# Models of computation (MOD) 2014/15

Appello straordinario – March 30, 2016

**[Ex. 1]**

Extend the syntax of IMP arithmetic expressions with the construct $a_0 \text{ or } a_1$ and $a_0 \text{ and } a_1$, whose operational semantics is defined by the rules

$$\frac{\langle a_0, \sigma\rangle \to n}{\langle a_0 \text{ or } a_1, \sigma\rangle \to n} \qquad \frac{\langle a_1, \sigma\rangle \to n}{\langle a_0 \text{ or } a_1, \sigma\rangle \to n} \qquad \frac{\langle a_0, \sigma\rangle \to n \quad \langle a_1, \sigma\rangle \to n}{\langle a_0 \text{ and } a_1, \sigma\rangle \to n}$$

1. Show that the operational semantics of arithmetic expressions is no longer deterministic.
2. Re-define the denotational semantics of arithmetic expressions in the form $\mathscr{A} : Aexp \to (\Sigma \to \wp(\mathbb{Z}))$, to account for non-determinism.
3. Show an expression $a$ such that $\forall\sigma.\ \mathscr{A}[\![a]\!]\sigma = \emptyset$.
4. Prove that $\forall a, \sigma, n$ it holds $P(\langle a, \sigma\rangle \to n) \stackrel{\text{def}}{=} n \in \mathscr{A}[\![a]\!]\sigma$
5. Prove that $\forall a$ it holds $Q(a) \stackrel{\text{def}}{=} \forall\sigma.\ \forall m.\ (\, m \in \mathscr{A}[\![a]\!]\sigma \Rightarrow \langle a, \sigma\rangle \to m\,)$.

**[Ex. 2]**

Let $(D, \sqsubseteq)$ be a $\text{CPO}_\perp$ and $f : D \to D$ be a continuous function on $D$.
It is immediate to check that $f^2 = f \circ f$ is also continuous.
Prove that $\text{fix } f = \text{fix } f^2$.

**[Ex. 3]**

Let us consider the HOFL term

$$t \stackrel{\text{def}}{=} \textbf{rec } f.\ \lambda x.\ \textbf{if } (f\ x) \textbf{ then } (f\ (x+1)) \textbf{ else } (f\ (x-1))$$

1. Find the principal type of $t$.
2. Compute the denotational semantics of $t$.

**[Ex. 4]**

Enrich the operational semantics of CCS with the additional rule for synchronizing multiple inputs

$$\frac{P \xrightarrow{a} P' \quad Q \xrightarrow{a} Q'}{P|Q \xrightarrow{a} P'|Q'}$$

Prove that in general $(P|Q)|R$ is no longer strong bisimilar to $P|(Q|R)$.