# Models of computation (MOD) 2013/14
Exam – June 4, 2014

[**Ex. 1**]

1. Prove that, according to the operational semantics of IMP, for any boolean expression $b$, command $c$ and stores $\sigma$, $\sigma'$

$$\langle \textbf{while } b \textbf{ do } c, \sigma \rangle \to \sigma' \text{ implies } \langle b, \sigma' \rangle \to \mathit{false}$$

   Explain which induction principle you exploit in the proof.

2. Use structural induction to show that an arithmetic expression $a$ evaluates to the same value when evaluated on two states that agree on the free variables occurring in $a$.

[**Ex. 2**] Let $t$ be the HOFL term

$$t \stackrel{\text{def}}{=} \textbf{fst}(\ (\lambda x.x)\ (\ 1\ ,\ ((\textbf{rec} f.\ \lambda y.(f\ y))\ 2)\ )\ ).$$

Compute the type of $t$, its canonical form and its denotational semantics.

[**Ex. 3**]

1. Let us consider the CCS processes

$$p \stackrel{\text{def}}{=} \textbf{rec}\ x.\ (a.x + a.\textbf{nil}) \qquad q \stackrel{\text{def}}{=} \textbf{rec}\ y.\ (a.a.y + a.\textbf{nil})$$

   Draw the LTS for $p$ and $q$ and prove that $p \not\simeq q$ by exhibiting a formula in HM-logic that distinguishes between the two. (Hint: exploit the sub-formula $[a]\mathit{false}$ and explain its meaning.)

2. Let us consider the CCS processes

$$\begin{aligned} r &\stackrel{\text{def}}{=} a.(b.c.\textbf{nil} + b.\tau.c.\textbf{nil} + \tau.b.\textbf{nil} + b.\textbf{nil}) \\ s &\stackrel{\text{def}}{=} a.(b.c.\tau.\textbf{nil} + \tau.b.\textbf{nil}) + a.b.\textbf{nil} \end{aligned}$$

   Draw the LTS for $r$ and $s$ and prove that they are weakly observational congruent by exploiting the axioms presented in the course. At each step of the proof explain which axiom is used and where it is applied.

[**Ex. 4**] Consider a simple system in which a process repeatedly carries out some task. In order to complete its task the process needs access to a resource for part, but not all, of the time. We want to model the process and the resource as two separate PEPA agents: *Process* and *Resource*, respectively. The *Process* will undertake two activities consecutively: *get* with some rate $rg$, in cooperation with the *Resource*, and *task* at rate $rt$, representing the remainder of its processing task. Similarly the *Resource* will engage in two activities consecutively: *get*, at a rate $rg' > 2rg$ and *update*, at rate $ru$.

1. Give the PEPA specification of a system composed with two *Process*es that compete for one shared *Resource*.

2. What is the apparent rate of action *get* in the initial state of the system?

3. Draw the complete LTS (eight states) of the system or list all its transitions.