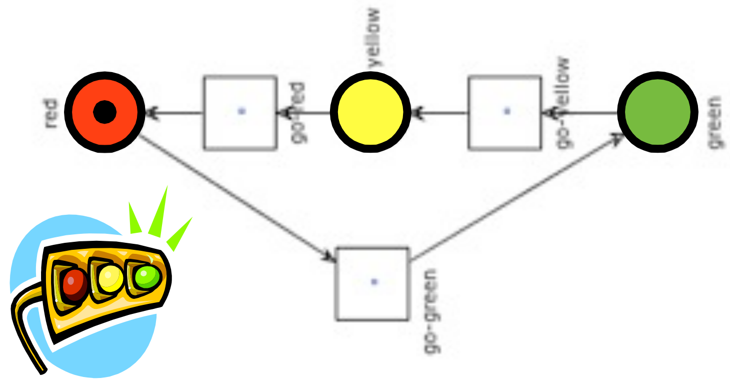

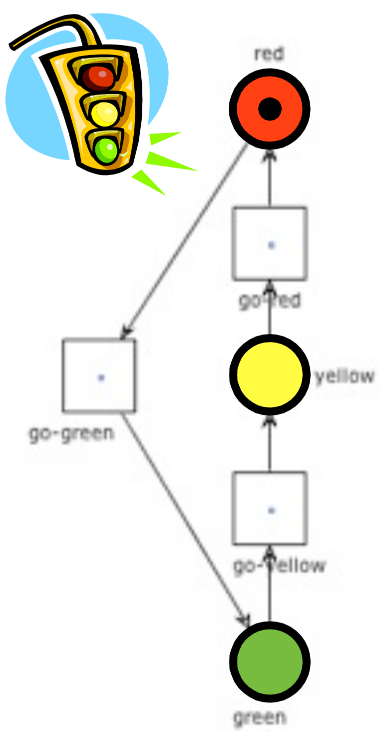

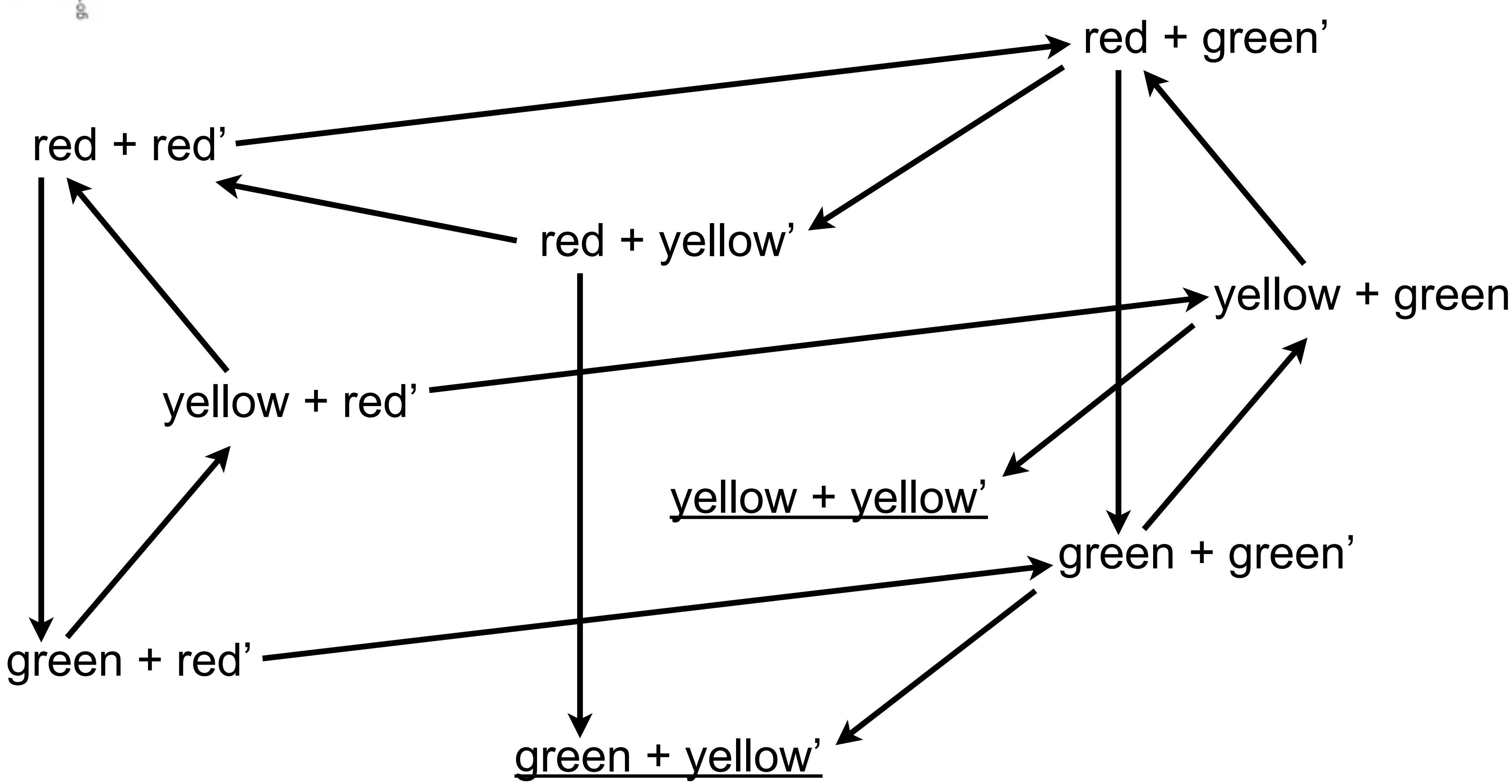

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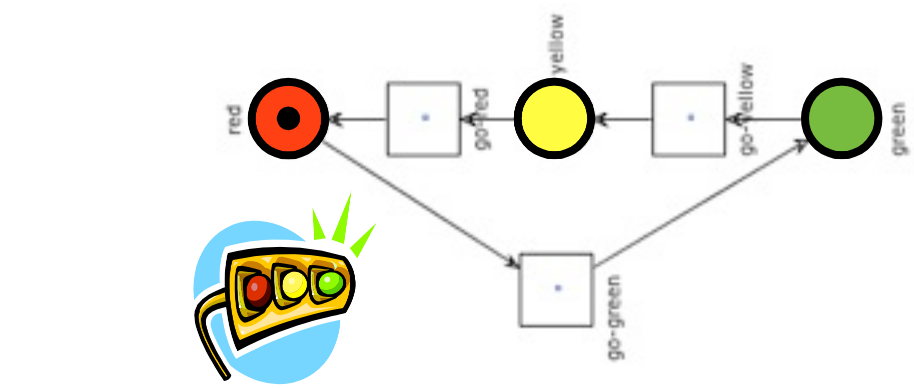

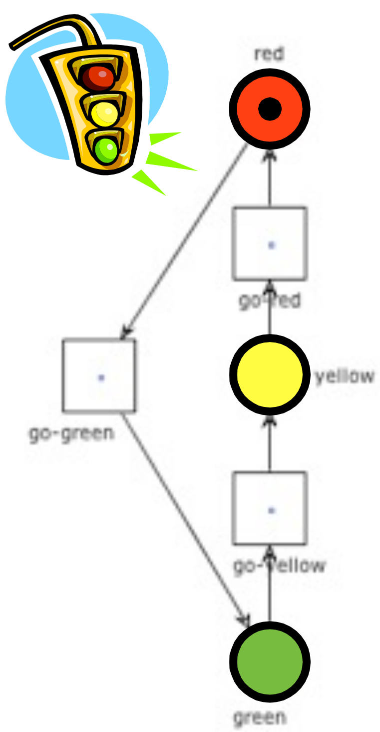

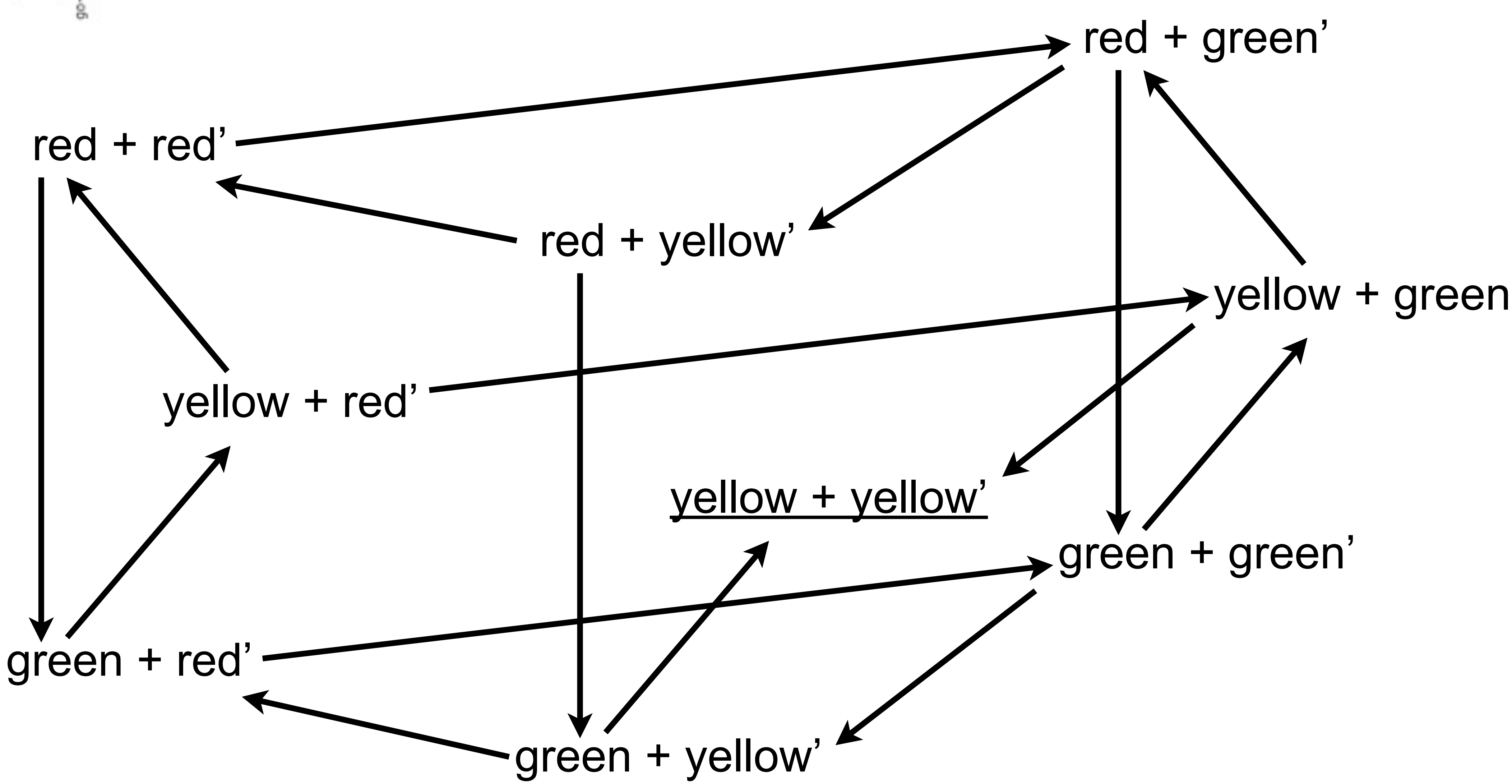

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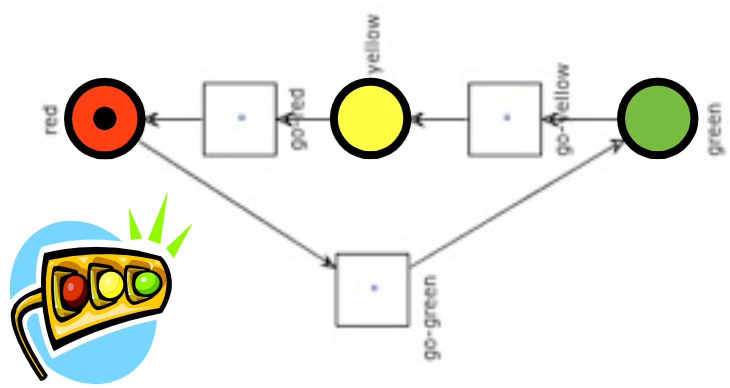

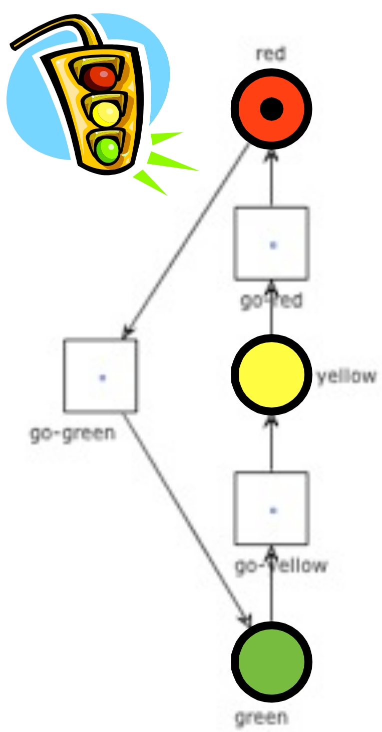

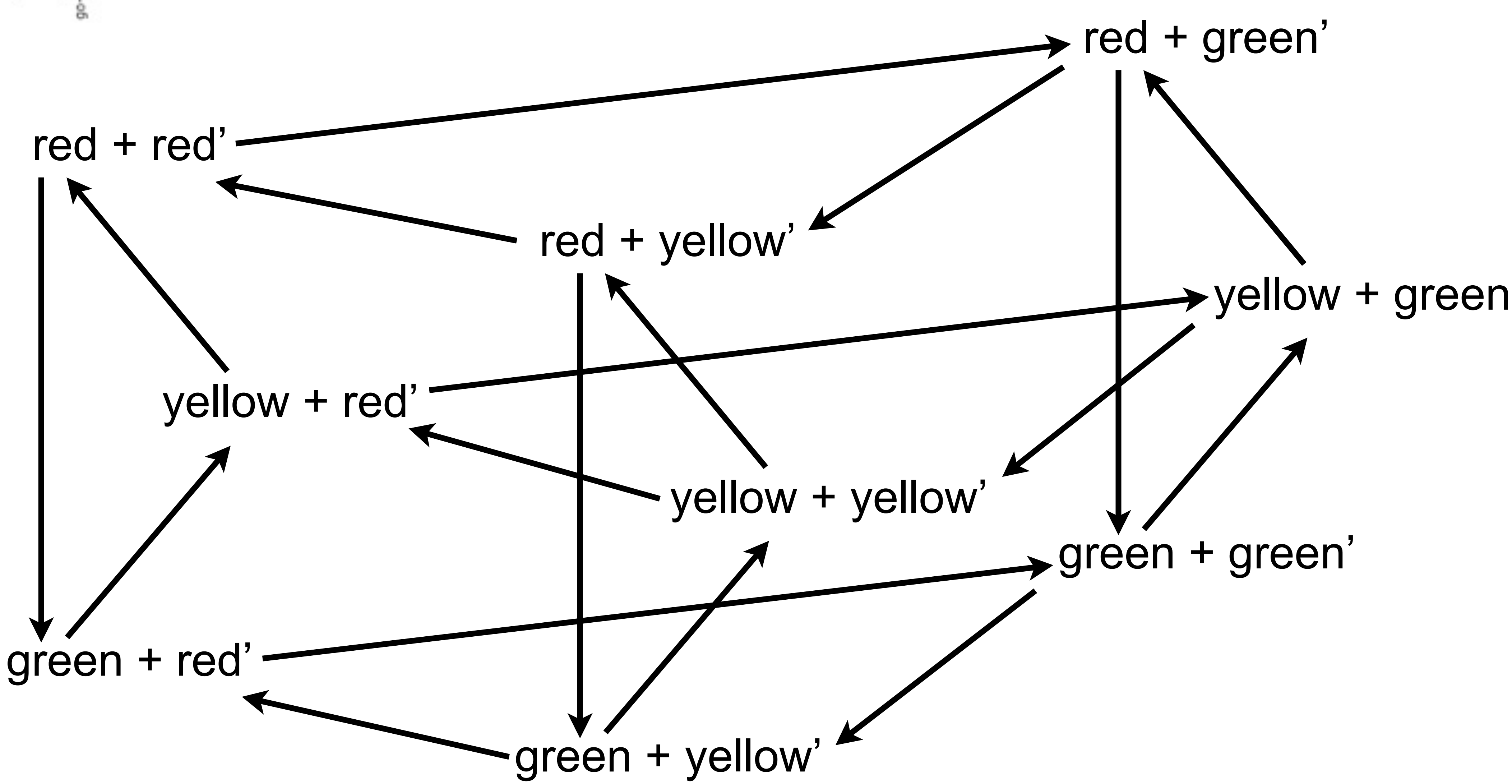

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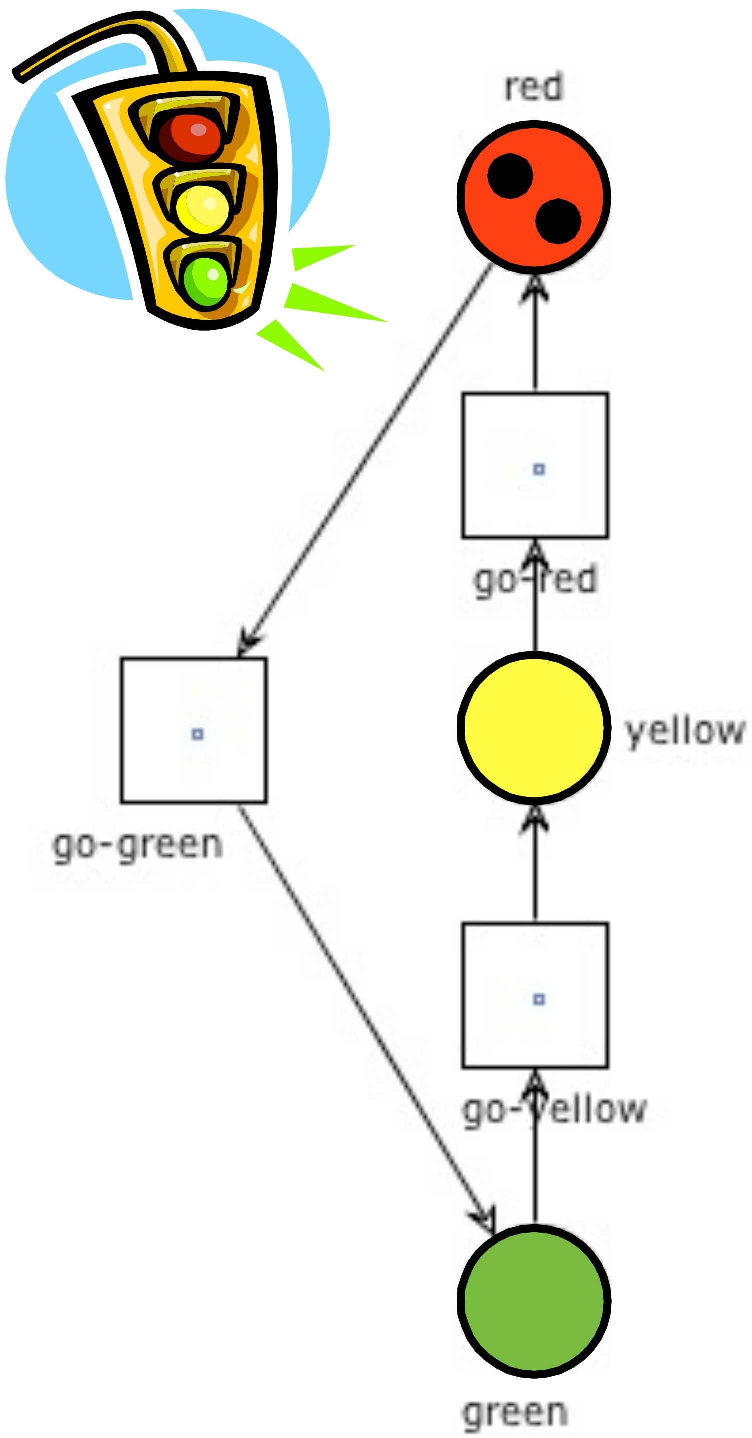
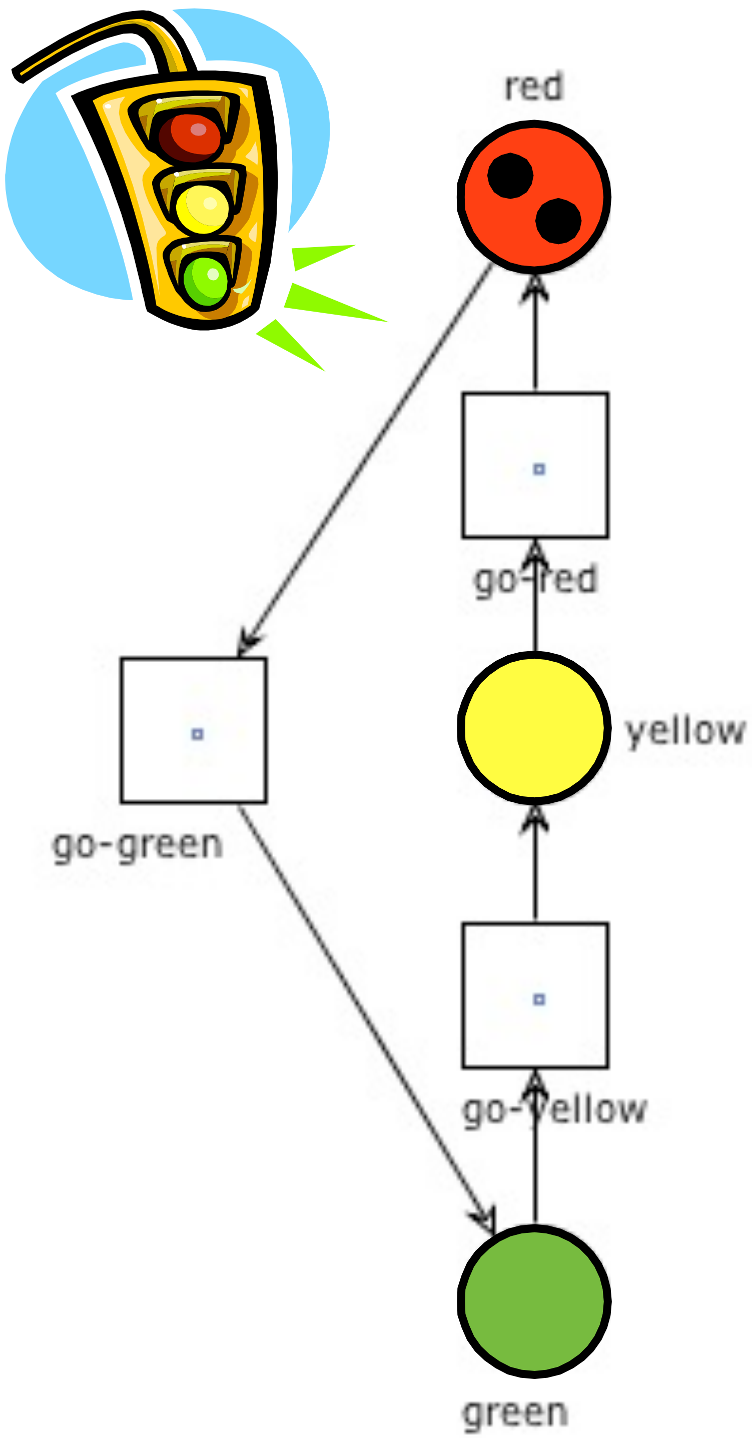


2 red

Example: two traffic lights

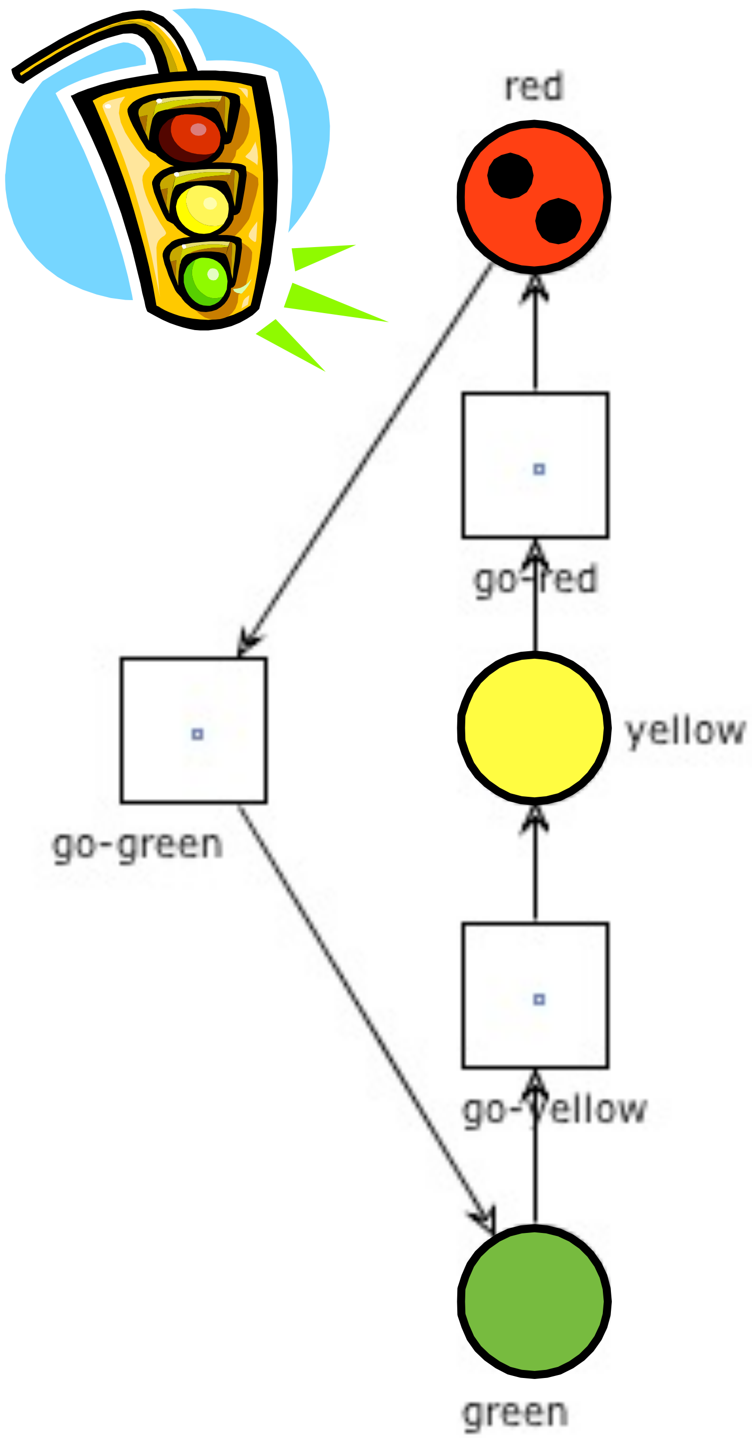


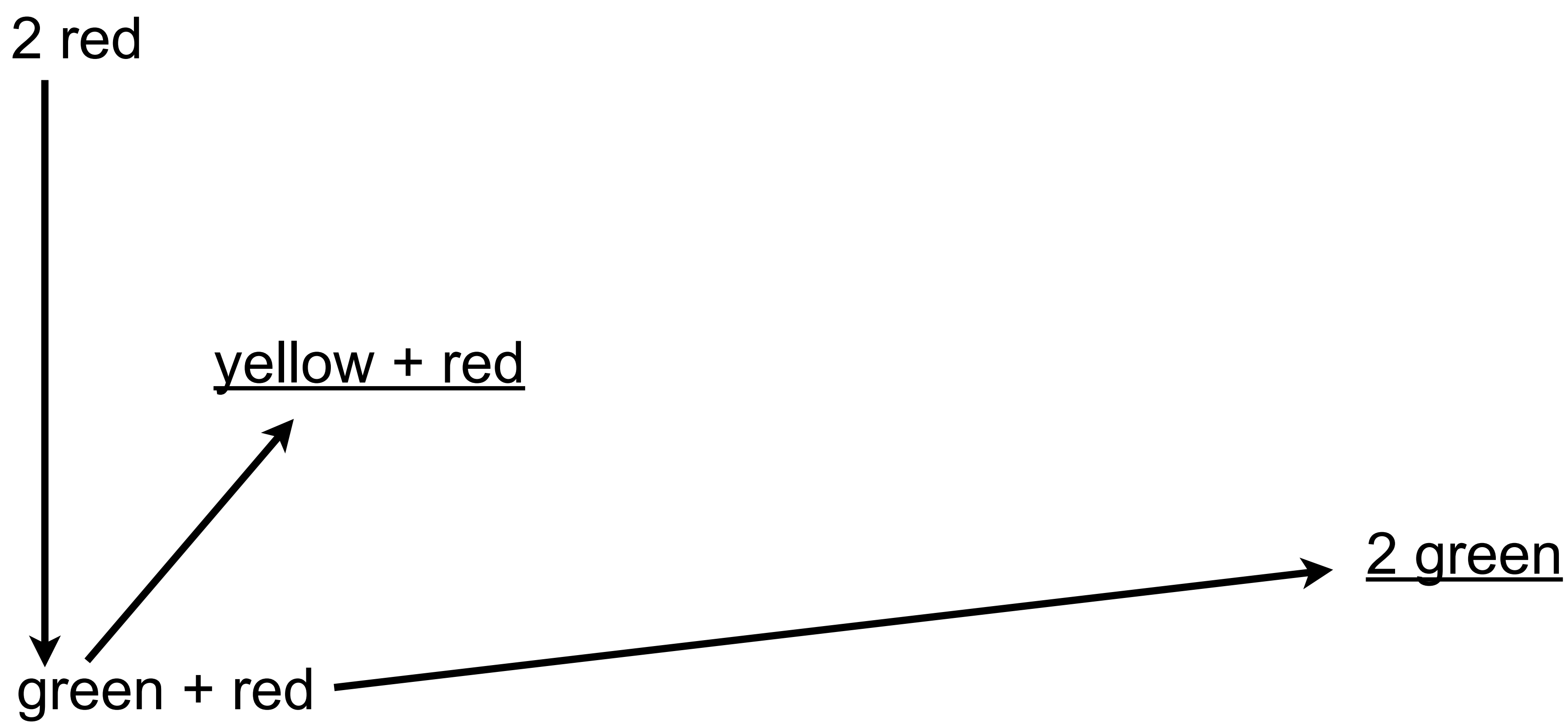
2 red

↓

green + red

Example: two traffic lights





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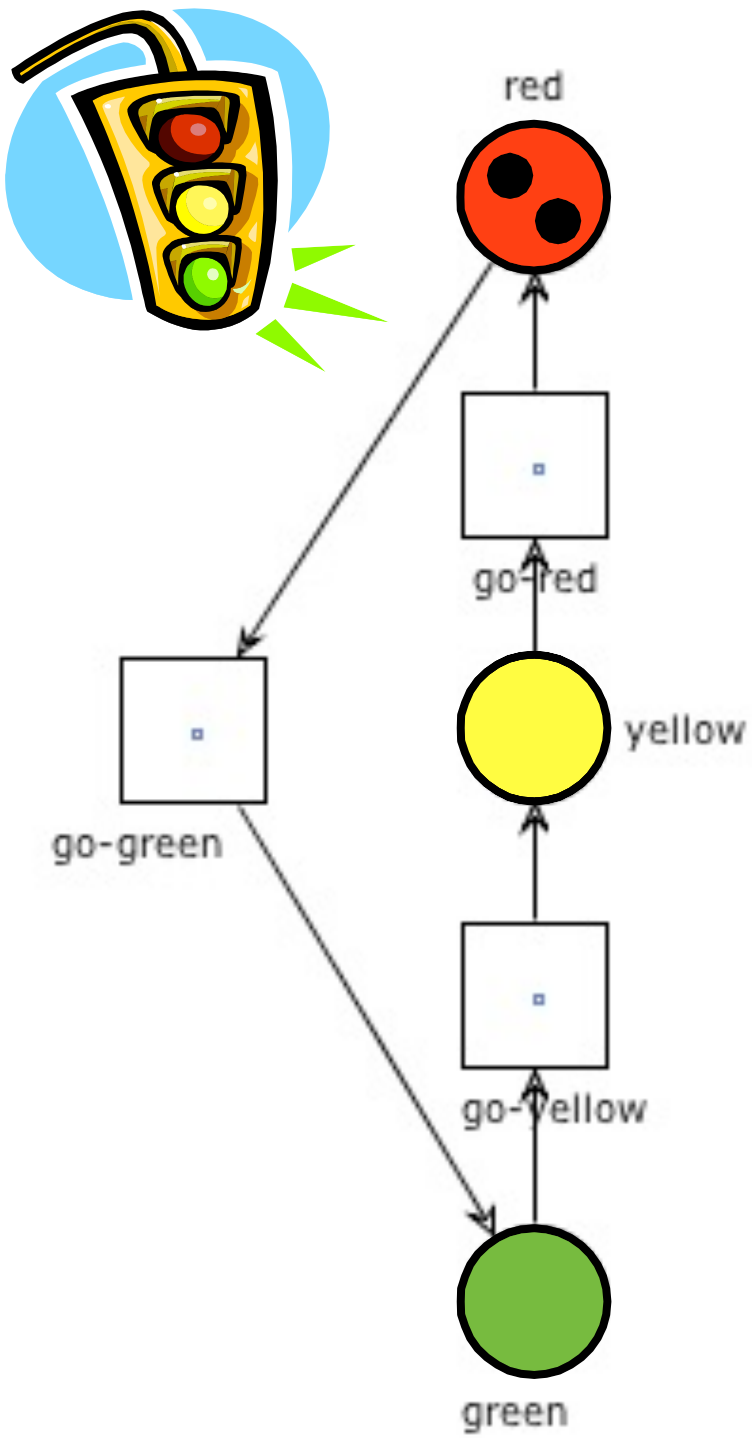

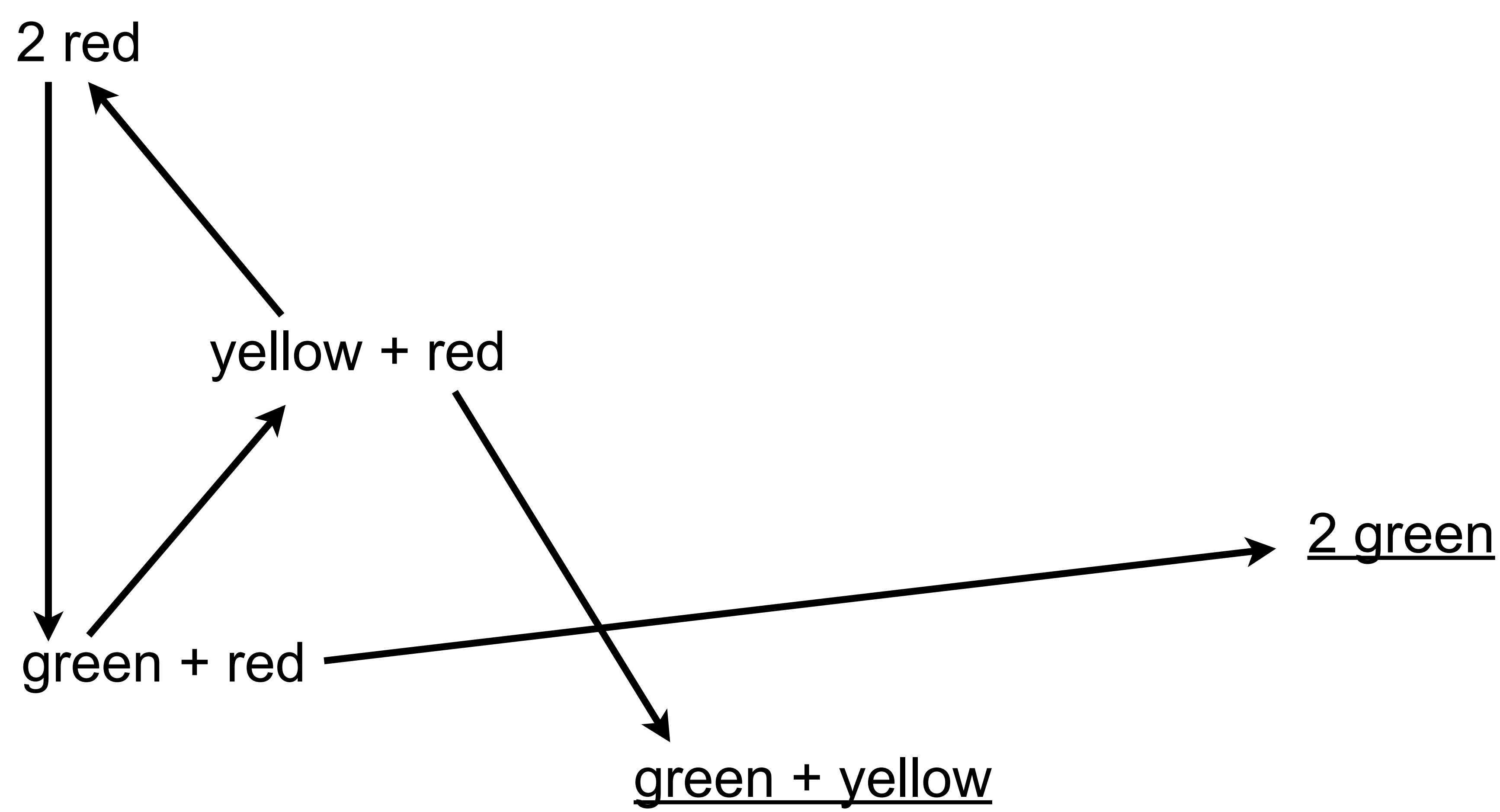

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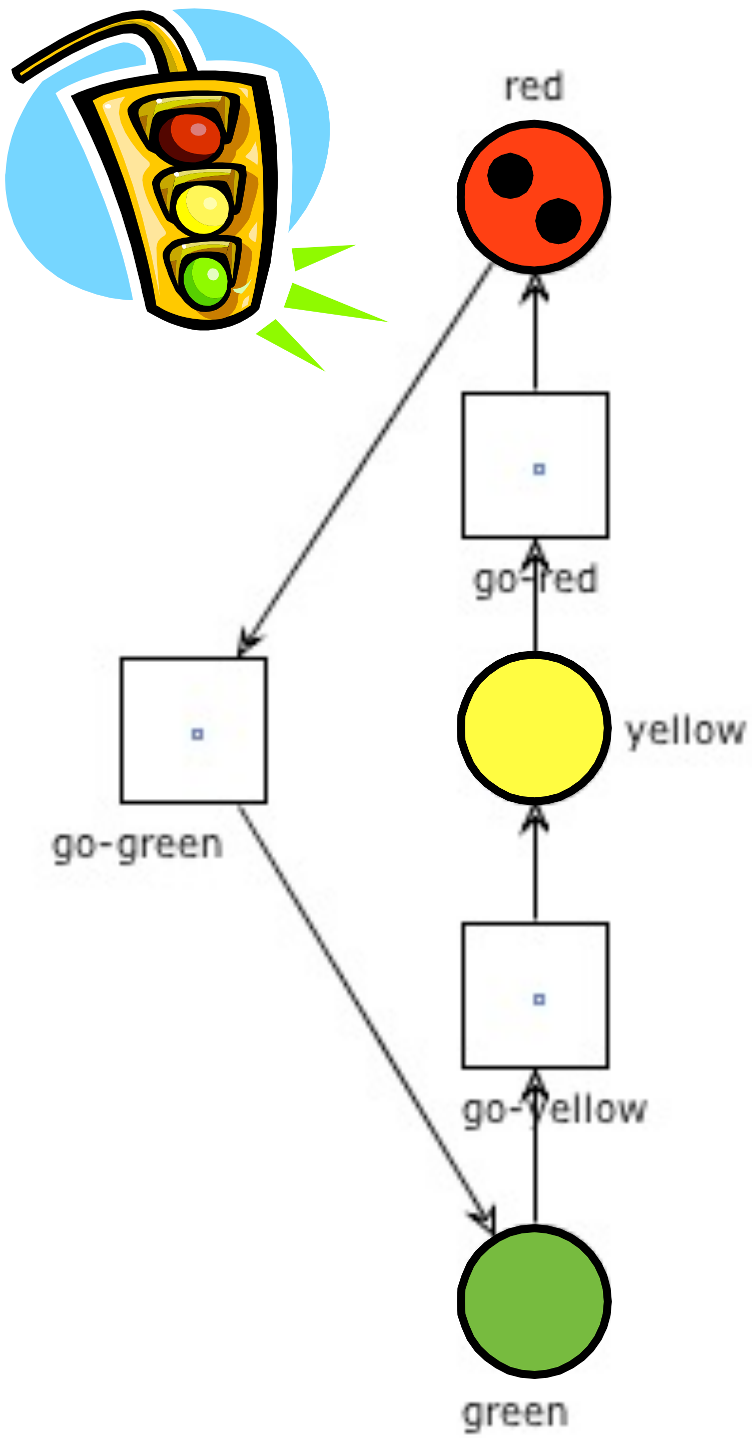

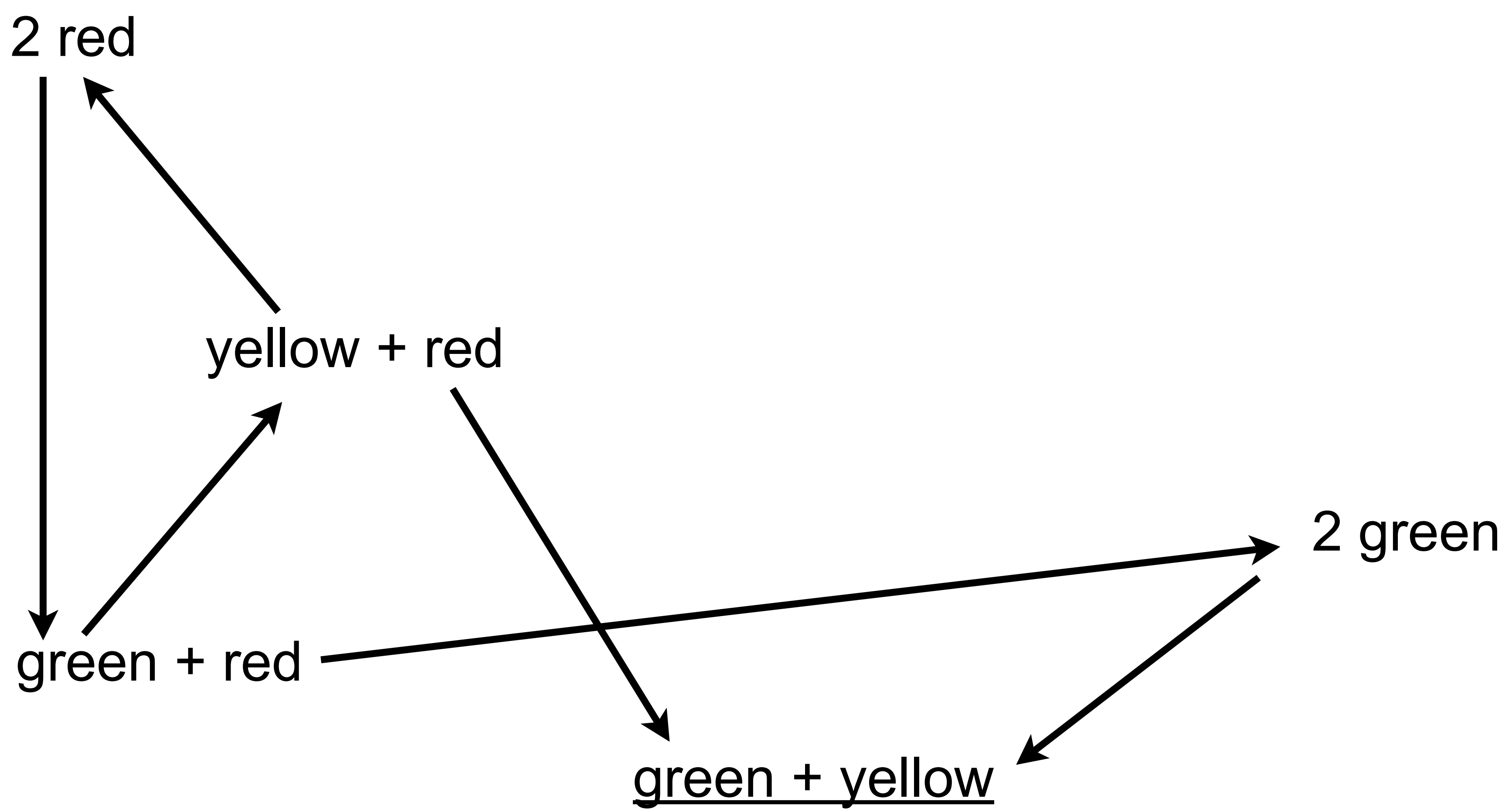

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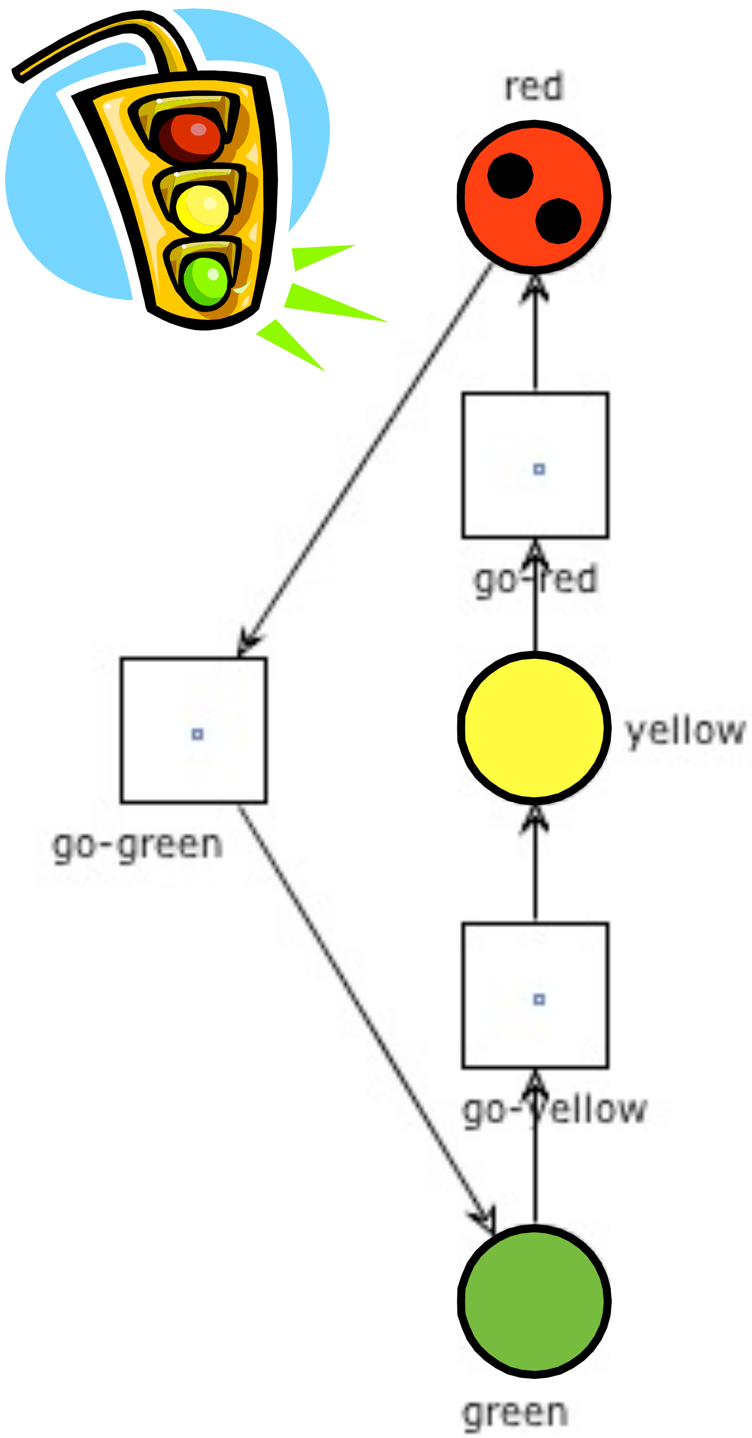
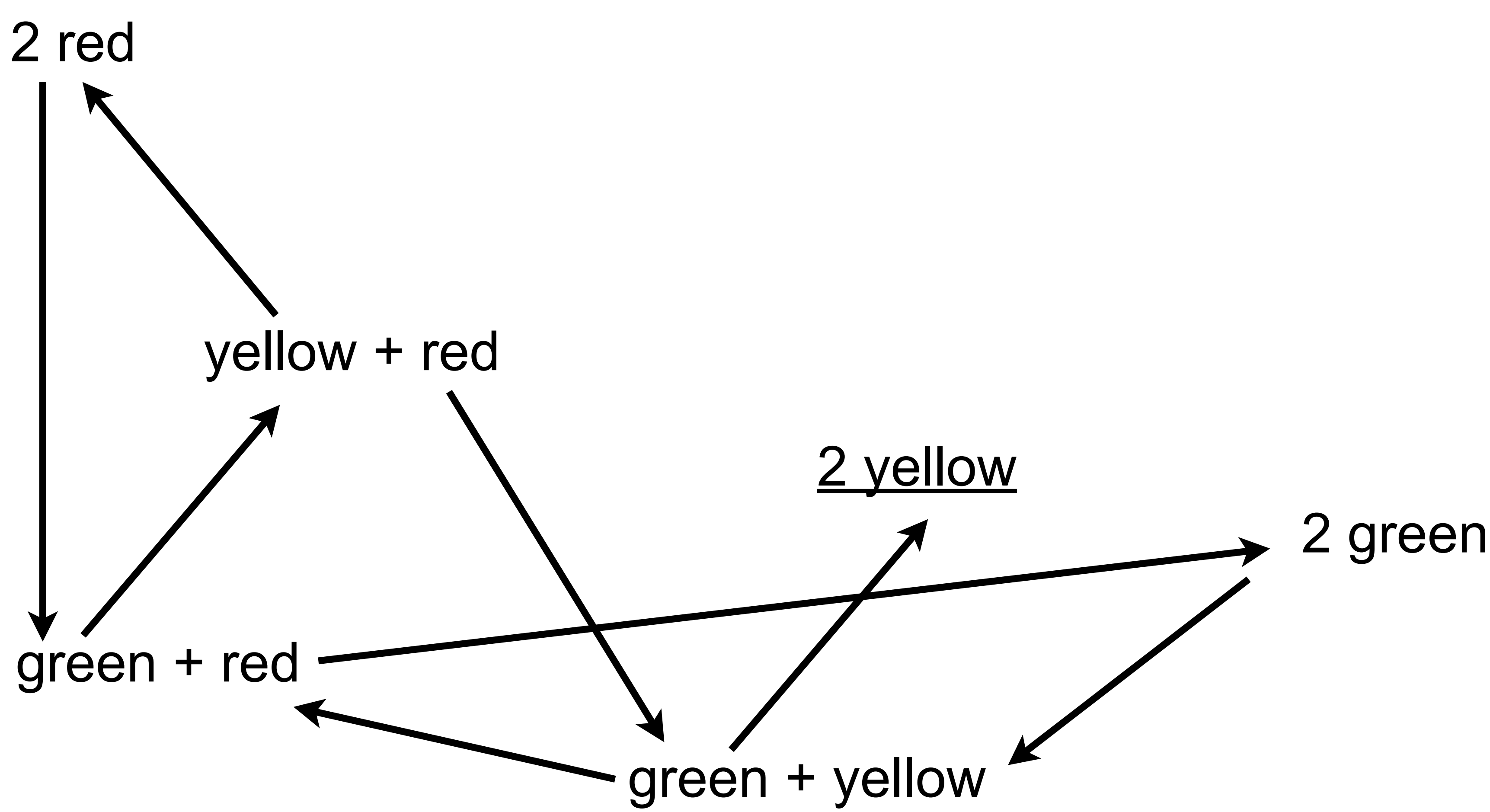
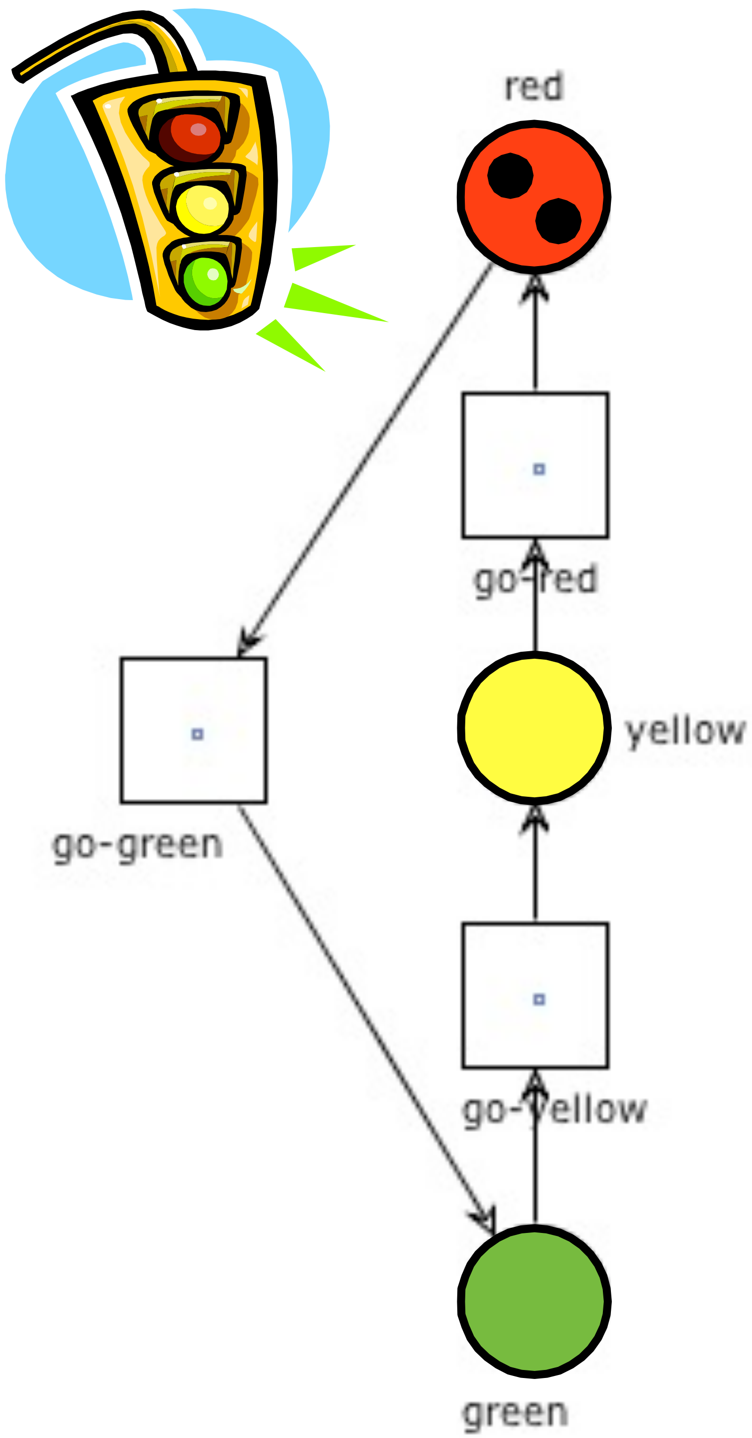
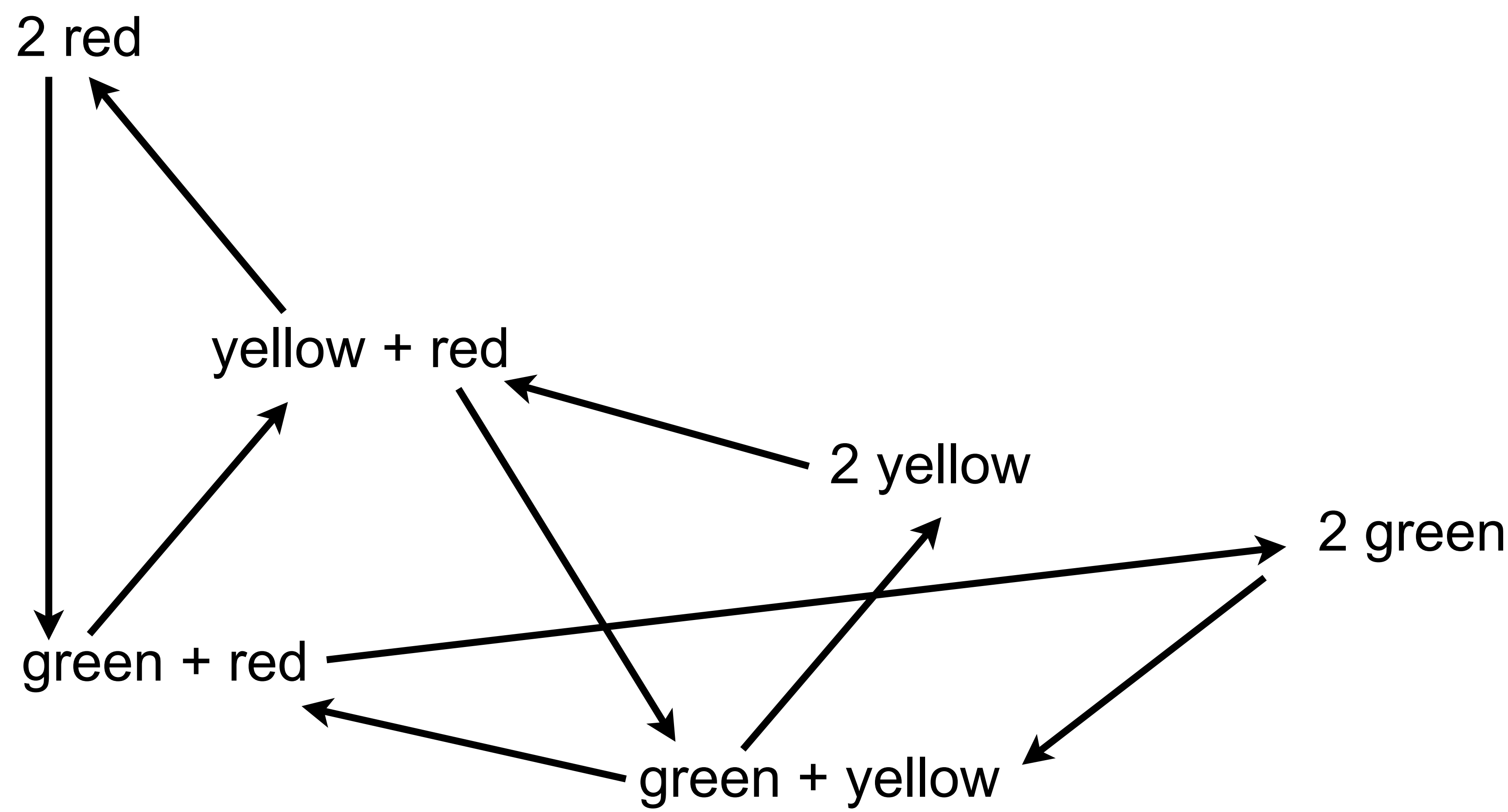


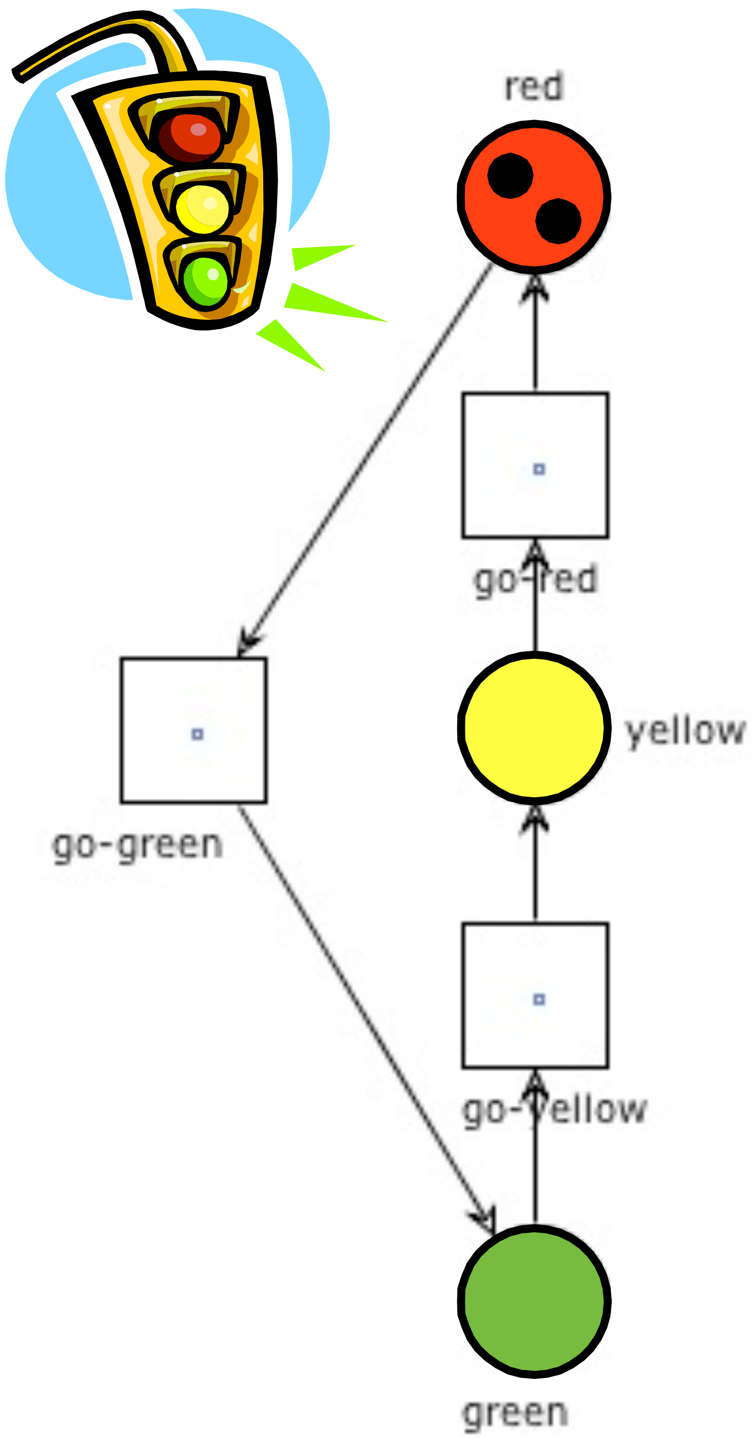


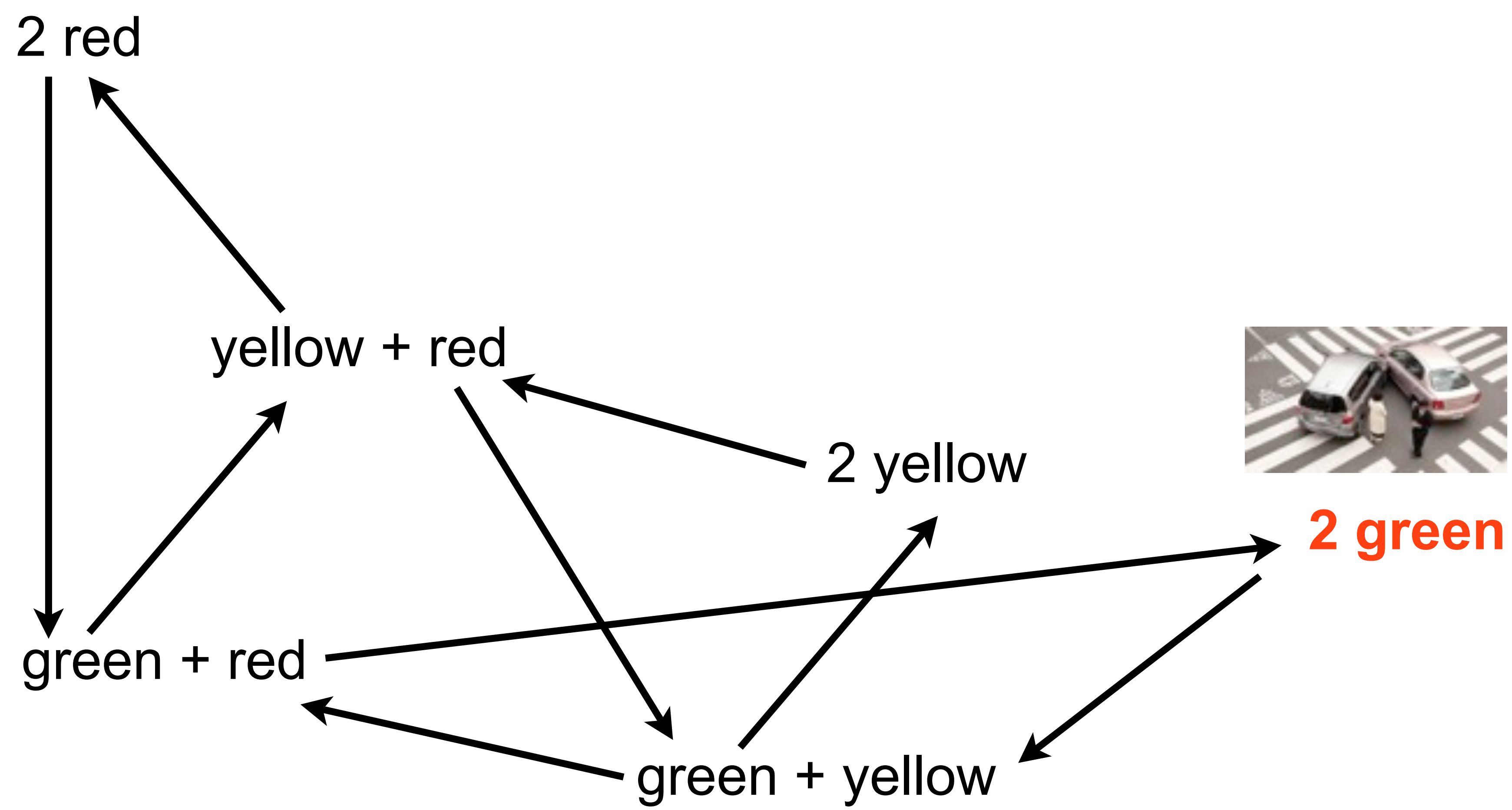
Example: two traffic lights



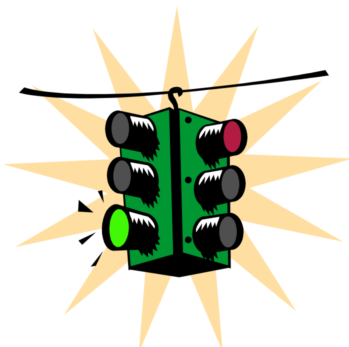


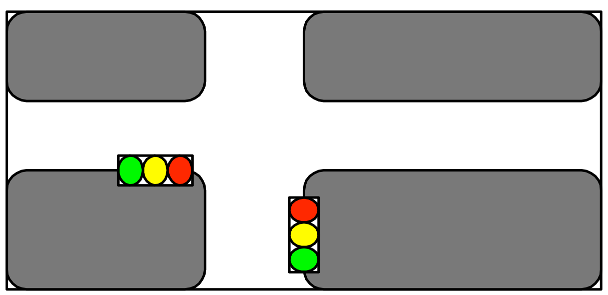
Example: two traffic lights

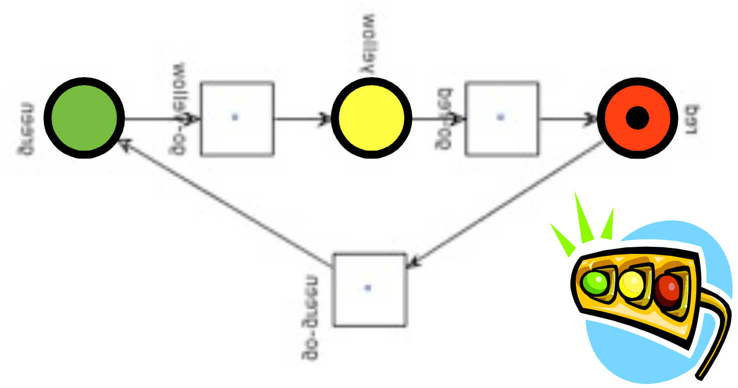


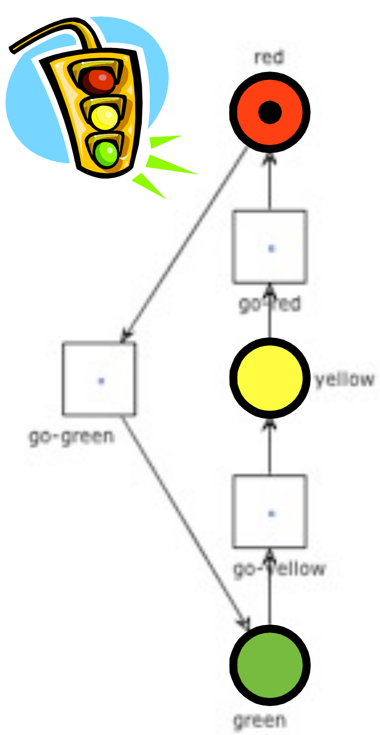


Exercise



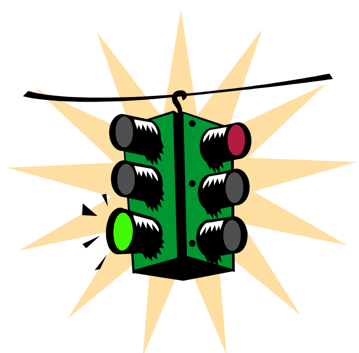


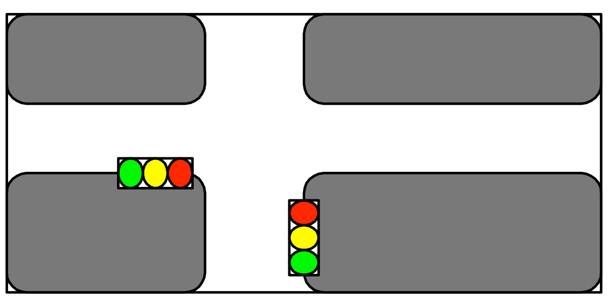




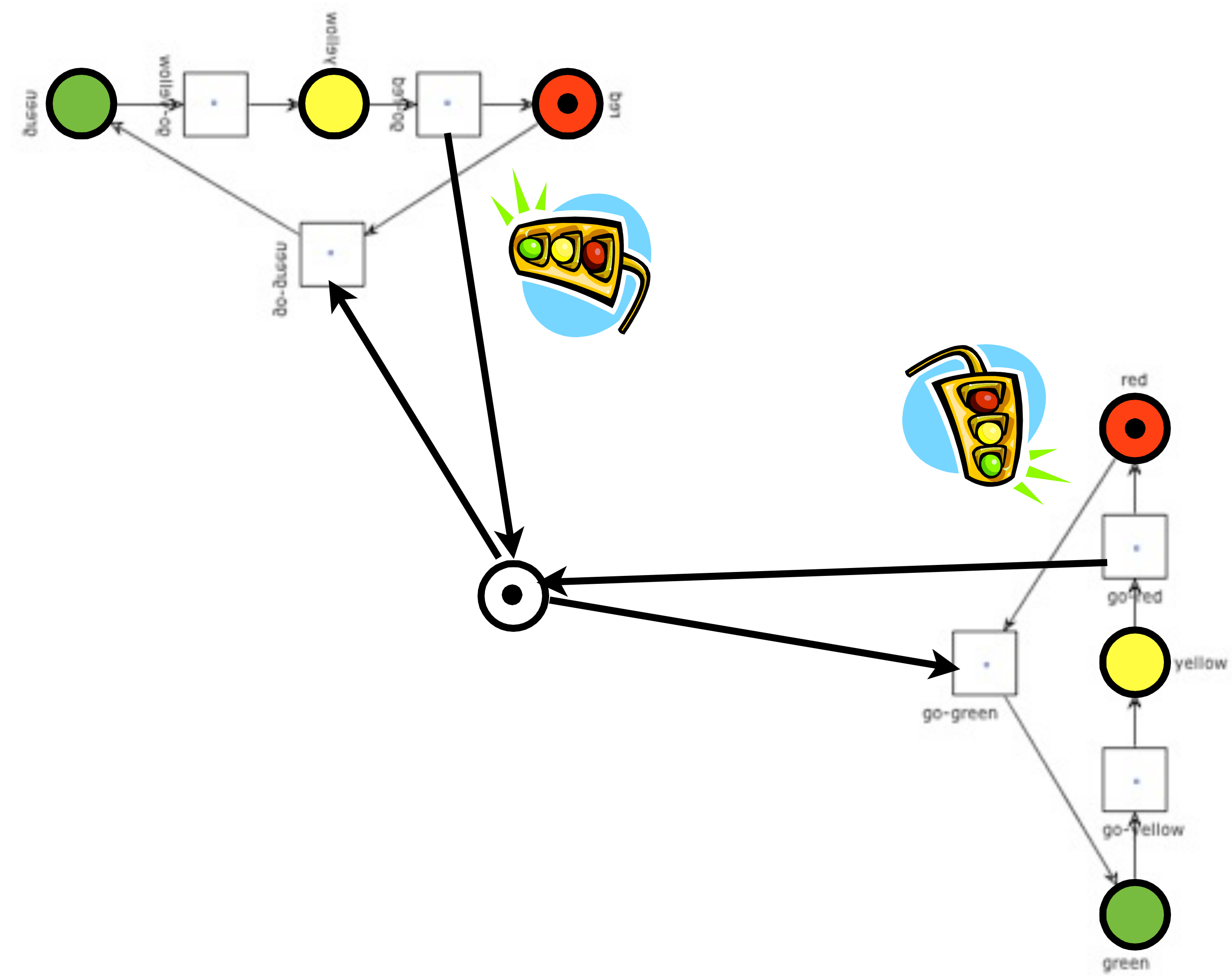
Complete the net in such a way that the two lights can never be green at the same time

Exercise





Complete the net in such a way that the two lights can never be green at the same time



Exercises

Draw the reachability graph of the last net

Modify the net so to guarantee that green alternate on the two traffic lights and then draw the reachability graph

Play the “token games” on the above nets
On the web (Petri net applet):
http://is.tm.tue.nl/staff/wvdaalst/workflowcourse/pn_applet/pn_applet.htm
On your PC (Workflow Petri net Designer):
http://www.woped.org

Exercise:
German traffic lights

German traffic lights have an extra phase: traffic lights turn not suddenly from red to green but give a red light together with a yellow light before turning to green.

Identify the possible states and model the transition system that lists all possible states and state transitions.

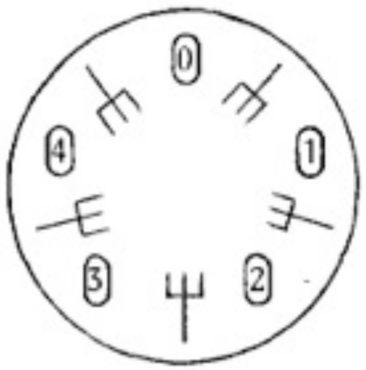
Provide a Petri net that is able to behave exactly like a German traffic light. There should be three places indicating the state of each light and make sure that the Petri net does not allow state transitions which should not be possible.

Exercise:

Producer and consumer

Model a process with one producer and one consumer:
Each one is either busy or free.
Each one alternates between these two states
After every production cycle the producer puts a product in a buffer and the consumer consumes one product from this buffer (when available) per cycle.

Draw the reachability graph
How to model 4 producers and 3 consumers connected through a single buffer?
How to limit the size of the buffer to 2 items?



Exercise: Dining philosophers

The problem is originally due to E.W. Dijkstra (and soon elaborated by T. Hoare) as an examination question on a synchronization problem where five computers competed for access to five shared tape drive peripherals.

It can be used to illustrate several important concepts in concurrency (mutual exclusion, deadlock, starvation)

Exercise: Dining philosophers

The life of a philosopher consists of an alternation of thinking and eating

Five philosophers are living in a house where the table laid for them, each philosopher having his own place at the table

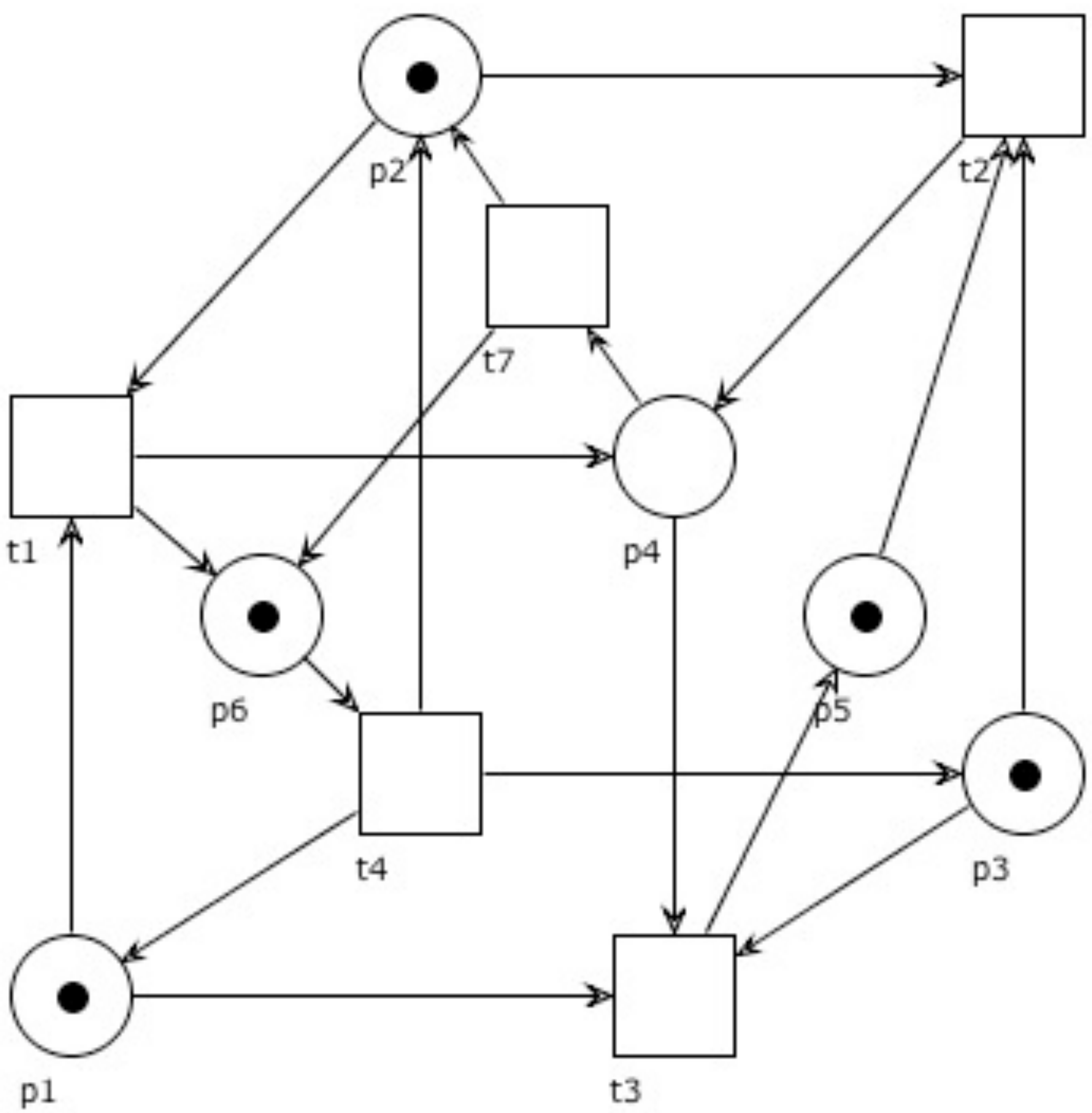
Their only problem (besides those of philosophy) is that the dish served is a very difficult kind of spaghetti, that has to be eaten with two forks. There are two forks next to each plate, so that presents no difficulty: as a consequence, however, no two neighbours may be eating simultaneously.

Exercise: Dining philosophers

Design a net for representing the dining philosophers problem, then use WoPeD to compute the reachability graph

Exercise

$$M_0 = p_1 + p_2 + p_3 + p_5 + p_6$$



Draw, at least in part, the reachability graph of the net

Properties of Petri nets

We describe, in an informal way, some of the properties of Petri nets that can play an important role in the verification of business processes

Liveness
Deadlock-freedom
Boundedness
Cyclicity (also Reversibility)

Liveness

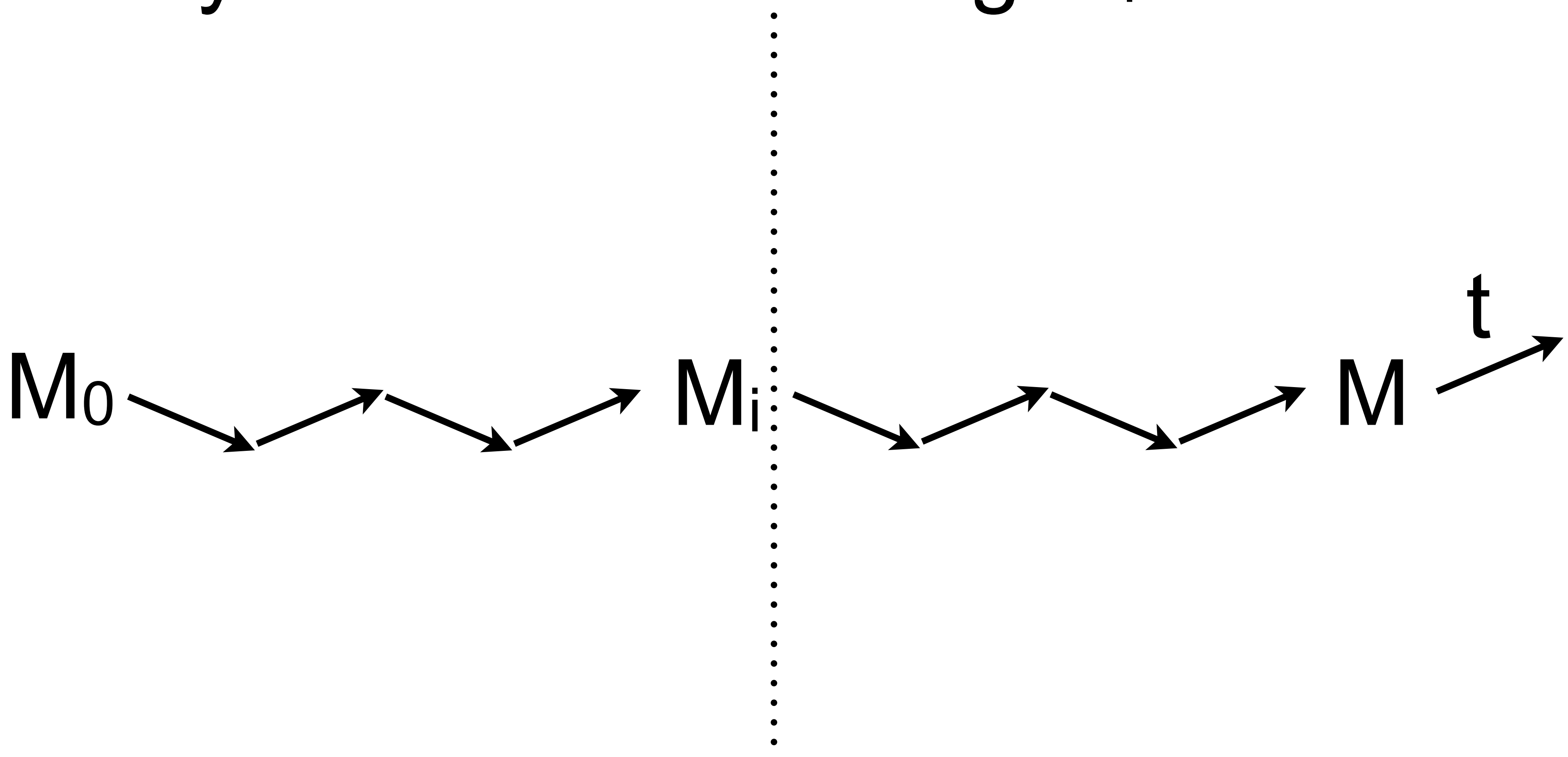
A transition t is **live**, if from any reachable marking M another marking M' can be reached where t is enabled

In other words, at any point in time of the computation, we cannot exclude that t will fire in the future

A Petri net is **live** if all of its transitions are live

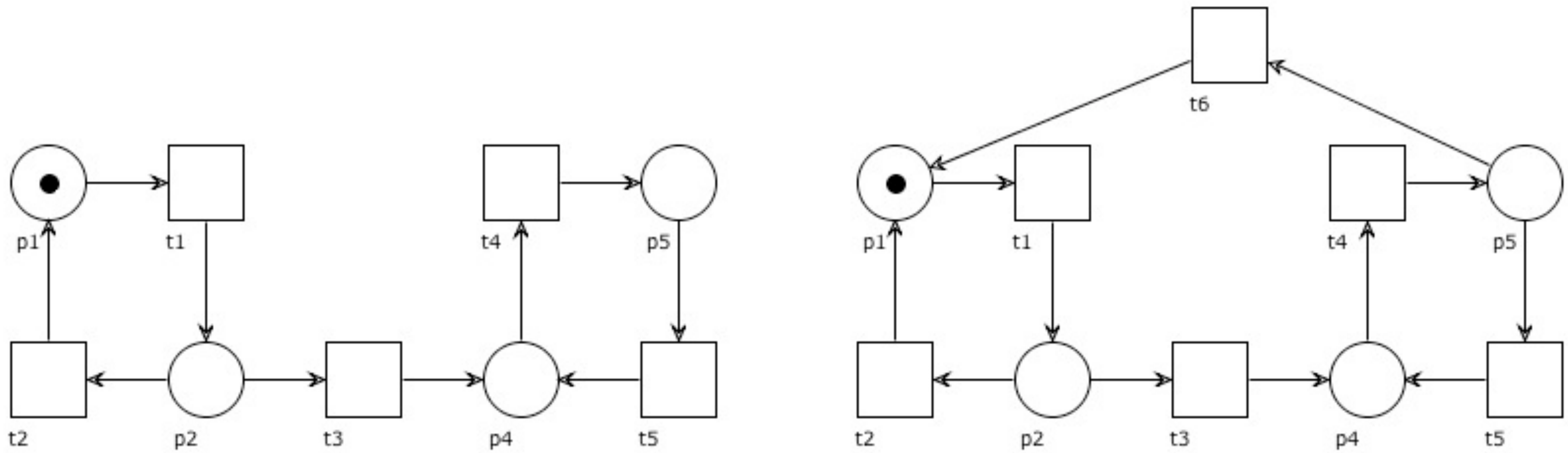
Liveness illustrated

For any reachable marking M_i



Can we find a way to enable t?

Liveness: example



Which transitions are live?
Which are not?
Is the net live?

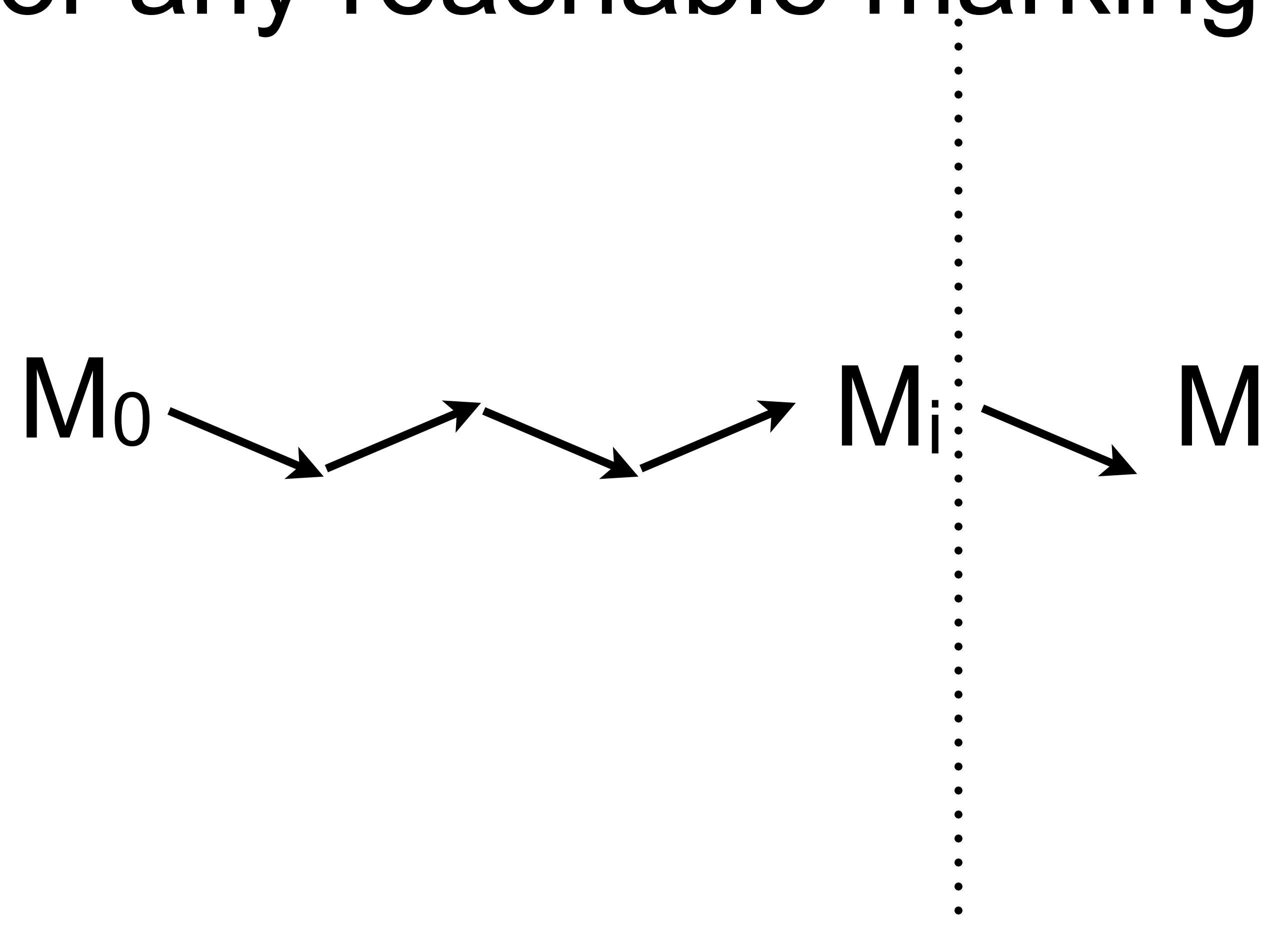
Deadlock-freedom

A Petri net is **deadlock free**, if every reachable marking enables some transition

In other words, we are guaranteed that at any point in time of the computation, some transition can be fired

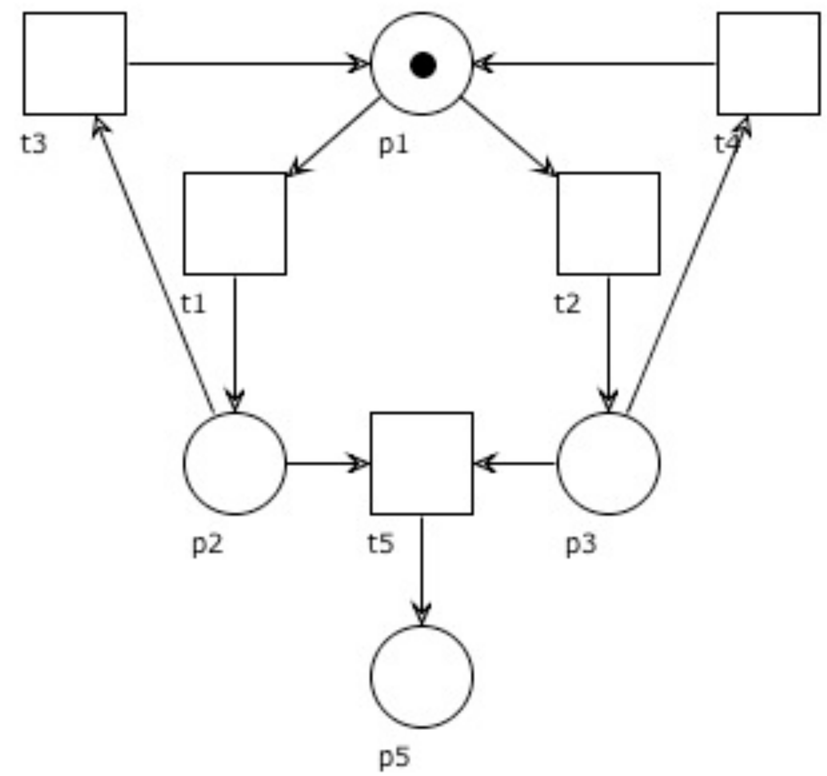
Deadlock-freedom illustrated

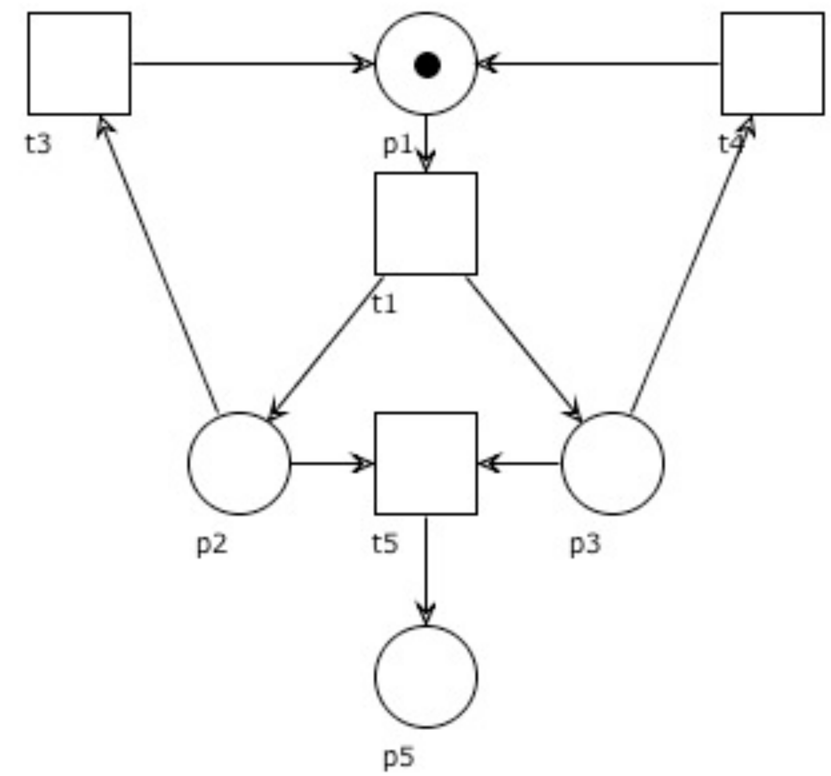
For any reachable marking M_i



Can we fire some transition?

Deadlock-freedom: example



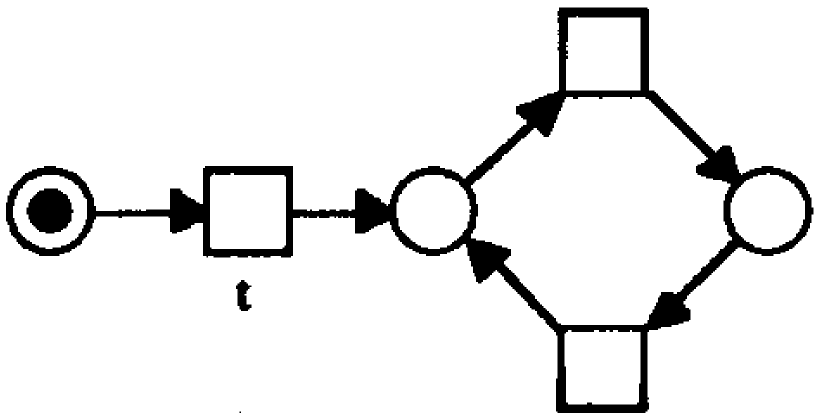


Is the net deadlock-free?

Question time

Does liveness imply deadlock-freedom? **YES**
(Can you exhibit a live Petri net that is not deadlock-free?)

Does deadlock-freedom imply liveness? **NO**
(Can you exhibit a deadlock-free net that is not live?)



k-Boundedness

Let k be a natural number

A place p is **k-bounded** if no reachable marking has more than k tokens in place p

A net is **k-bounded** if all of its places are k-bounded

In other words, if a net is k-bounded, then k is a capacity constraint that can be imposed over places without any risk of causing “overflow”

Safe nets

A place p is **safe** if it is 1-bounded

A net is **safe** if all of its places are safe

In other words, if the net is safe, then we know that, in any reachable marking, each place contains one token at most

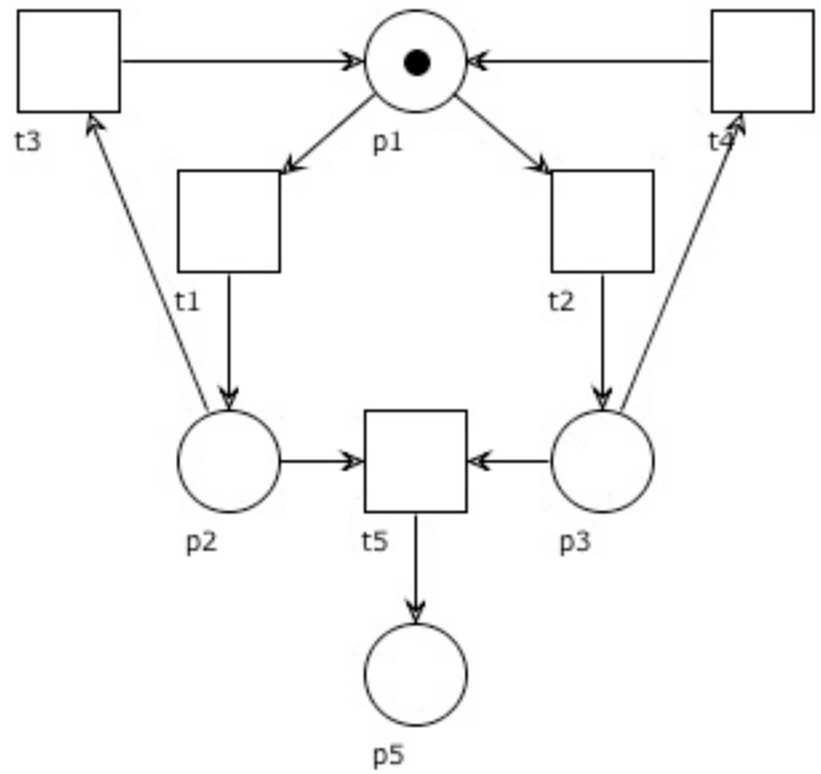
Boundedness

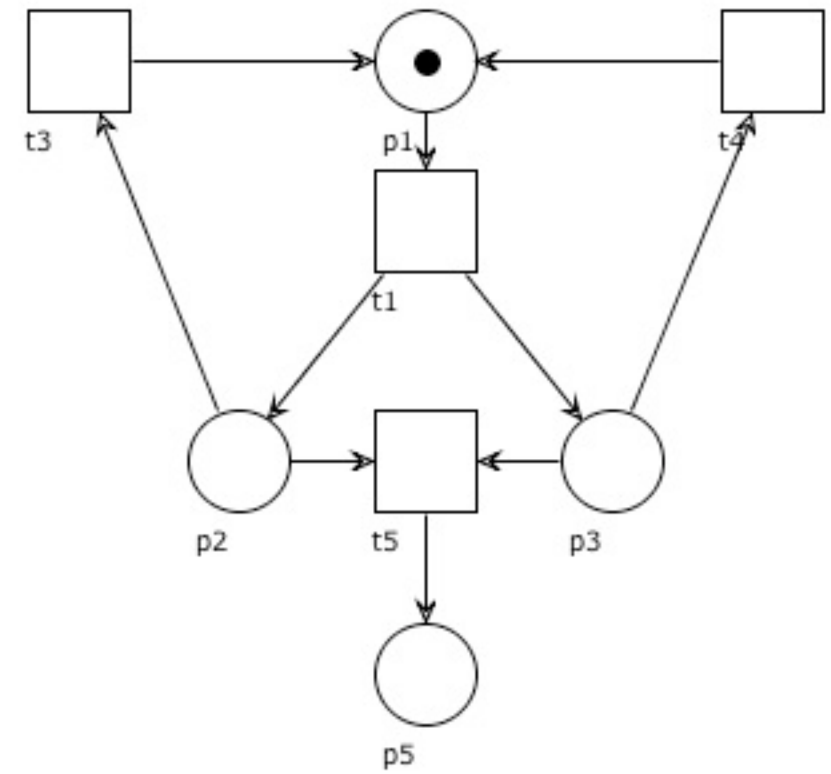
A place p is **bounded** if it is k-bounded for some natural number k

A net is **bounded** if all of its places are bounded

A net is **unbounded** if it is not bounded

Boundedness: example





Which places are bounded?
Is the net bounded?
Which places are safe?
Is the net safe?

Cyclicity (aka Reversibility)

A marking M is a **home marking** if it can be reached from every reachable marking

A net is **cyclic** (or **reversible**) if its initial marking is a home marking

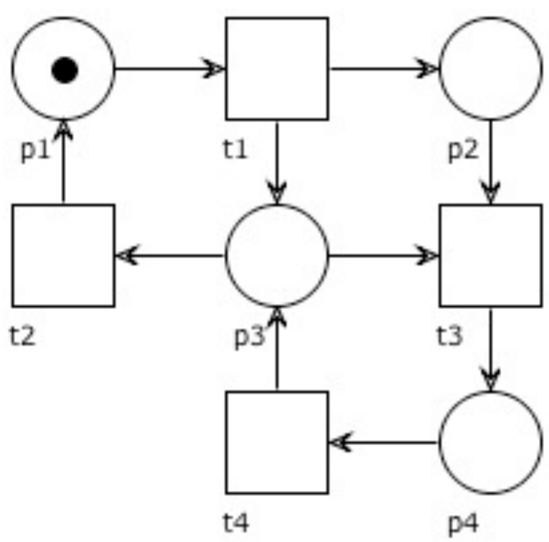
Orthogonal properties

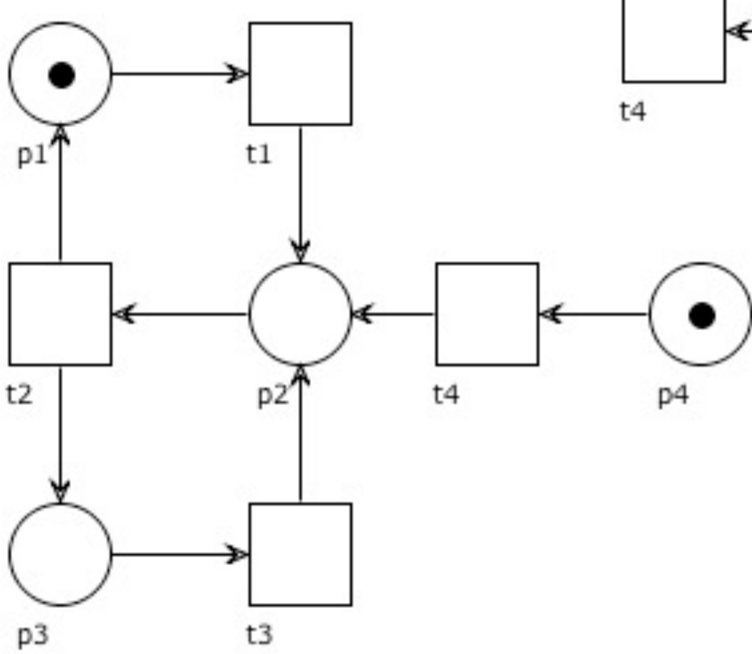
Liveness, boundedness and cyclicity are independent of each other

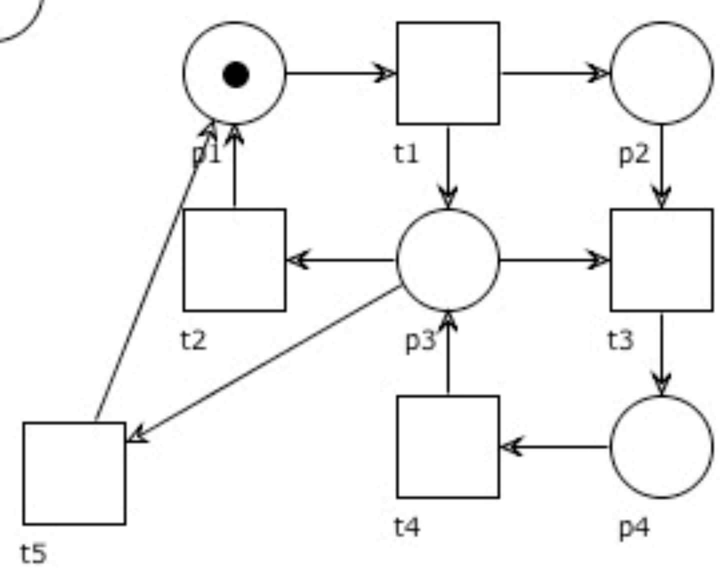
In other words, you can find nets that satisfy any arbitrary combination of the above three properties (and not the others)

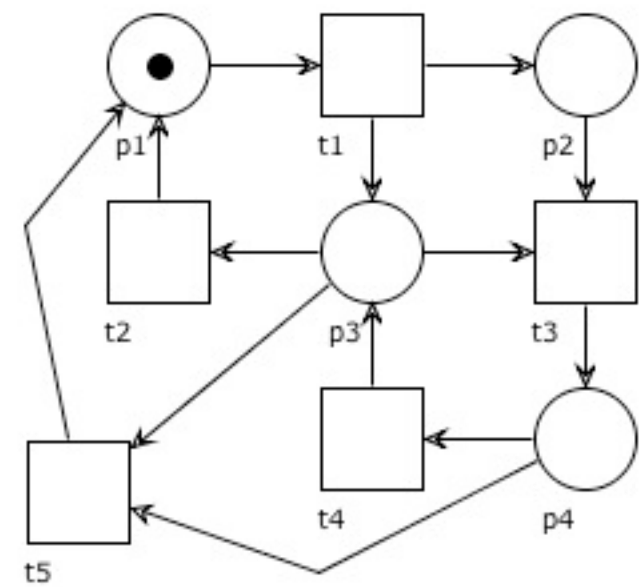
Exercises

For each of the following nets, say if they are live, deadlock-free, bounded, safe, cyclic



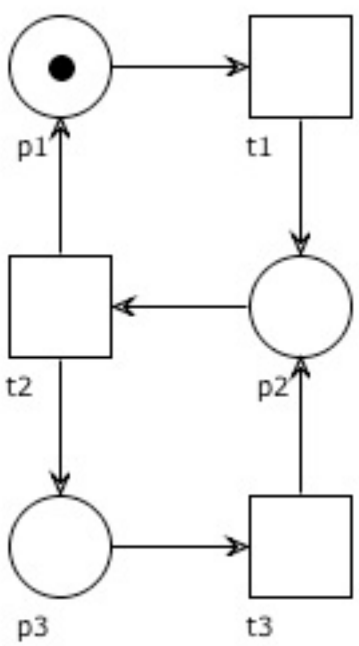


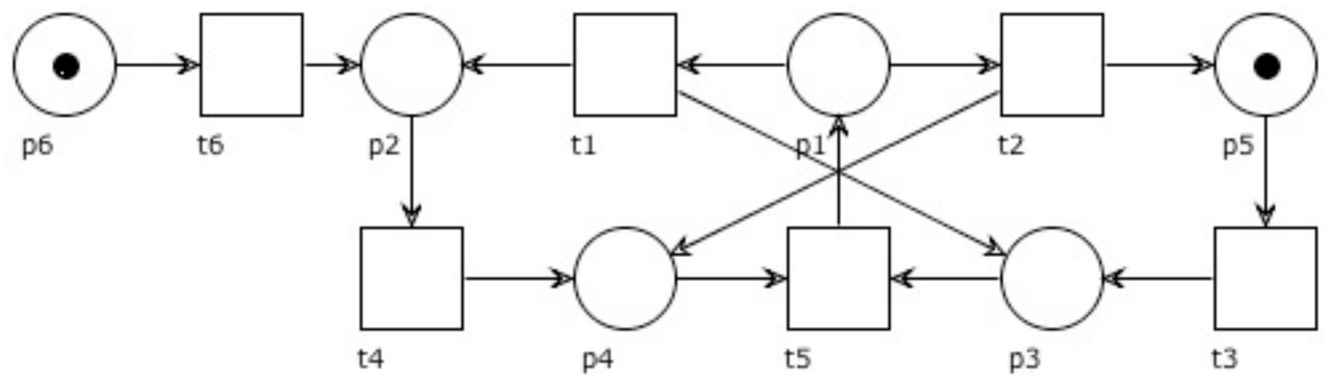


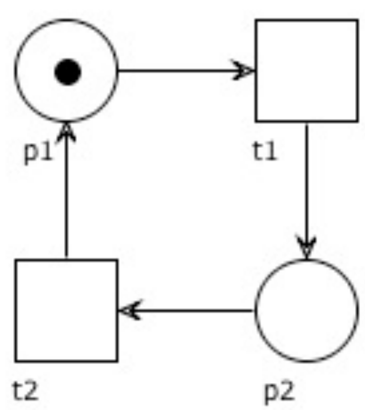


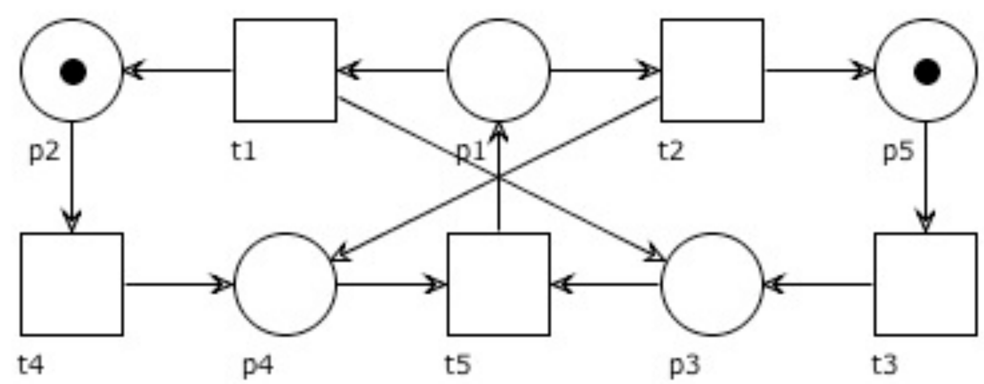
Exercises

For each of the following nets, say if they are live, deadlock-free, bounded, safe, cyclic









Live and dead places

Live place

Definition: Let (P, T, F, M_0) be a net system.

A place $p \in P$ is **live** if $\forall M \in [\, M_0 \,\rangle.\, \exists M' \in [\, M \,\rangle.\, M'(p) > 0$

Live place, intuitively

A place p is live

if every time it becomes unmarked

there is still the possibility to be marked in the future

(or if it is always marked)

Place liveness

Definition:
A net system (P, T, F, M_0) is **place-live** if every place $p \in P$ is live

Liveness implies place-liveness

Proposition: Live systems are place-live

Take any p and any $t \in \bullet p \cup p \bullet$

Let $M \in [\, M_0 \,\rangle$

By liveness: there is $M', M'' \in [\, M \,\rangle$ s.t. $M' \xrightarrow{t} M''$

Then $M'(p) > 0$ or $M''(p) > 0$

Dead nodes

Definition: Let (P, T, F) be a net system.

A transition $t \in T$ is **dead** at M if $\forall M' \in [\, M \,\rangle .\, M' \not\xrightarrow{t}$

A place $p \in P$ is **dead** at M if $\forall M' \in [\, M \,\rangle .\, M'(p) = 0$

Some obvious facts

If a system is not live, it has a transition dead at some reachable marking

If a system is not place-live, it has a place dead at some reachable marking

If a place / transition is dead at M, then it remains dead at any marking reachable from M
(the set of dead nodes can only increase during a run)

Every transition in the pre- or post-set of a dead place is also dead

Behavioural vs Structural Properties

Structural properties

All the properties we have seen so far are **behavioural** (or **dynamic**) (i.e. they depend on the initial marking and firing rules)

It is sometimes interesting to connect them to **structural** properties (i.e. the shape of the graph representing the net)

This way we can give **structural characterization** of behavioural properties for a class of nets (computationally less expensive to check)

A matter of terminology

To better reflect the above distinction, it is frequent:

to use the term **net system** for denoting a Petri net with a given initial marking
(we study behavioural properties of systems)

to use the term **net** for denoting a Petri net without specifying any initial marking
(we study structural properties of nets)

Paths and circuits

A **path** of a net (P, T, F) is a non-empty sequence $x_1 x_2 ... x_k$ such that

$$(x_i, x_{i+1}) \in F \quad \text{for every } 1 \leq i < k$$

(and we say that it lead**s** from x_1 to x_k)

A path from x to y is called a **circuit** if:
no element occurs more than once in it and $(y, x) \in F$
(since for any x we have $(x, x) \notin F$, hence a circuit involves at least two nodes)

Connectedness

A net (P,T,F) is **weakly connected** iff it does not fall into (two or more) unconnected parts (i.e. no two subnets (P_1,T_1,F_1) and (P_2,T_2,F_2) with disjoint and non-empty sets of elements can be found that partition (P,T,F))

A weakly connected net is **strongly connected** iff for every arc (x,y) there is a path from y to x

Connectedness, formally

A net (P, T, F) is **weakly connected** if every two nodes x, y satisfy

$$(x, y) \in (F \cup F^{-1})^*$$

(i.e. if there is an undirected path from x to y)

It is **strongly connected** if $(x, y) \in F^*$

A note

In the following we will consider (implicitly) weakly connected nets only

(if they are not, then we can study each of their subsystems separately)

S-systems

A Petri net is called **S-system** if every transition has one input place and one output place (S comes from *Stellen*, the German word for place)

This way any synchronization is ruled out

The theory of S-systems is very simple

T-systems

A Petri net is called **T-system** if every place has one input transition and one output transition

This way all conflicts are ruled out

T-systems have been studied extensively since the early Seventies

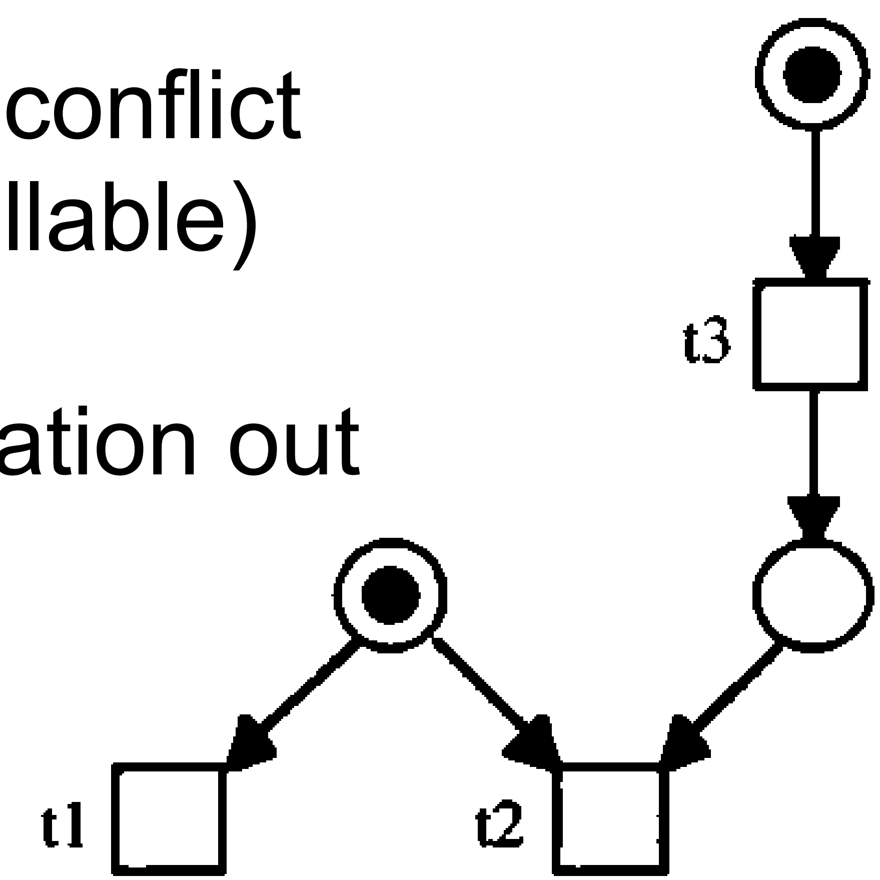
Interference of conflicts and synch

Typical situation:

initially t1 and t2 are not in conflict

but when t3 fires they are in conflict
(the firing of t3 is not controllable)

Free-choice nets rule this situation out



Free-choice nets

The aim is to avoid that a choice between transition is influenced by the rest of the system

Easiest way:
keep places with more than one output transition apart from transitions with more than one input place

In other words, if (p,t) is an arc, then it means that t is the only output transition of p (no conflict)
OR
p is the only input place of t (no synch)

Free-choice nets

But we can study a slightly more general class of nets
by requiring a weaker constraint

A Petri net is **free-choice** if
whenever there is an arc (p,t), then there is an arc
from any input place of t
to any output transition of p

Exercise

Is the net an S-system, a T-system, free-choice?