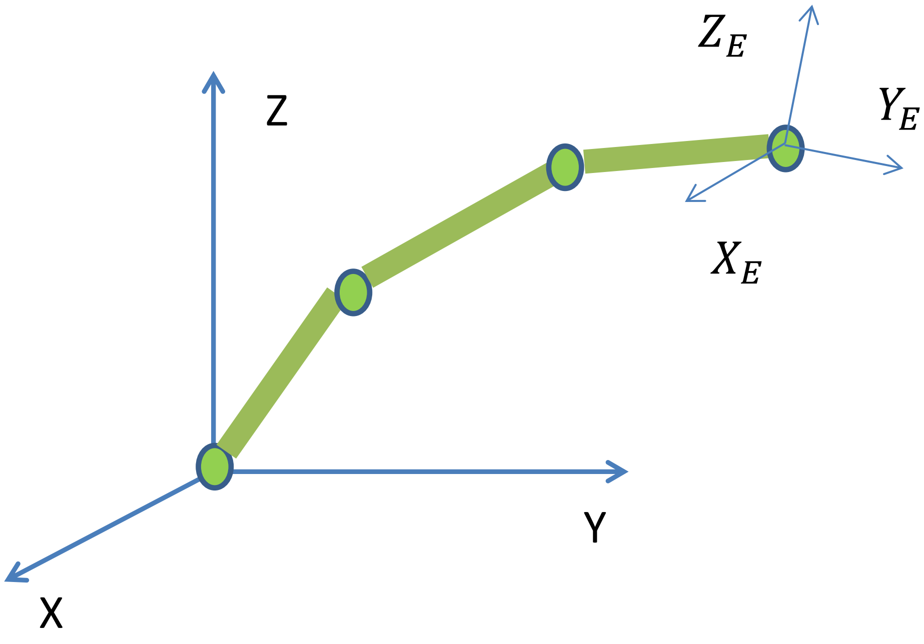
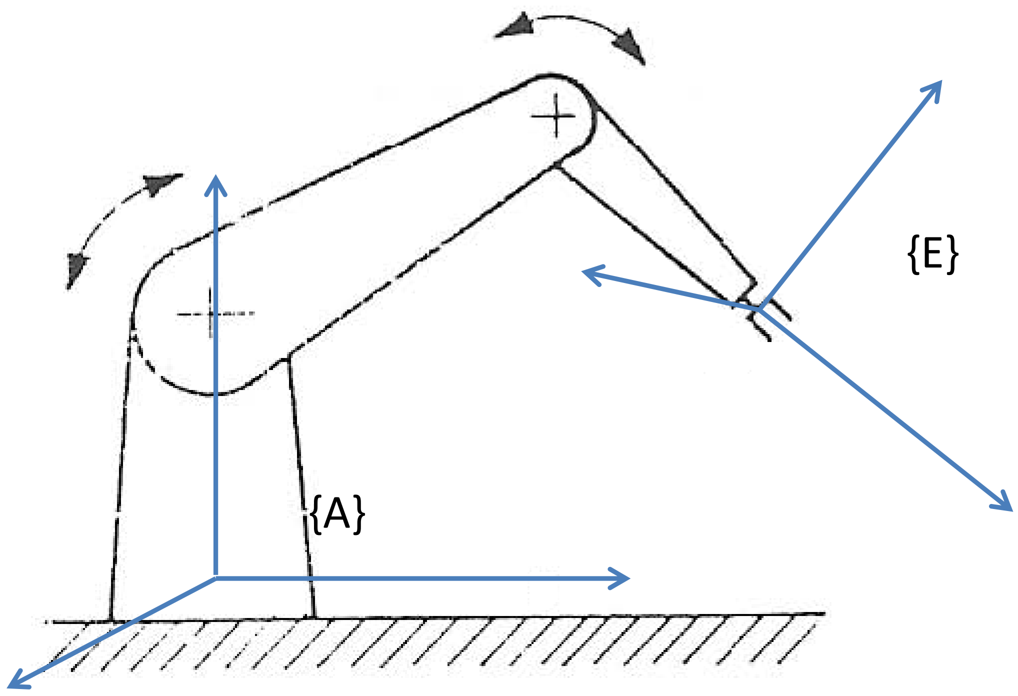
Robot kinematics

- End effector positioning in space with respect to a predefined reference frame



Robot kinematics(II)



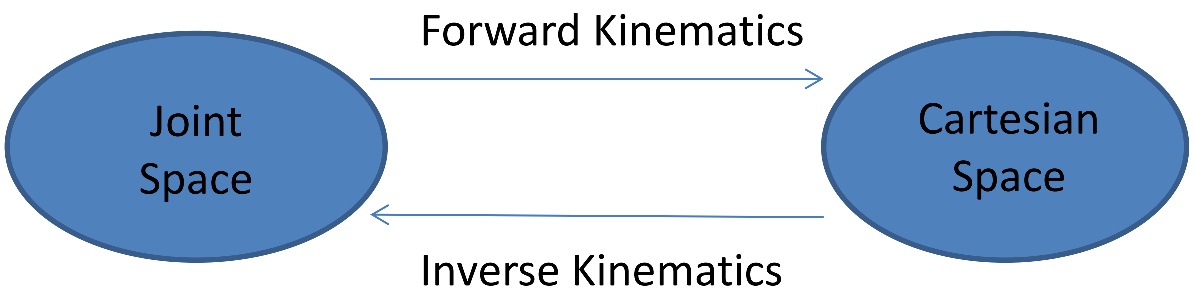
Forward Kinematics $\quad {}^{A}T_{E}(\theta_1, \ldots, \theta_n)$

Inverse Kinematics $\quad \theta_i = f_i\,(x, y, z, \alpha, \beta, \gamma)$

Cartesian space and joint space

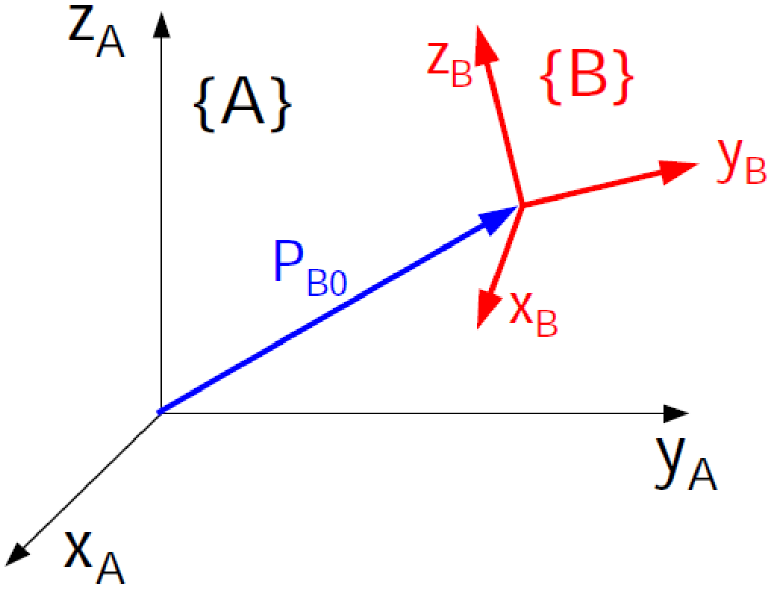
Cartesian Space: $P \in \Re^6$ $\textit{position and orientation vector related to the end effector}$

Spazio dei giunti: $P \in \Re^N$ $\textit{Joint values vector}$



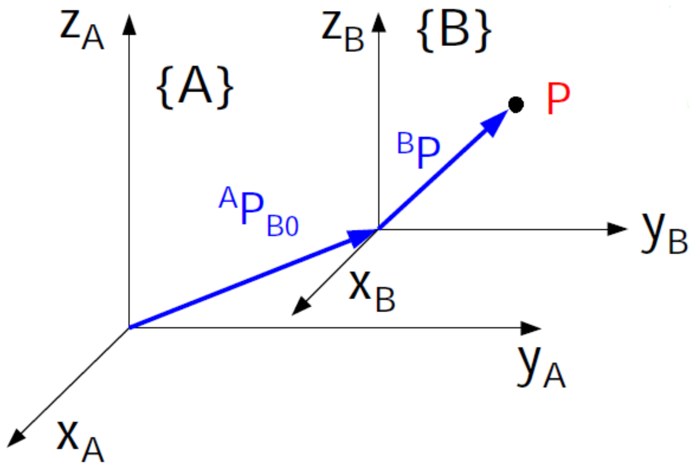
Reference Frame

A reference frame {B} can be defined by the position of the frame origin and the rotation of its axes relative to {A}

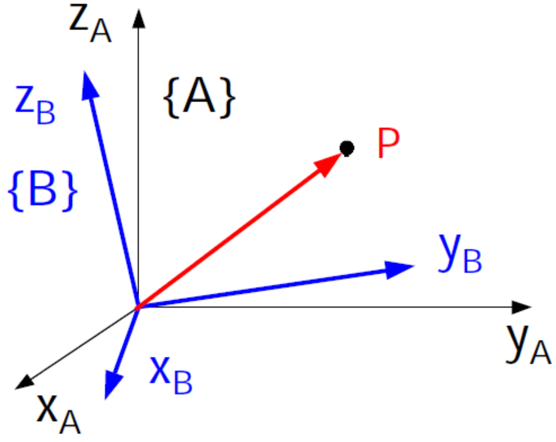


$$\{^{B}R_{A},\ ^{A}P_{B0}\}$$

Rotations and translations



$$^AP = {^BP} + {^AP_{B0}}$$



$$^AP = {^AR_B} \; {^BP}$$

Elemantary rotations aboue t frame axes

The rotation of a frame by a θ angle about the three rotational axes is given by:

$$R_x(\theta) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & -\sin\theta \\ 0 & \sin\theta & \cos\theta \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$$R_z(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Rotation about the z-axis of the vector [0 0 1]

$$R_z(90°) \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \cos 90° & -\sin 90° & 0 \\ \sin 90° & \cos 90° & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

Homogeneous Transformations

- Homogeneous transformations allow to compute roto-traslations through a matrix operation

$$ {}^{A}P = {}^{A}R_{B} \; {}^{B}P + {}^{A}P_{B0} \quad \Longrightarrow \quad {}^{A}P = {}^{A}T_{B} \; {}^{B}P $$

- In the homogeneous space:

$$ {}^{A}P = {}^{A}T_{B} \; {}^{B}P $$

$$ {}^{A}T_{B} = \begin{pmatrix} & {}^{A}R_{B} & & | & {}^{A}P_{B0} \\ & \text{—} & & \text{—} & \text{—} \\ 0 & 0 & 0 & | & 1 \end{pmatrix} $$

Homogeneous Transformations(II)

TRANSLATION

$$^{A}Trasl_{B} = \begin{pmatrix} 1 & 0 & 0 & dx \\ 0 & 1 & 0 & dy \\ 0 & 0 & 1 & dz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

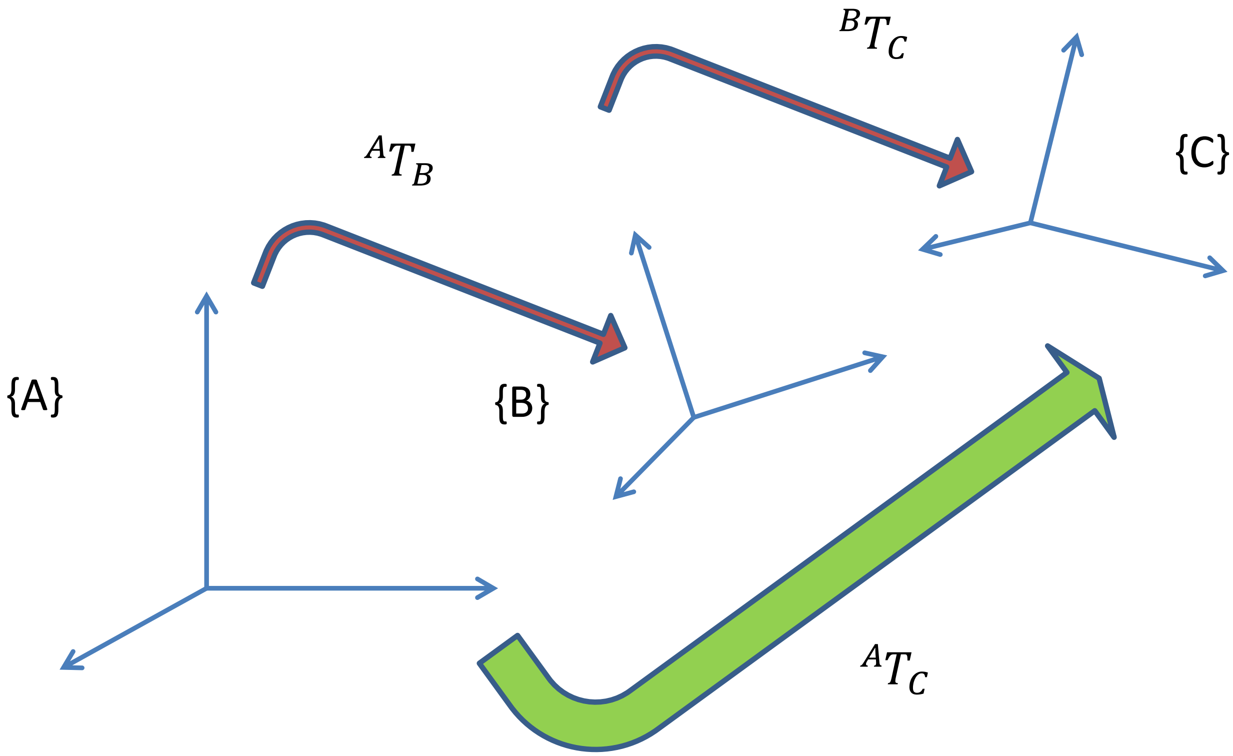
ROTATION

$$^{A}Rot_{B} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & 0 \\ r_{21} & r_{22} & r_{23} & 0 \\ r_{31} & r_{32} & r_{33} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

ROTO-TRASLATIONS

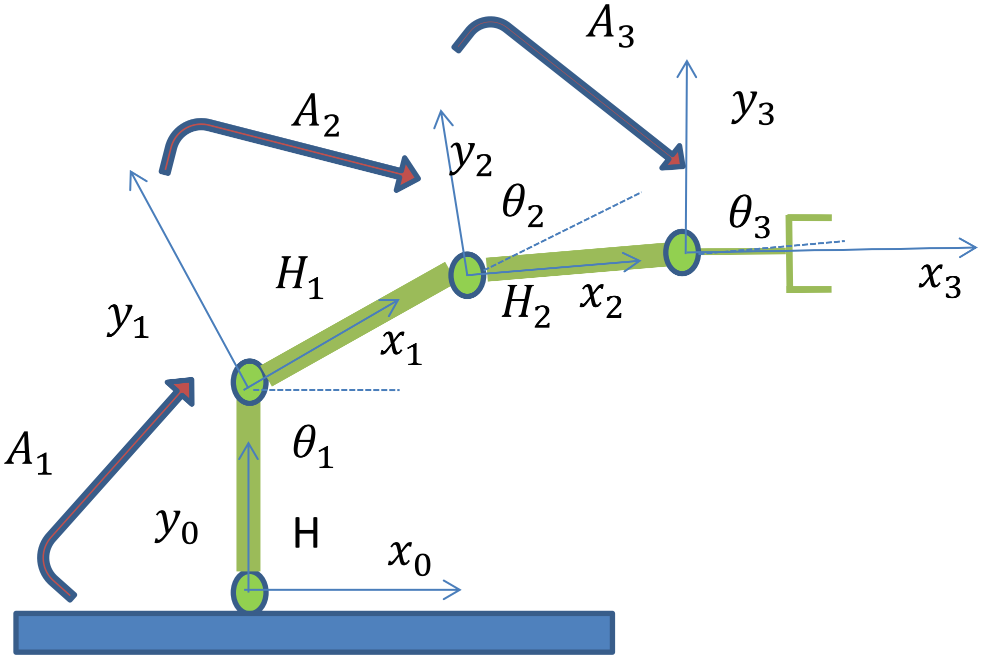
$$^{A}Rot - Trasl_{B} = \begin{pmatrix} r_{11} & r_{12} & r_{13} & dx \\ r_{21} & r_{22} & r_{23} & dy \\ r_{31} & r_{32} & r_{33} & dz \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Homogeneous Transformations(III) Composition…



$$^{A}T_{C} = {}^{A}T_{B}\,{}^{B}T_{C}$$

Exercise 2D



$$A_1 = \begin{pmatrix} \cos\theta_1 & -sin\theta_1 & 0 \\ sin\theta_1 & \cos\theta_1 & H \\ 0 & 0 & 1 \end{pmatrix}$$

$$A_2 = \begin{pmatrix} \cos\theta_2 & -sin\theta_2 & H_1 \\ sin\theta_2 & \cos\theta_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

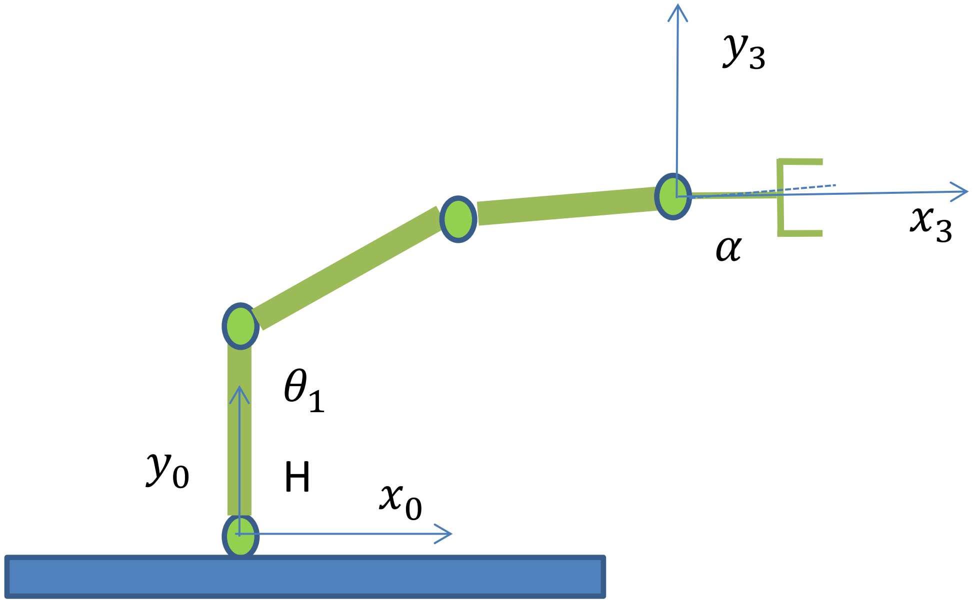
$$A_3 = \begin{pmatrix} \cos\theta_3 & -sin\theta_3 & H_2 \\ sin\theta_3 & \cos\theta_3 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$T_3 = A_1 A_2 A_3$$

$$T_3 = \begin{pmatrix} \cos(\theta_1 + \theta_2 + \theta_3) & -sin(\theta_1 + \theta_2 + \theta_3) & \cos(\theta_1 + \theta_2)\, H_2 + \cos(\theta_1)\, H_1 \\ sin(\theta_1 + \theta_2 + \theta_3) & \cos(\theta_1 + \theta_2 + \theta_3) & \mathrm{H} + \sin(\theta_1 + \theta_2)\, H_2 + \sin(\theta_1)\, H_1 \\ 0 & 0 & 1 \end{pmatrix}$$

Exercise 2D - Inverse kinematics

Computing end-effector coordinates with respect to the joint
T^* is the end-effector transformation



$$T^* = \begin{pmatrix} \cos\alpha & -\sin\alpha & x \\ sin\,\alpha & \cos\alpha & y \\ 0 & 0 & 1 \end{pmatrix}$$

With $T^* = T_3$ we can obtain:

$$\begin{cases} \alpha = \theta_1 + \theta_2 + \theta_3 \\ x = \cos(\theta_1 + \theta_2)\, H_2 + \cos(\theta_1)\, H_1 \\ y - H = \sin(\theta_1 + \theta_2)\, H_2 + \sin(\theta_1)\, H_1 \end{cases}$$

Execise 2D Inverse Kinematics(II)

By summing the suare values:

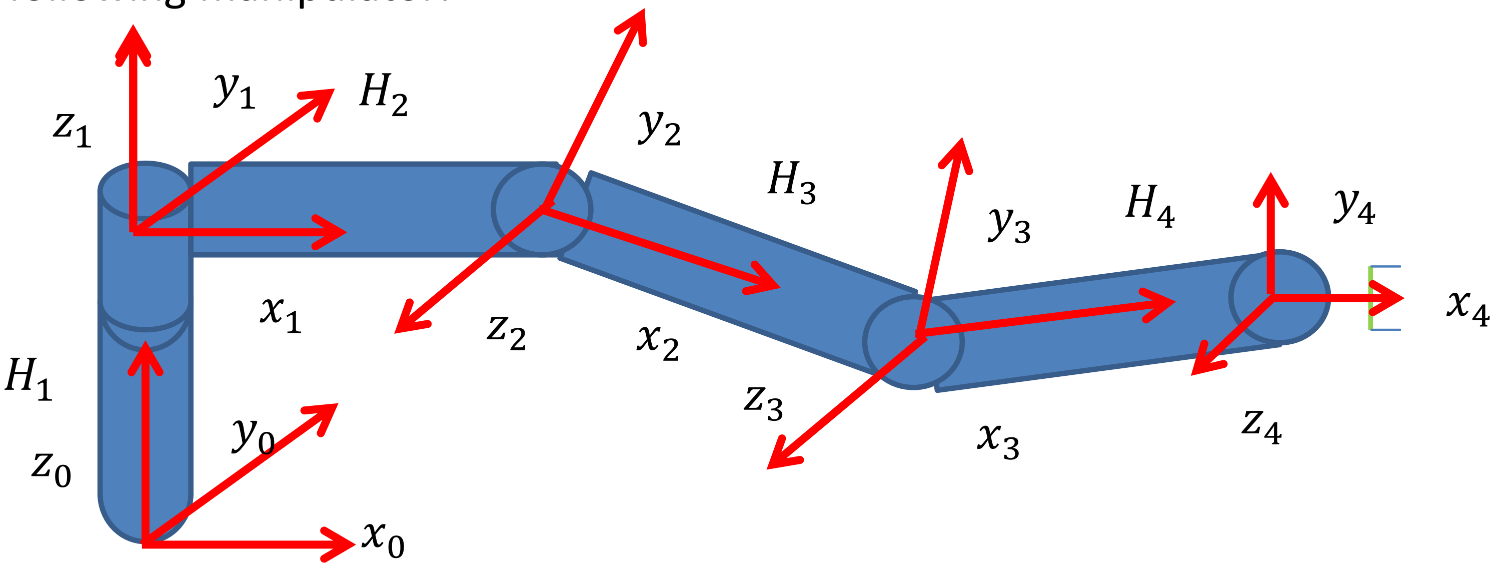
$$x^2 + (y - H)^2 = H_2^2 + H_1^2 + 2H_1H_2 \cos\theta_2$$

$$\begin{cases} \cos\theta_2 = (x^2 + (y - H)^2 - H_2^2 - H_1^2 \ )/2H_1H_2 \\ \sin\theta_2 = \pm\sqrt{1 - (\cos\theta_2)^2} \end{cases}$$

Then using θ_2 it is possible to compute the values of θ_1 e θ_3

Exercise 3D - Kinematics

Given the following manipulator:

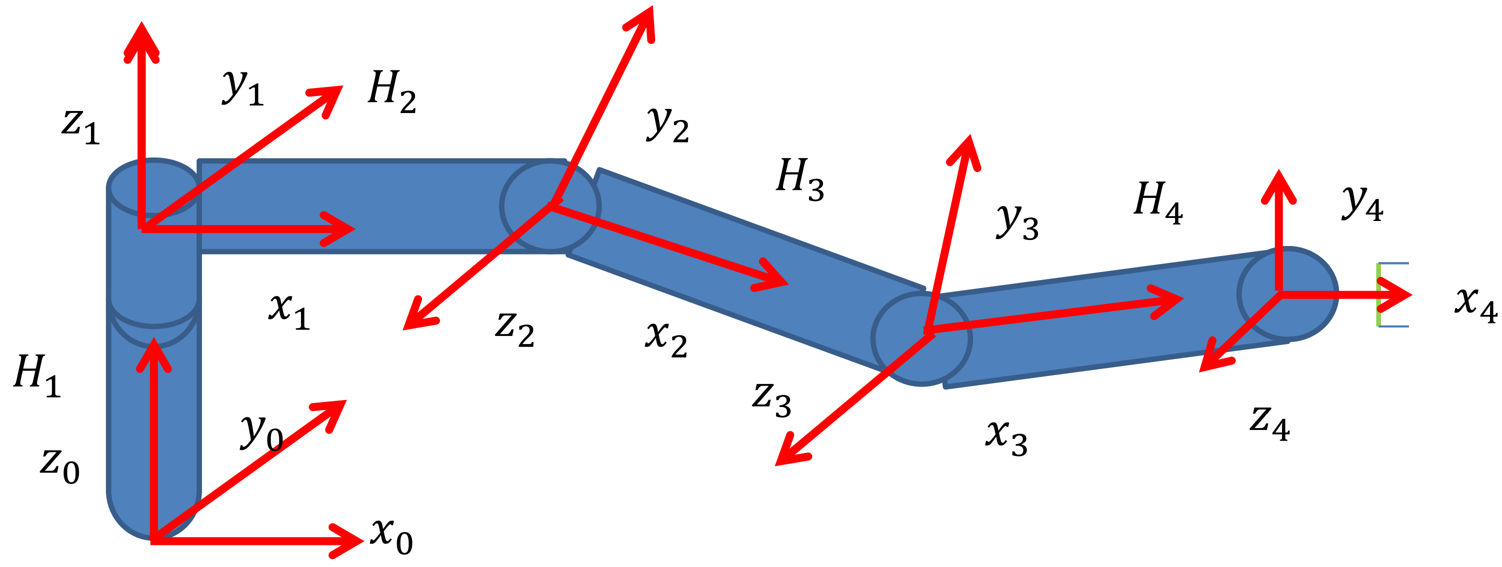


Compute the transformation matrix from one reference frame to the next one

$$A_1 = \begin{pmatrix} \cos\theta_1 & -sin\theta_1 & 0 & 0 \\ sin\theta_1 & \cos\theta_1 & 0 & 0 \\ 0 & 0 & 1 & H_1 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$A_3 = \begin{pmatrix} \cos\theta_3 & -sin\theta_3 & 0 & H_3 \\ sin\theta_3 & \cos\theta_3 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Exercise 3D kinematics (II)



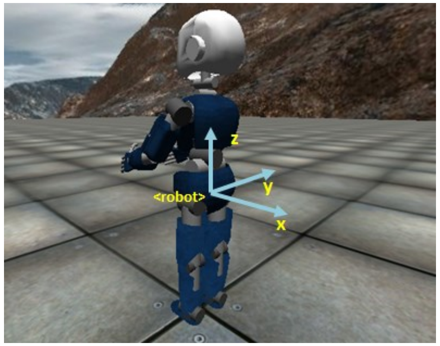
$$A_2 = \begin{pmatrix} \cos\theta_2 & -sin\theta_2 & 0 & H_2 \\ 0 & 0 & -1 & 0 \\ sin\theta_2 & \cos\theta_2 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \qquad A_4 = \begin{pmatrix} \cos\theta_4 & -sin\theta_4 & 0 & H_4 \\ sin\theta_4 & \cos\theta_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

T_4 transformation is:

$$T_4 = A_1 A_2 A_3 A_4$$

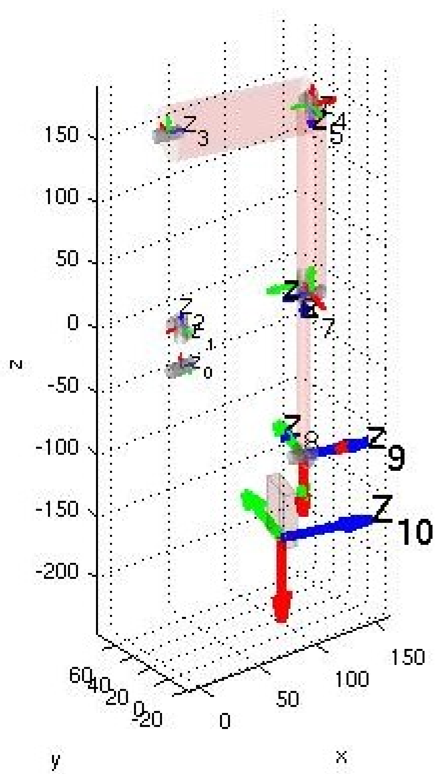
iCub robot kinematics

Reference frame

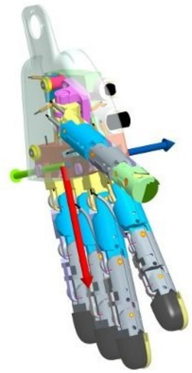


Denavit-Hartemberg parameters

Link i / H – D	Ai (mm)	d_i (mm)	alpha_i (rad)	theta_i (deg)
i = 0	32	0	pi/2	-22 -> 84
i = 1	0	-5.5	pi/2	-90 + (-39 -> 39)
i = 2	23.3647	-143.3	-pi/2	105 + (-59 -> 59)
i = 3	0	107.74	-pi/2	90 + (5 -> -95)
i = 4	0	0	pi/2	-90 + (0 -> 160.8)
i = 5	15	152.28	-pi/2	75 + (-37 -> 100)
i = 6	-15	0	pi/2	5.5 -> 106
i = 7	0	137.3	pi/2	-90 + (-50 -> 50)
i = 8	0	0	pi/2	90 + (10 -> -65)
i = 9	62.5	-16	0	(-25 -> 25)



Left arm



Reference frame placed on the end effector

- http://wiki.icub.org/wiki/ICubFowardKinematics_left