Models of computation (MOD) 2015/16
Exam – July 25, 2016

[**Ex. 1**] Suppose we extend IMP with the new arithmetic expression $c \Rightarrow a$, whose operational semantics is defined by the rules:

$$\frac{\langle c, \sigma\rangle \to \sigma' \quad \langle a, \sigma'\rangle \to n}{\langle c \Rightarrow a, \sigma\rangle \to n} \qquad \frac{\langle a, \sigma\rangle \to n}{\langle c \Rightarrow a, \sigma\rangle \to n}$$

1. Extend the proof of termination of arithmetic expressions (normalisation) by taking into account the new construct, namely prove that:

$$\forall a.\ \forall \sigma.\ \exists m.\ \langle a, \sigma\rangle \to m$$

2. Prove that the introduction of the new construct makes no longer valid the determinacy property of arithmetic expressions, namely prove that

$$\neg\,(\,\forall a.\ \forall \sigma.\ \forall n, m.\ (\,(\,\langle a, \sigma\rangle \to n \wedge \langle a, \sigma\rangle \to m\,)\ \Rightarrow\ n = m\,)\,)$$

[**Ex. 2**] Consider the HOFL term

$$t \stackrel{\text{def}}{=} \textbf{rec}\ g.\ \lambda x.\ \lambda y.\ \textbf{if}\ x\ \textbf{then}\ ((g\ x)\ y)\ \textbf{else}\ y$$

1. Under which hypothesis is t typable?
2. Let $t_0, t_1 : \mathit{int}$ be two closed terms. Under which hypotheses $((t\ t_0)\ t_1)\downarrow$?
3. Compute the (lazy) denotational semantics of t.

[**Ex. 3**] Consider the CCS processes

$$\begin{array}{llll} p \stackrel{\text{def}}{=} \textbf{rec}\ x.\ (\alpha.x + \gamma.\textbf{nil}) & & r \stackrel{\text{def}}{=} (p|q)\backslash\gamma \\ q \stackrel{\text{def}}{=} \textbf{rec}\ y.\ (\beta.y + \overline{\gamma}.\textbf{nil}) & & s \stackrel{\text{def}}{=} \textbf{rec}\ z.\ (\tau.\alpha.z + \tau.\beta.z + \tau.r) \end{array}$$

1. Draw the LTSs of processes r and s.
2. Are r and s weak bisimilar?

[**Ex. 4**] In the Dark Ages, Harvard, Dartmouth, and Yale admitted only male students. Assume that, at that time, 80 percent of the sons of Harvard men went to Harvard and the rest went to Yale, 40 percent of the sons of Yale men went to Yale, and the rest split evenly between Harvard and Dartmouth; and of the sons of Dartmouth men, 70 percent went to Dartmouth, 20 percent to Harvard, and 10 percent to Yale.
Use DTMC to find the probability that the grandson of a man from Harvard went to Harvard.[1]

[1] Exercise taken from some notes on stochastic processes by Takis Konstantopoulos.