Models of computation (MOD) 2016/17
Exam – July 25, 2017

[Ex. 1] Extend IMP with the command **let** $x = a$ **in** c whose operational semantics if defined by the rule

$$\frac{\langle a, \sigma\rangle \to n \quad \langle c, \sigma[^n/_x]\rangle \to \sigma'}{\langle \mathbf{let}\ x = a\ \mathbf{in}\ c, \sigma\rangle \to \sigma'[^{\sigma(x)}/_x]}$$

1. Write a command c such that for any memory σ we have:

   $$\langle c, \sigma\rangle \to \sigma[^{\sigma(y)}/_z, ^{\sigma(z)}/_y]$$

   (i.e., the values of y and z are switched without changing the content of a third variable)

2. Define the denotational semantics of the new construct.

3. Extend the proof of correctness between the operational and denotational semantics of commands to take the new construct into account.

4. Extend the proof of completeness between the operational and denotational semantics of commands to take the new construct into account.

[Ex. 2] Let $(D, \sqsubseteq_D)$ be a $\mathrm{CPO}_\perp$, and $f, g : D \to D$ be two monotone functions. Let $(f \odot g) : D \to D$ be the function that assigns to each element $d \in D$ the smallest element between $f(d)$ and $g(d)$, i.e.

$$(f \odot g)(d) \stackrel{\text{def}}{=} \begin{cases} f(d) & \text{if } f(d) \sqsubseteq_D g(d) \\ g(d) & \text{if } g(d) \sqsubseteq_D f(d) \\ \perp_D & \text{otherwise} \end{cases}$$

1. Show that in general $(f \odot g)$ is not monotone.

2. Prove that if D is totally ordered then $(f \odot g)$ is a monotone function.

[Ex. 3] Consider that the pi-calculus process

$$p \stackrel{\text{def}}{=} (x)\,(\ \overline{x}x.\mathbf{nil} \mid !x(y).\overline{y}y.\mathbf{nil}\ )$$

1. Draw the LTS of p assuming that $\mathbf{nil}|q = q$ and $q|!q = !q$ for any q.

2. Prove that p is weak early/late ground bisimilar to **nil**.

[Ex. 4] Three dogs live in a house with two couches and a front garden. Let $couch_{i,j}$ represent the predicate "the dog i sits on couch j" and $garden_i$ represent the predicate "the dog i plays in the front garden".

1. Write an LTL formula expressing the fact that whenever dog 1 plays in the garden then he keeps playing until he sits on some couch (but he may also play forever).

2. Write a CTL formula expressing the fact that dog 2 eventually plays in the garden whenever couch 1 is occupied by another dog.

3. Write a μ-calculus formula expressing the fact that no more than one couch is occupied at any time by dog 3.