Models of computation (MOD)

Appello straordinario – April 3, 2019

[Ex. 1] Suppose we add to IMP the command **repeat** c **until** b, whose denotational semantics is defined recursively as:

$$\mathcal{C}[\![\mathbf{repeat}\ c\ \mathbf{until}\ b]\!]\sigma = (\lambda\sigma'.\mathcal{B}[\![b]\!]\sigma' \ \rightarrow\ \sigma'\ ,\ \mathcal{C}[\![\mathbf{repeat}\ c\ \mathbf{until}\ b]\!]\sigma')^*\ (\mathcal{C}[\![c]\!]\sigma)$$

1. Define the operational semantics of the new construct.
2. Extend the proof of determinacy of the operational semantics taking into account the new construct.
3. Define the function $\Gamma_{c,b}$ such that $\mathcal{C}[\![\mathbf{repeat}\ c\ \mathbf{until}\ b]\!] = \mathit{fix}\ \Gamma_{c,b}$.
4. Compute the denotational semantics of **repeat** $x := x + 1$ **until true**.

[Ex. 2] Consider the $\mathrm{CPO}_\perp$ $\mathcal{D} \stackrel{\text{def}}{=} (\wp(\mathbb{N}), \subseteq)$ and the function $f : \wp(\mathbb{N}) \to \wp(\mathbb{N})$ such that $f(X) \stackrel{\text{def}}{=} \{y \mid \exists x \in X.\ y \leq x\}$, where $\leq$ is the usual total order on $\mathbb{N}$.

1. Is f monotone?
2. Is f continuous?
3. What is the least fixpoint of f? Does f have other fixpoints?

[Ex. 3] Let us consider the CCS processes

$$p \stackrel{\text{def}}{=} \mathbf{rec}\ x.(\tau.x + \beta.\mathbf{nil} + \alpha.(\mathbf{rec}\ y.\ \alpha.x + \tau.y)) \qquad r \stackrel{\text{def}}{=} \mathbf{rec}\ u.(\tau.u)$$

$$q \stackrel{\text{def}}{=} \mathbf{rec}\ z.(\alpha.\alpha.z + \beta.\mathbf{nil})$$

1. Draw the LTSs of the processes p and $s \stackrel{\text{def}}{=} q|r$.
2. Show that p and s are not strong bisimilar.
3. Prove that p and s are weak bisimilar.

[Ex. 4] Let snd_v and rec_v two predicates representing the fact that the value v is sent and received, respectively.

1. Write a LTL formula expressing the property that everytime the value 1 is sent then it is eventually received.
2. Write a CTL formula expressing the property that there is a future where the value 2 is sent until it is received.
3. Write a μ-calculus formula expressing the property that the value 3 is never received.