Models of computation (MOD) 2013/14

Mid-term exam – May 28, 2014

[Ex. 1] A *simulation relation* is a relation S such that if $p\ S\ q$ then

$$p \xrightarrow{\mu} p' \text{ implies } \exists q'.q \xrightarrow{\mu} q' \text{ with } p'\ S\ q'$$

1. Prove that the union $S_1 \cup S_2$ and the composition $S_1 \circ S_2$ of two simulations S_1 and S_2 are simulations, where

$$S_1 \circ S_2 \stackrel{\text{def}}{=} \{\ (p,q)\ \mid\ \exists r.\ p\ S_1\ r\ \wedge\ r\ S_2\ q\ \}$$

2. Let us write $p \precsim q$ if there exists a simulation S such that $p\ S\ q$. Given the CCS process $p \stackrel{\text{def}}{=} \alpha.\beta.\mathbf{nil} + \alpha.\mathbf{nil}$ define a CCS process q such that $p \precsim q$, $q \precsim p$ and $p \not\simeq q$ (where $\simeq$ denotes bisimilarity).

[Ex. 2] Given a natural number $n \geq 1$, let us define the family of CCS processes B^n_k for $0 \leq k \leq n$ by letting:

$$B^n_0 \stackrel{\text{def}}{=} in.B^n_1 \qquad B^n_k \stackrel{\text{def}}{=} in.B^n_{k+1} + \overline{out}.B^n_{k-1} \text{ for } 0 < k < n \qquad B^n_n \stackrel{\text{def}}{=} \overline{out}.B^n_{n-1}$$

Intuitively B^n_k represents a buffer with n positions of which k are occupied.

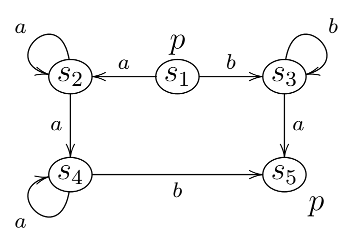
Prove that $B^n_0 \simeq \underbrace{B^1_0|B^1_0|\cdots|B^1_0}_{n}$ by providing a suitable bisimulation.

[Ex. 3] Let us extend the μ-calculus with the formulas $\langle A\rangle\phi$ and $[A]\phi$, where A is a set of labels: they represent, respectively, the ability to perform a transition with some label $a \in A$ and reach a state that satisfies ϕ, and the necessity to reach a state that satisfies ϕ after performing any transition with label $a \in A$.

1. Define the semantics $[\![\langle A\rangle\phi]\!]\,\rho$ and $[\![[A]\phi]\!]\,\rho$.
2. Compute the denotational semantics of the formulas

$$\phi_1 \stackrel{\text{def}}{=} \nu x.(((\langle\{a\}\rangle true \wedge \langle\{b\}\rangle true) \vee p) \wedge [\{a,b\}]x) \qquad \phi_2 \stackrel{\text{def}}{=} \mu x.p \vee \langle\{a,b\}\rangle x$$

and evaluate them on the LTS below:



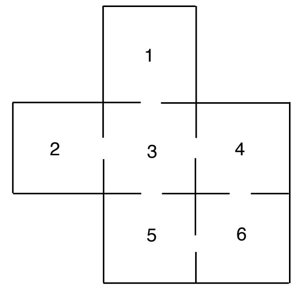


Figure 1: Maze

[Ex. 4] A mouse runs through the maze shown in Figure 1. At each step it stays in the room it is in or leaves the room by choosing at random one of the doors out of the room (all choices have equal probability).

1. Draw the transition graph and give the matrix P for this DTMC.
2. Show that it is ergodic and compute the steady state distribution.
3. Assuming the mouse is initially in room 1, what is the probability that it is in room 6 after three steps?