# Chapter 5
# **Association Analysis: Basic Concepts**

Introduction to Data Mining, 2$^{nd}$ Edition

by

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# Association Rule Mining

- Given a set of transactions, find rules that will predict the occurrence of an item based on the occurrences of other items in the transaction

**Market-Basket transactions**

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

**Example of Association Rules**

{Diaper} $\rightarrow$ {Beer},
{Milk, Bread} $\rightarrow$ {Eggs,Coke},
{Beer, Bread} $\rightarrow$ {Milk},

Implication means co-occurrence, not causality!

**Find groups of items which are frequently purchased together**

# Definition: Frequent Itemset

- **Itemset**
  - A collection of one or more items
    - Example: {Milk, Bread, Diaper}
  - k-itemset
    - An itemset that contains k items
- **Support count ($\sigma$)**
  - Frequency of occurrence of an itemset
  - E.g. $\sigma$({Milk, Bread,Diaper}) = 2
- **Support**
  - Fraction of transactions that contain an itemset
  - E.g. s({Milk, Bread, Diaper}) = 2/5
- **Frequent Itemset**
  - An itemset whose support is greater than or equal to a *minsup* threshold

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

# Definition: Association Rule

- **Association Rule**
  - An implication expression of the form X → Y, where X and Y are itemsets
  - Example:
    {Milk, Diaper} → {Beer}

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

- **Rule Evaluation Metrics**
  - Support (s)
    - Fraction of transactions that contain both X and Y
  - Confidence (c)
    - Measures how often items in Y appear in transactions that contain X

Example:

$$\{\text{Milk}, \text{Diaper}\} \Rightarrow \{\text{Beer}\}$$

$$s = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{|\text{T}|} = \frac{2}{5} = 0.4$$

$$c = \frac{\sigma(\text{Milk}, \text{Diaper}, \text{Beer})}{\sigma(\text{Milk}, \text{Diaper})} = \frac{2}{3} = 0.67$$

# Association Rule Mining Task

- Given a set of transactions T, the goal of association rule mining is to find all rules having
  - support ≥ *minsup* threshold
  - confidence ≥ *minconf* threshold

- Brute-force approach:
  - List all possible association rules
  - Compute the support and confidence for each rule
  - Prune rules that fail the *minsup* and *minconf* thresholds

  $\Rightarrow$ Computationally prohibitive!

# Mining Association Rules

| TID | Items |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

## Example of Rules:

{Milk,Diaper} $\rightarrow$ {Beer} (s=0.4, c=0.67)
{Milk,Beer} $\rightarrow$ {Diaper} (s=0.4, c=1.0)
{Diaper,Beer} $\rightarrow$ {Milk} (s=0.4, c=0.67)
{Beer} $\rightarrow$ {Milk,Diaper} (s=0.4, c=0.67)
{Diaper} $\rightarrow$ {Milk,Beer} (s=0.4, c=0.5)
{Milk} $\rightarrow$ {Diaper,Beer} (s=0.4, c=0.5)

## Observations:

- All the above rules are binary partitions of the same itemset:
  {Milk, Diaper, Beer}
- Rules originating from the same itemset have identical support but can have different confidence
- Thus, we may decouple the support and confidence requirements

# Mining Association Rules

- Two-step approach:
  1. Frequent Itemset Generation
     - Generate all itemsets whose support $\geq$ minsup
  2. Rule Generation
     - Generate high confidence rules from each frequent itemset, where each rule is a binary partitioning of a frequent itemset

- Frequent itemset generation is still computationally expensive

# Basic Apriori Algorithm

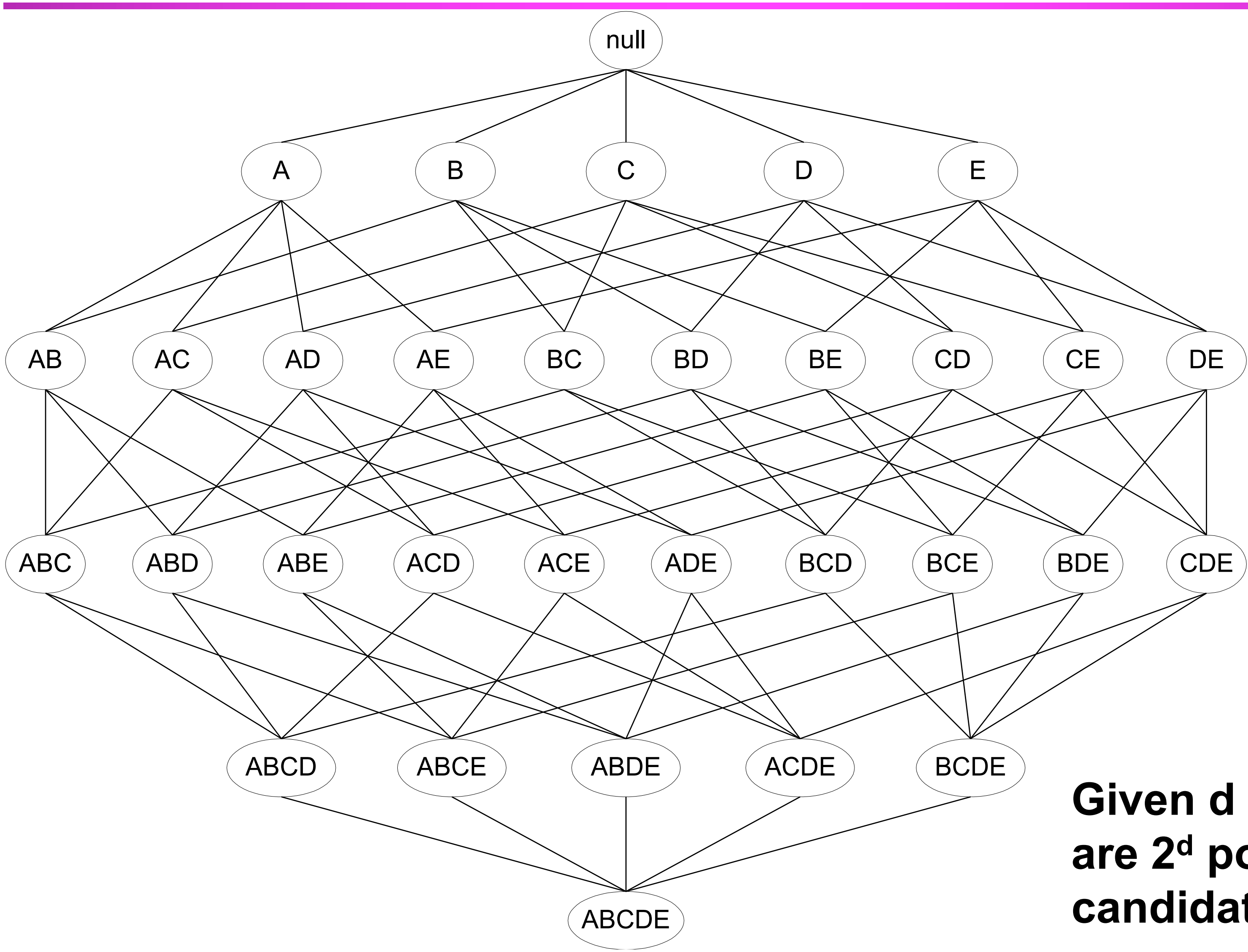
## Problem Decomposition

① Find the *frequent itemsets*: the sets of items that satisfy the support constraint

- A subset of a frequent itemset is also a frequent itemset, i.e., if {*A*,*B*} is a frequent itemset, both {*A*} and {*B*} should be a frequent itemset
- Iteratively find frequent itemsets with cardinality from 1 to *k* (*k*-itemset)

② Use the frequent itemsets to generate association rules.

# Frequent Itemset Generation




**Given d items, there are $2^d$ possible candidate itemsets**

# Frequent Itemset Generation

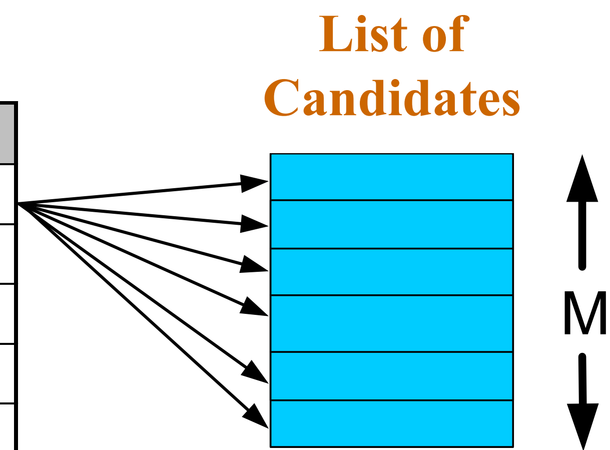
- Brute-force approach:
  - Each itemset in the lattice is a candidate frequent itemset
  - Count the support of each candidate by scanning the database

**Transactions**

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

N

w




  - Match each transaction against every candidate
  - Complexity ~ O(NMw) => Expensive since $M = 2^d$ !!!

# Frequent Itemset Generation Strategies

- Reduce the number of candidates (M)
  - Complete search: $M=2^d$
  - Use pruning techniques to reduce M
- Reduce the number of transactions (N)
  - Reduce size of N as the size of itemset increases
- Reduce the number of comparisons (NM)
  - Use efficient data structures to store the candidates or transactions
  - No need to match every candidate against every transaction

# Reducing Number of Candidates

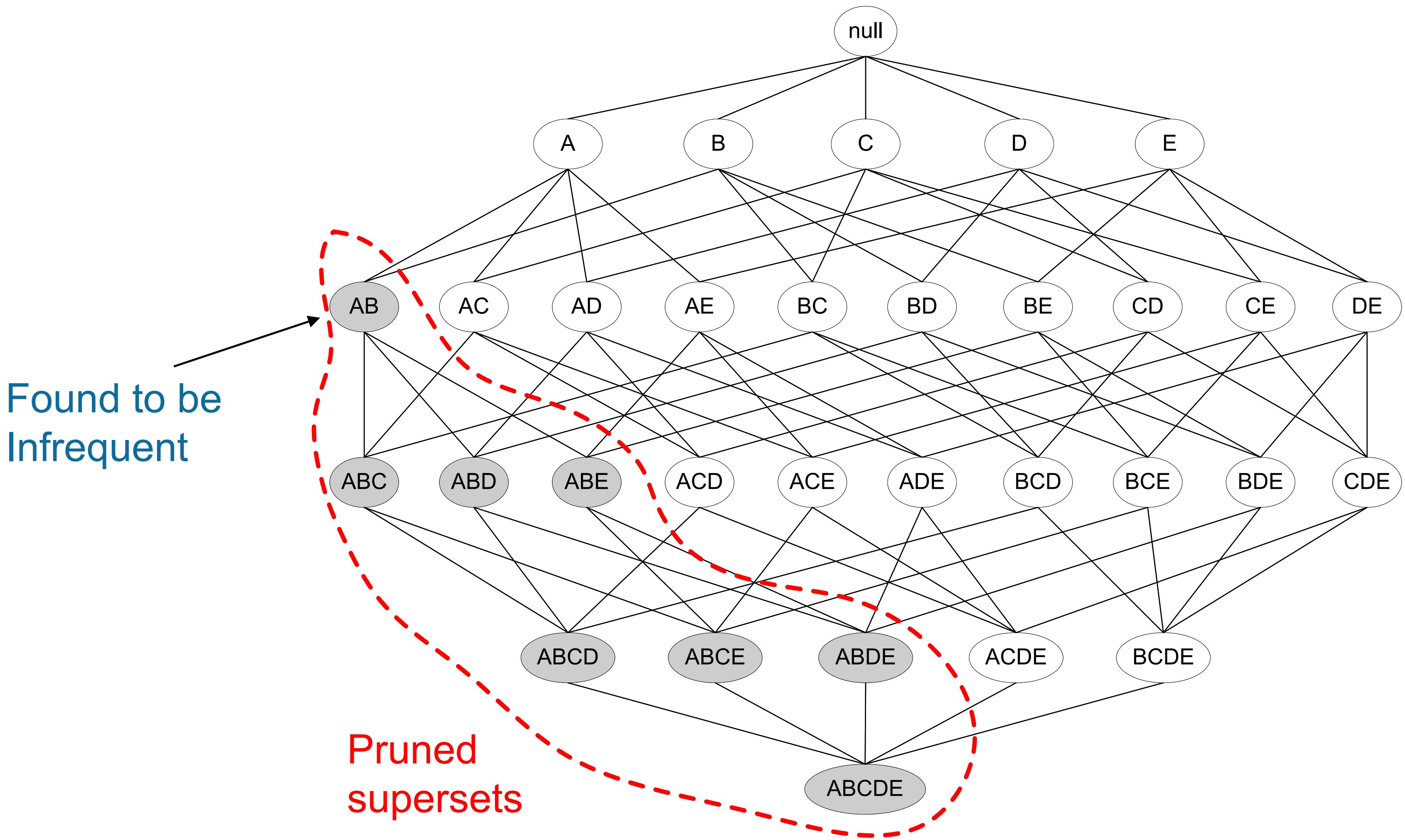
- Apriori principle:
  - If an itemset is frequent, then all of its subsets must also be frequent

- Apriori principle holds due to the following property of the support measure:

$$\forall X, Y : (X \subseteq Y) \Rightarrow s(X) \geq s(Y)$$

  - Support of an itemset never exceeds the support of its subsets
  - This is known as the anti-monotone property of support

# Illustrating Apriori Principle



# Illustrating Apriori Principle

| TID | Items |
|---|---|
| 1 | Bread, Milk |
| 2 | Beer, Bread, Diaper, Eggs |
| 3 | Beer, Coke, Diaper, Milk |
| 4 | Beer, Bread, Diaper, Milk |
| 5 | Bread, Coke, Diaper, Milk |

Items (1-itemsets)

| Item | Count |
|---|---|
| Bread | 4 |
| Coke | 2 |
| Milk | 4 |
| Beer | 3 |
| Diaper | 4 |
| Eggs | 1 |

Minimum Support = 3

# Illustrating Apriori Principle

| TID | Items |
|---|---|
| 1 | Bread, Milk |
| 2 | Beer, Bread, Diaper, Eggs |
| 3 | Beer, Coke, Diaper, Milk |
| 4 | Beer, Bread, Diaper, Milk |
| 5 | Bread, Coke, Diaper, Milk |

Items (1-itemsets)

| Item | Count |
|---|---|
| **Bread** | **4** |
| **Coke** | **2** |
| **Milk** | **4** |
| **Beer** | **3** |
| **Diaper** | **4** |
| **Eggs** | **1** |

Minimum Support = 3

# Illustrating Apriori Principle

| Item | Count |
|---|---|
| **Bread** | **4** |
| **Coke** | **2** |
| **Milk** | **4** |
| **Beer** | **3** |
| **Diaper** | **4** |
| **Eggs** | **1** |

Items (1-itemsets)

| Itemset |
|---|
| **{Bread,Milk}** |
| **{Bread, Beer }** |
| **{Bread,Diaper}** |
| **{Beer, Milk}** |
| **{Diaper, Milk}** |
| **{Beer,Diaper}** |

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

# Illustrating Apriori Principle

| Item | Count |
|---|---|
| **Bread** | **4** |
| **Coke** | **2** |
| **Milk** | **4** |
| **Beer** | **3** |
| **Diaper** | **4** |
| **Eggs** | **1** |

Items (1-itemsets)

| Itemset | Count |
|---|---|
| **{Bread,Milk}** | **3** |
| **{Beer, Bread}** | **2** |
| **{Bread,Diaper}** | **3** |
| **{Beer,Milk}** | **2** |
| **{Diaper,Milk}** | **3** |
| **{Beer,Diaper}** | **3** |

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

# Illustrating Apriori Principle

| Item | Count |
|---|---|
| Bread | 4 |
| Coke | 2 |
| Milk | 4 |
| Beer | 3 |
| Diaper | 4 |
| Eggs | 1 |

Items (1-itemsets)

| Itemset | Count |
|---|---|
| {Bread,Milk} | 3 |
| {Bread,Beer} | 2 |
| {Bread,Diaper} | 3 |
| {Milk,Beer} | 2 |
| {Milk,Diaper} | 3 |
| {Beer,Diaper} | 3 |

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Triplets (3-itemsets)

| Itemset |
|---|
| { Beer, Diaper, Milk} |
| { Beer,Bread,Diaper} |
| {Bread,Diaper,Milk} |
| { Beer, Bread, Milk} |

# Illustrating Apriori Principle

| Item | Count |
|---|---|
| Bread | 4 |
| Coke | 2 |
| Milk | 4 |
| Beer | 3 |
| Diaper | 4 |
| Eggs | 1 |

Items (1-itemsets)

| Itemset | Count |
|---|---|
| {Bread,Milk} | 3 |
| {Bread,Beer} | 2 |
| {Bread,Diaper} | 3 |
| {Milk,Beer} | 2 |
| {Milk,Diaper} | 3 |
| {Beer,Diaper} | 3 |

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Triplets (3-itemsets)

| Itemset | Count |
|---|---|
| { Beer, Diaper, Milk} | 2 |
| { Beer,Bread, Diaper} | 2 |
| {Bread, Diaper, Milk} | 2 |
| {Beer, Bread, Milk} | 1 |

# Illustrating Apriori Principle

| Item | Count |
|---|---|
| Bread | 4 |
| Coke | 2 |
| Milk | 4 |
| Beer | 3 |
| Diaper | 4 |
| Eggs | 1 |

Items (1-itemsets)

| Itemset | Count |
|---|---|
| {Bread,Milk} | 3 |
| {Bread,Beer} | 2 |
| {Bread,Diaper} | 3 |
| {Milk,Beer} | 2 |
| {Milk,Diaper} | 3 |
| {Beer,Diaper} | 3 |

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Triplets (3-itemsets)

| Itemset | Count |
|---|---|
| { Beer, Diaper, Milk} | 2 |
| { Beer,Bread, Diaper} | 2 |
| {Bread, Diaper, Milk} | 2 |
| {Beer, Bread, Milk} | 1 |

# Apriori Algorithm

- $F_k$: frequent k-itemsets
- $L_k$: candidate k-itemsets

- Algorithm
  - Let k=1
  - Generate $F_1$ = {frequent 1-itemsets}
  - Repeat until $F_k$ is empty
    - **Candidate Generation**: Generate $L_{k+1}$ from $F_k$
    - **Candidate Pruning**: Prune candidate itemsets in $L_{k+1}$ containing subsets of length k that are infrequent
    - **Support Counting**: Count the support of each candidate in $L_{k+1}$ by scanning the DB
    - **Candidate Elimination**: Eliminate candidates in $L_{k+1}$ that are infrequent, leaving only those that are frequent => $F_{k+1}$

# Candidate Generation: $F_{k-1} \times F_{k-1}$ Method

- Merge two frequent (k-1)-itemsets if their first (k-2) items are identical

- $F_3$ = {ABC,ABD,ABE,ACD,BCD,BDE,CDE}
  - Merge(**AB**C, **AB**D) = **AB**CD
  - Merge(**AB**C, **AB**E) = **AB**CE
  - Merge(**AB**D, **AB**E) = **AB**DE

  - Do not merge(**A**BD,**A**CD) because they share only prefix of length 1 instead of length 2

# Candidate Pruning

- Let $F_3$ = {ABC,ABD,ABE,ACD,BCD,BDE,CDE} be the set of frequent 3-itemsets

- $L_4$ = {ABCD,ABCE,ABDE} is the set of candidate 4-itemsets generated (from previous slide)

- Candidate pruning
  - Prune ABCE because ACE and BCE are infrequent
  - Prune ABDE because ADE is infrequent

- After candidate pruning: $L_4$ = {ABCD}

# Illustrating Apriori Principle

| Item | Count |
|---|---|
| **Bread** | **4** |
| **Coke** | **2** |
| **Milk** | **4** |
| **Beer** | **3** |
| **Diaper** | **4** |
| **Eggs** | **1** |

Items (1-itemsets)

| Itemset | Count |
|---|---|
| **{Bread,Milk}** | **3** |
| **{Bread,Beer}** | **2** |
| **{Bread,Diaper}** | **3** |
| **{Milk,Beer}** | **2** |
| **{Milk,Diaper}** | **3** |
| **{Beer,Diaper}** | **3** |

Pairs (2-itemsets)

(No need to generate candidates involving Coke or Eggs)

Minimum Support = 3

Triplets (3-itemsets)

| Itemset | Count |
|---|---|
| **{Bread, Diaper, Milk}** | **2** |

**Use of $F_{k-1}xF_{k-1}$ method for candidate generation results in only one 3-itemset. This is eliminated after the support counting step.**

# Alternate $F_{k-1}$ x $F_{k-1}$ Method

- Merge two frequent (k-1)-itemsets if the last (k-2) items of the first one is identical to the first (k-2) items of the second.

- $F_3$ = {ABC,ABD,ABE,ACD,BCD,BDE,CDE}
  - Merge(A**BC**, **BC**D) = A**BC**D
  - Merge(A**BD**, **BD**E) = A**BD**E
  - Merge(A**CD**, **CD**E) = A**CD**E
  - Merge(B**CD**, **CD**E) = B**CD**E

# Candidate Pruning for Alternate $F_{k-1} \times F_{k-1}$ Method

- Let $F_3$ = {ABC,ABD,ABE,ACD,BCD,BDE,CDE} be the set of frequent 3-itemsets
- $L_4$ = {ABCD,ABDE,ACDE,BCDE} is the set of candidate 4-itemsets generated (from previous slide)
- Candidate pruning
  - Prune ABDE because ADE is infrequent
  - Prune ACDE because ACE and ADE are infrequent
  - Prune BCDE because BCE
- After candidate pruning: $L_4$ = {ABCD}

# Support Counting of Candidate Itemsets

- Scan the database of transactions to determine the support of each candidate itemset
  - Must match every candidate itemset against every transaction, which is an expensive operation

| TID | Items |
|---|---|
| 1 | Bread, Milk |
| 2 | Beer, Bread, Diaper, Eggs |
| 3 | Beer, Coke, Diaper, Milk |
| 4 | Beer, Bread, Diaper, Milk |
| 5 | Bread, Coke, Diaper, Milk |

| Itemset |
|---|
| { Beer, Diaper, Milk} |
| { Beer,Bread,Diaper} |
| {Bread, Diaper, Milk} |
| { Beer, Bread, Milk} |

# Support Counting of Candidate Itemsets

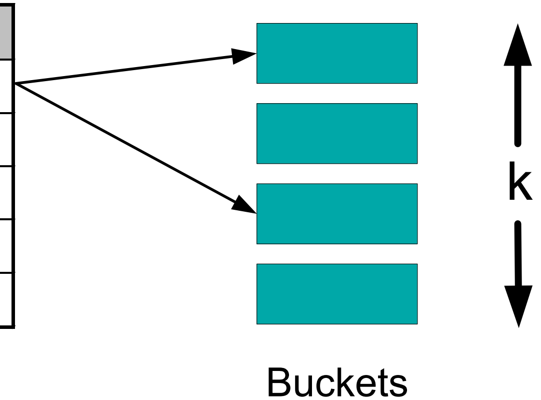
- To reduce number of comparisons, store the candidate itemsets in a hash structure
  - Instead of matching each transaction against every candidate, match it against candidates contained in the hashed buckets

**Transactions**

N

| *TID* | *Items* |
|---|---|
| 1 | Bread, Milk |
| 2 | Bread, Diaper, Beer, Eggs |
| 3 | Milk, Diaper, Beer, Coke |
| 4 | Bread, Milk, Diaper, Beer |
| 5 | Bread, Milk, Diaper, Coke |

**Hash Structure**

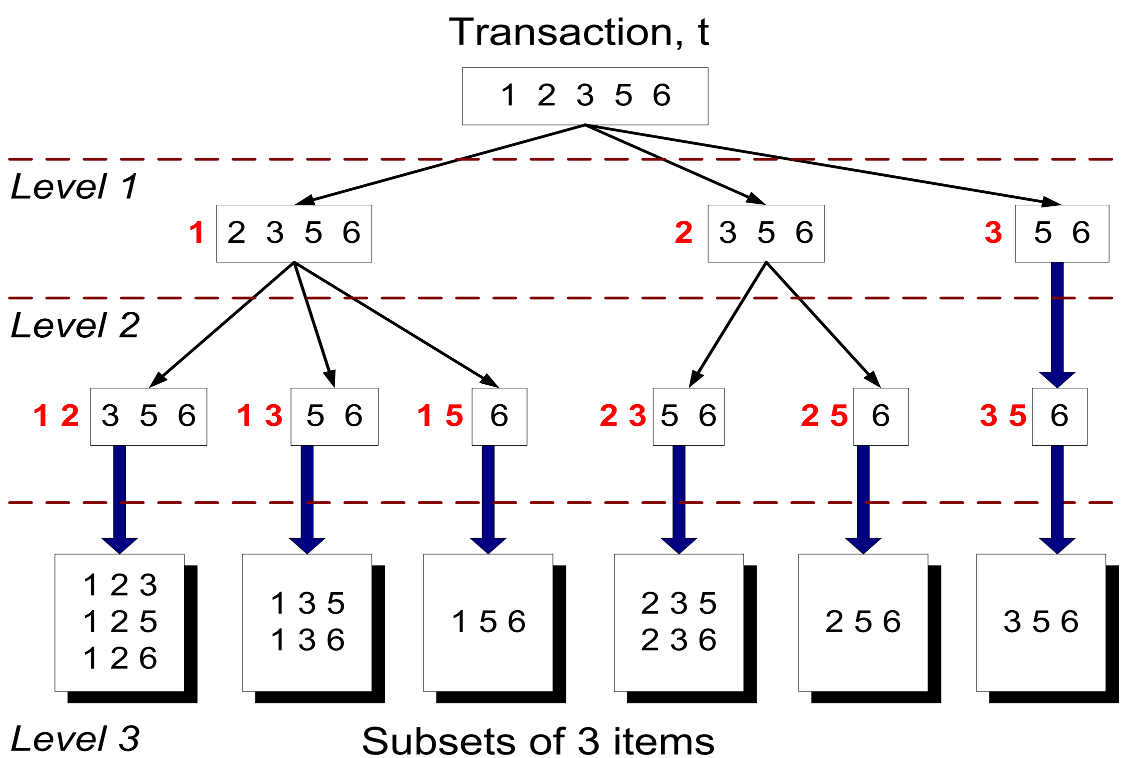


# Support Counting: An Example

**Suppose you have 15 candidate itemsets of length 3:**

**{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}**

**How many of these itemsets are supported by transaction (1,2,3,5,6)?**



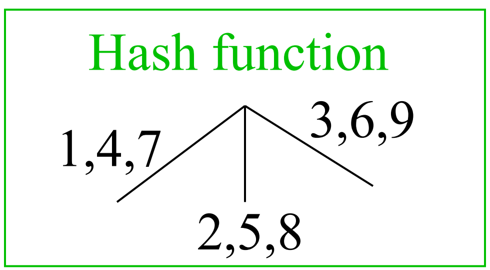
# Support Counting Using a Hash Tree

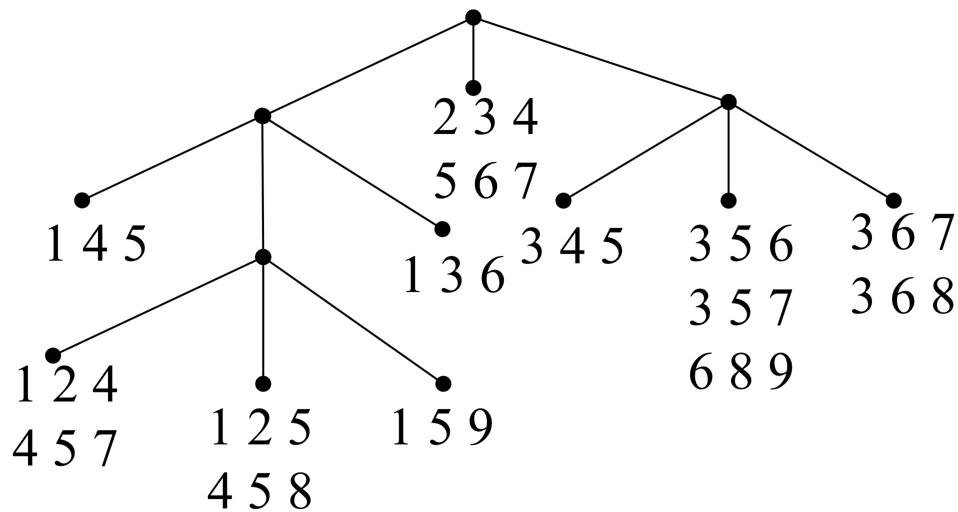
Suppose you have 15 candidate itemsets of length 3:

{1 4 5}, {1 2 4}, {4 5 7}, {1 2 5}, {4 5 8}, {1 5 9}, {1 3 6}, {2 3 4}, {5 6 7}, {3 4 5}, {3 5 6}, {3 5 7}, {6 8 9}, {3 6 7}, {3 6 8}

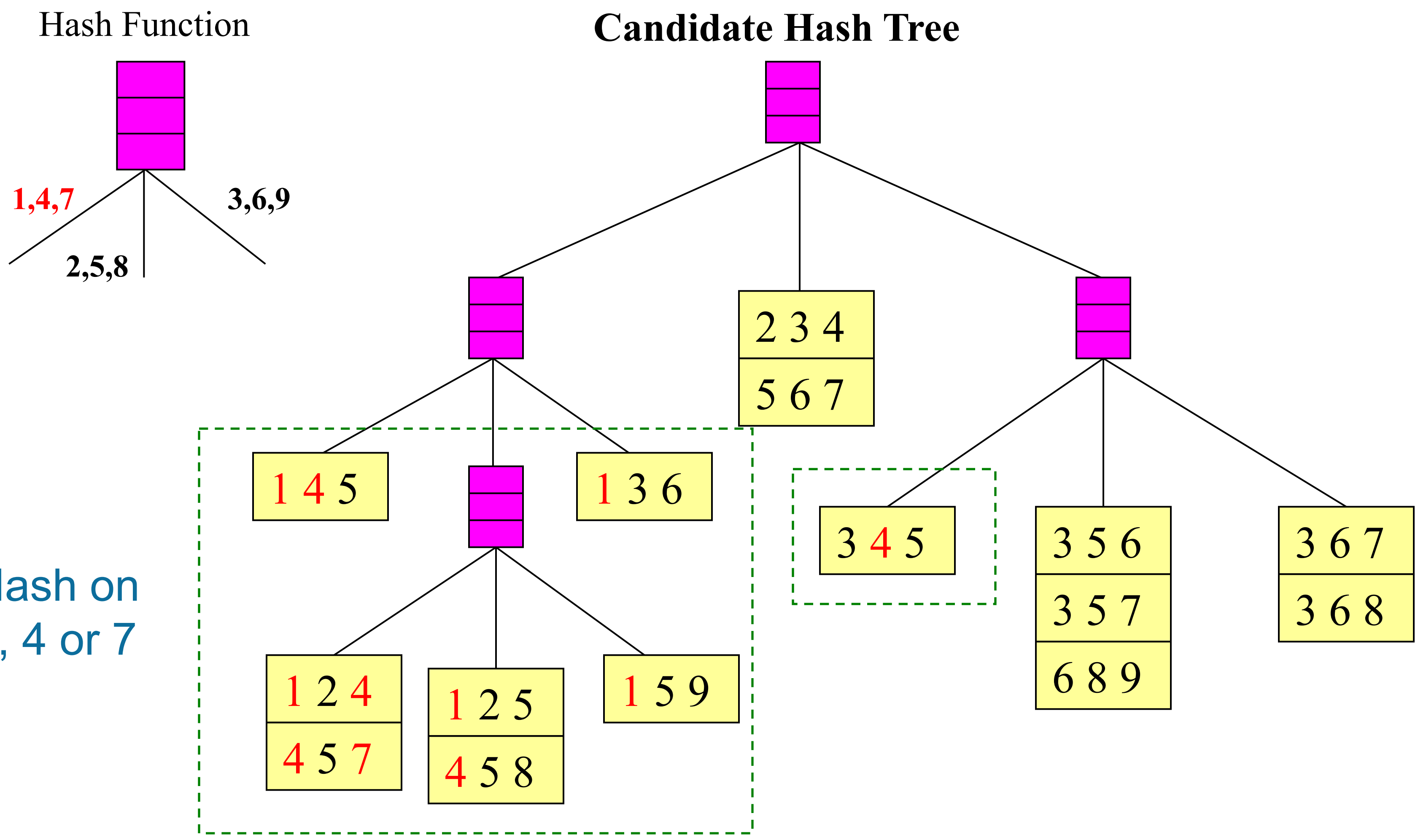
You need:

- Hash function
- Max leaf size: max number of itemsets stored in a leaf node (if number of candidate itemsets exceeds max leaf size, split the node)



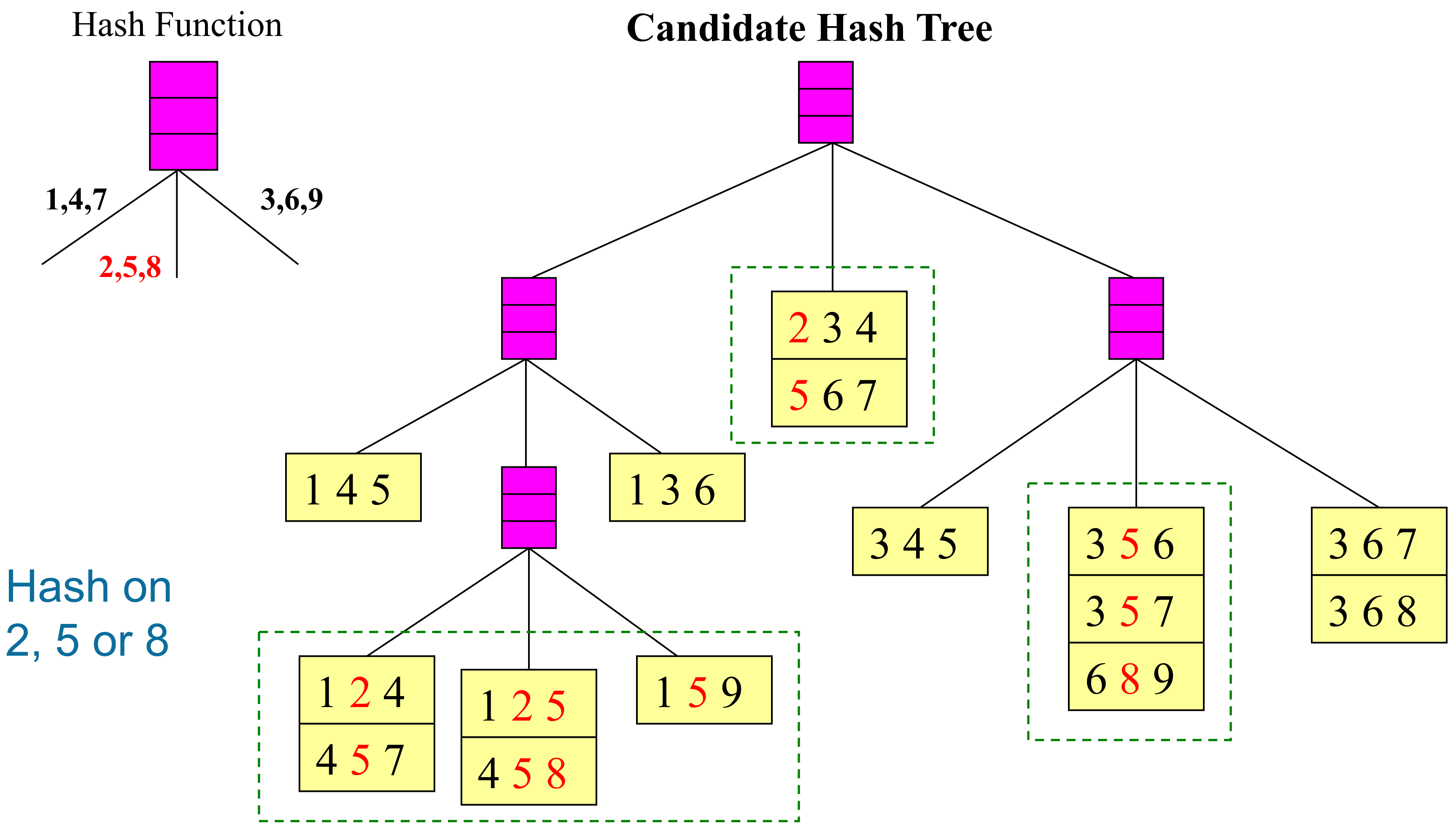



# Support Counting Using a Hash Tree

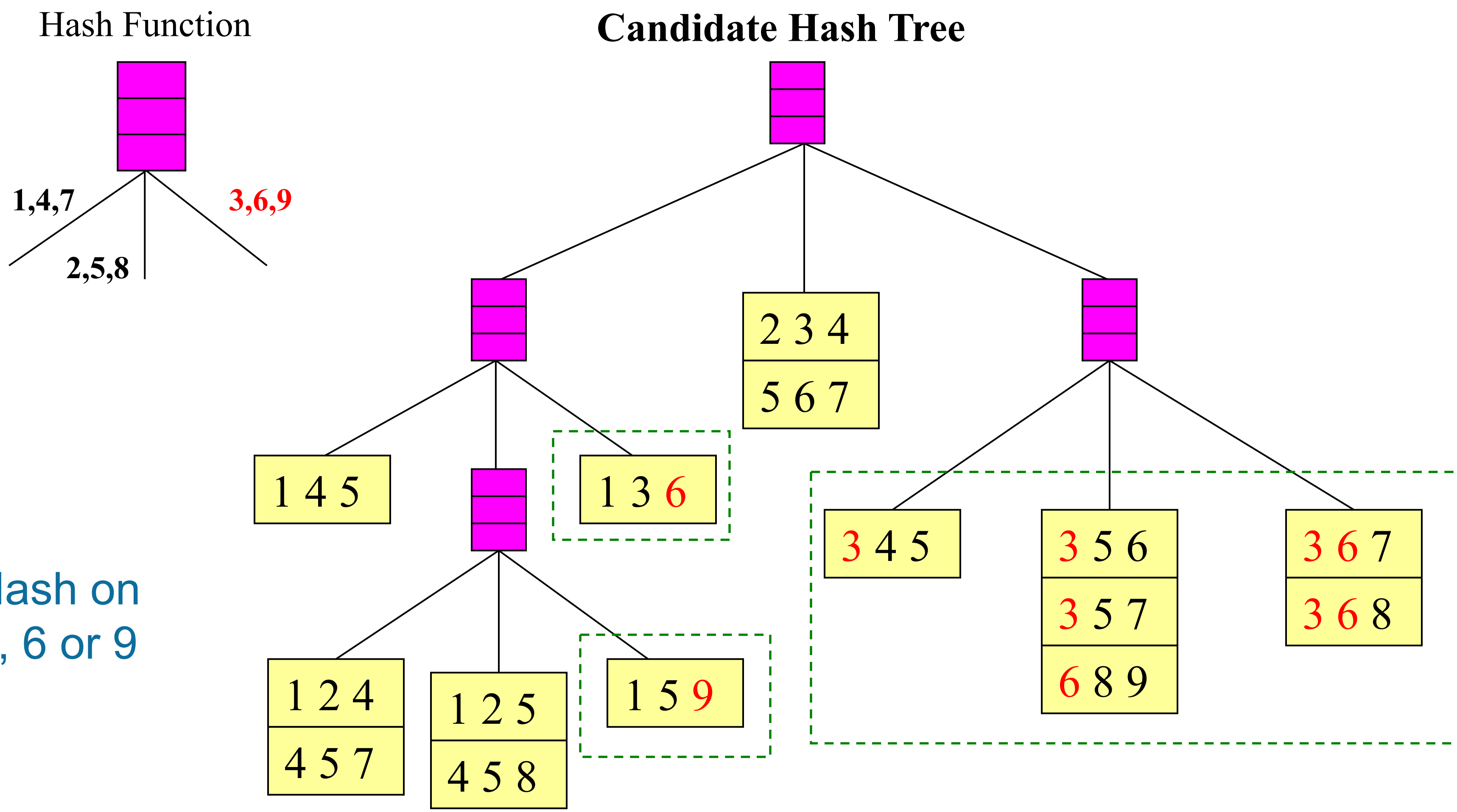



Hash on
1, 4 or 7

# Support Counting Using a Hash Tree

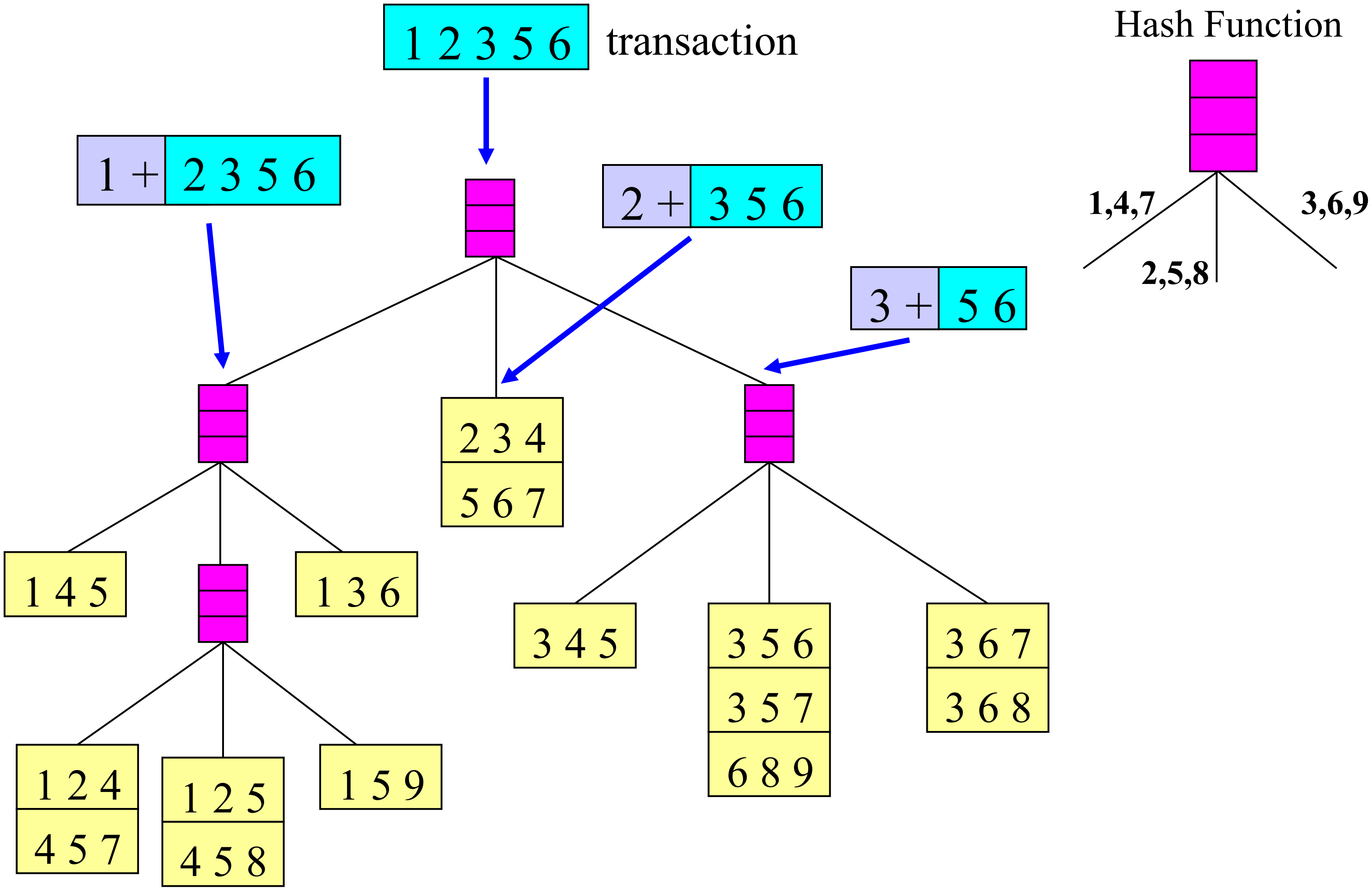


# Support Counting Using a Hash Tree

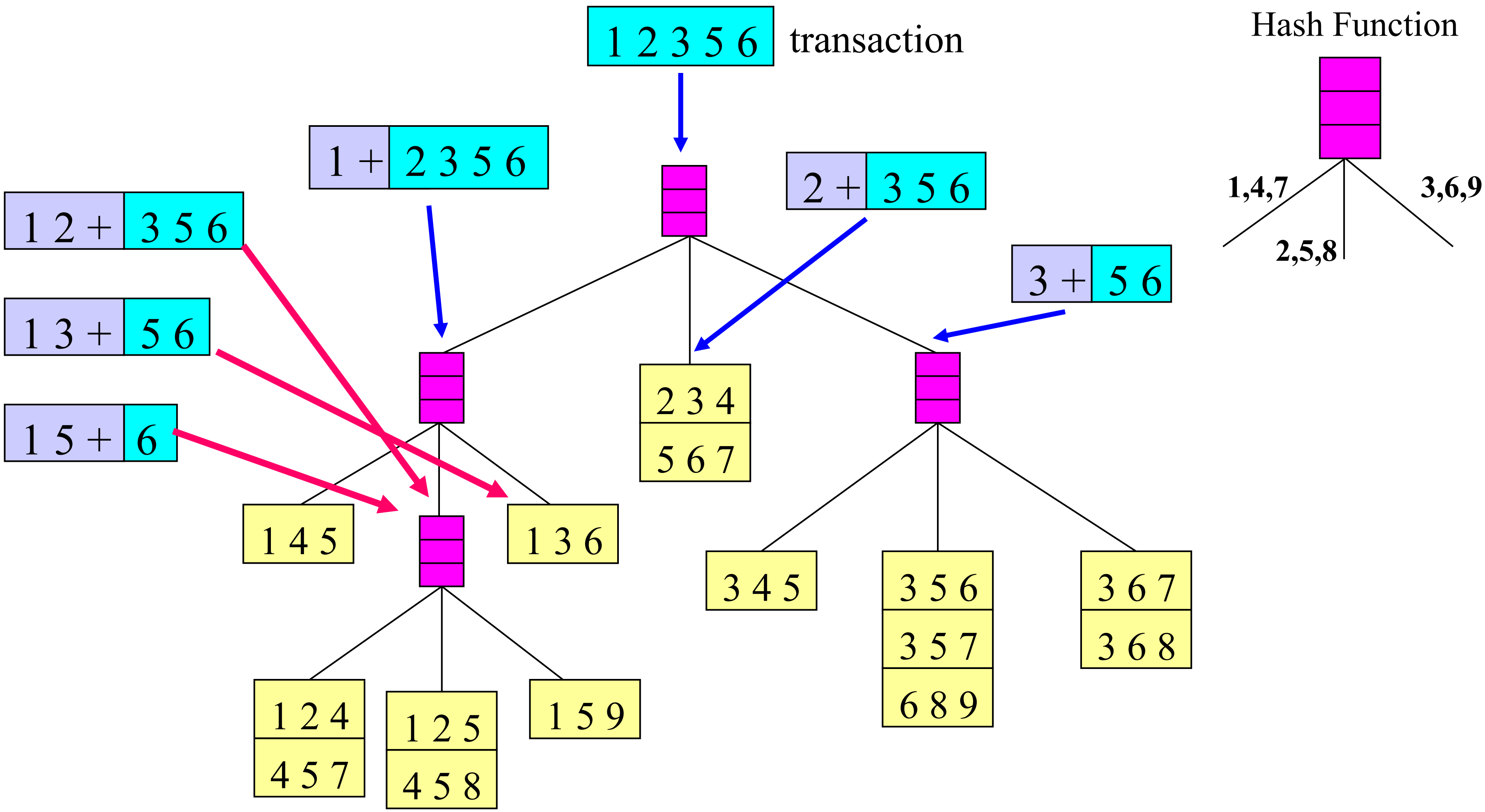



Hash on
3, 6 or 9

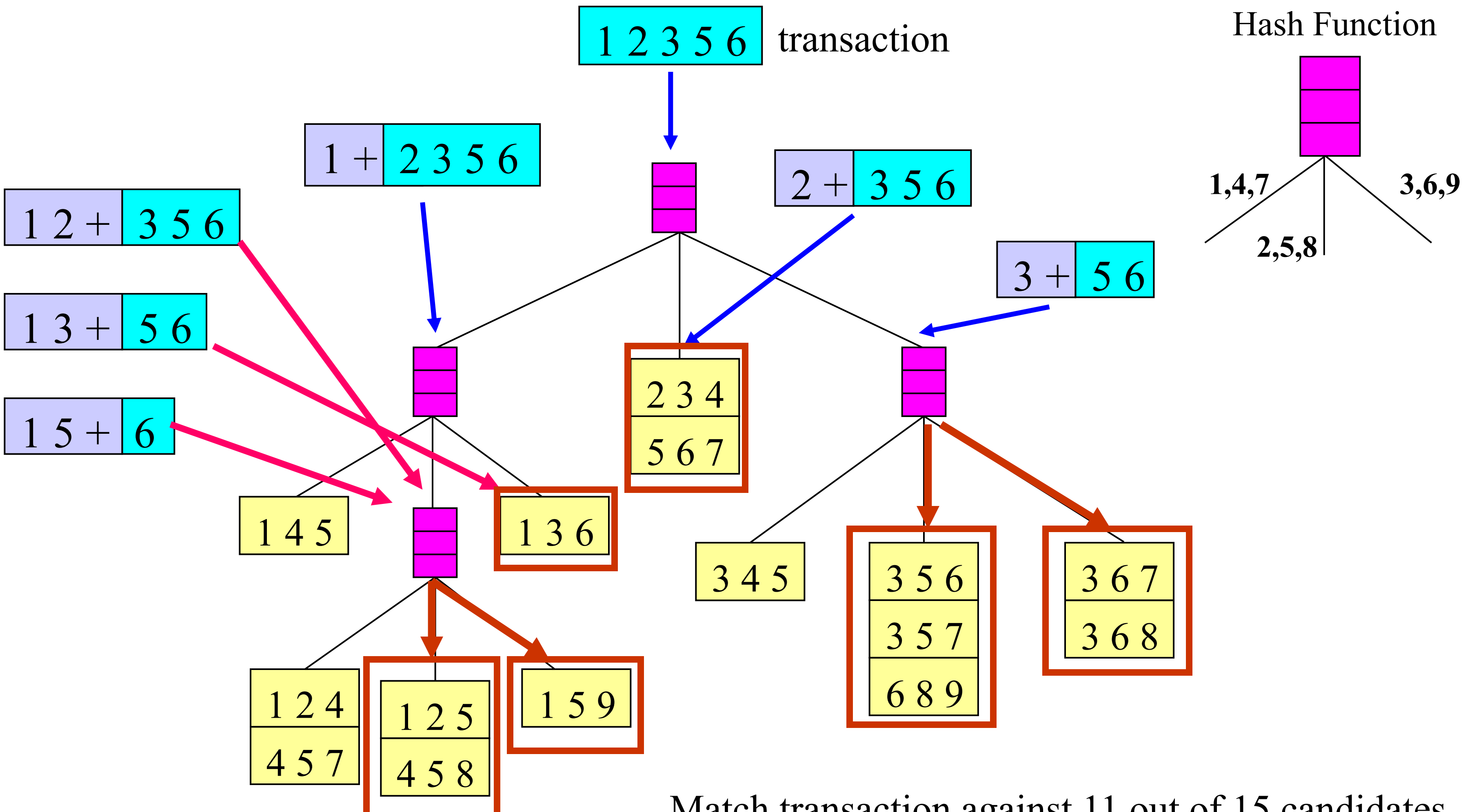
# Support Counting Using a Hash Tree



# Support Counting Using a Hash Tree



# Support Counting Using a Hash Tree




Match transaction against 11 out of 15 candidates

# Rule Generation

- Given a frequent itemset L, find all non-empty subsets $f \subset L$ such that $f \rightarrow L - f$ satisfies the minimum confidence requirement
  - If {A,B,C,D} is a frequent itemset, candidate rules:

    ABC →D, ABD →C, ACD →B, BCD →A,
    A →BCD, B →ACD, C →ABD, D →ABC
    AB →CD, AC → BD, AD → BC, BC →AD,
    BD →AC, CD →AB,

- If $|L| = k$, then there are $2^k - 2$ candidate association rules (ignoring $L \rightarrow \emptyset$ and $\emptyset \rightarrow L$)

# Rule Generation

- In general, confidence does not have an anti-monotone property

  $c(ABC \rightarrow D)$ can be larger or smaller than $c(AB \rightarrow D)$

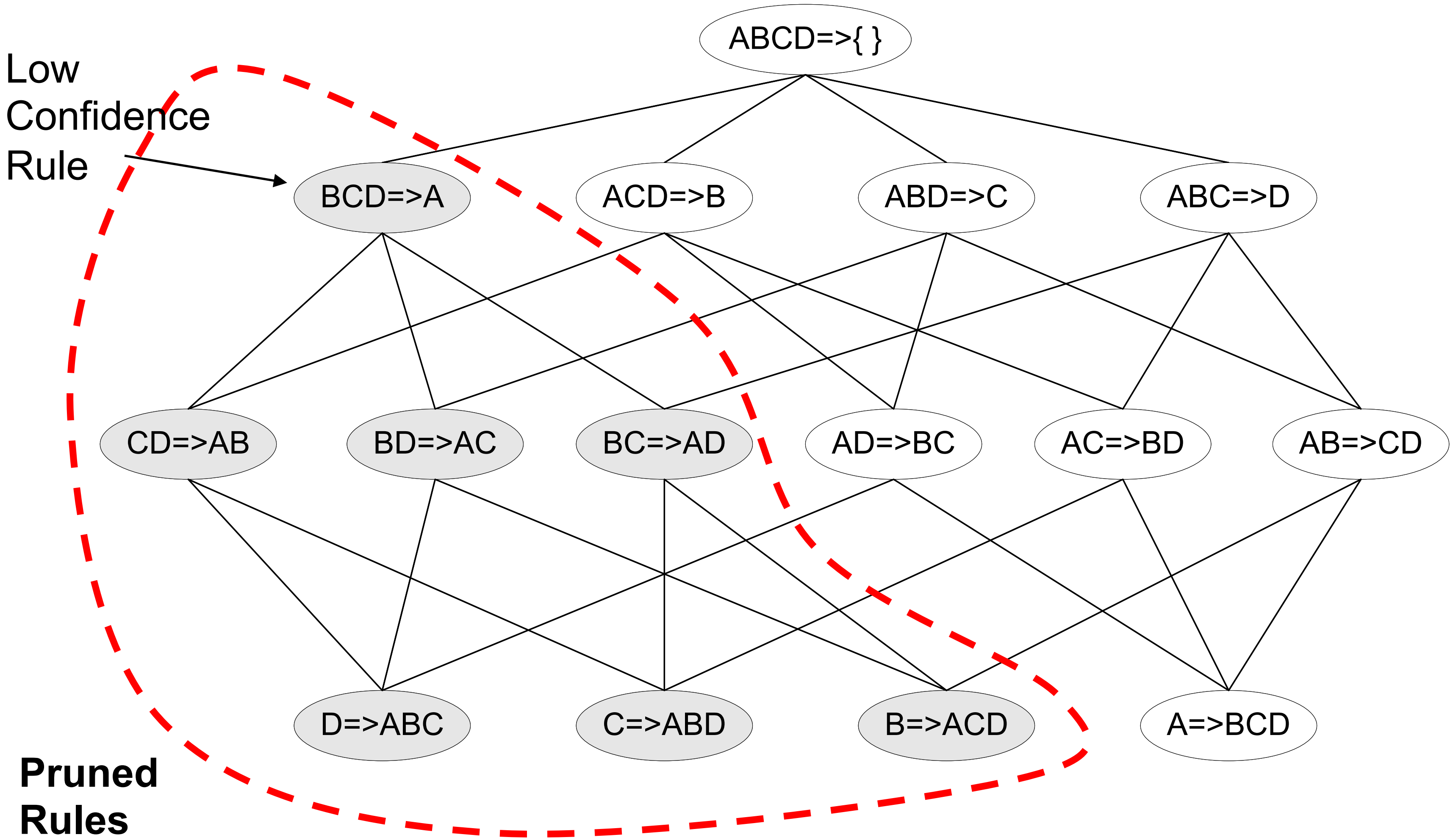
- But confidence of rules generated from the same itemset has an anti-monotone property
  - E.g., Suppose {A,B,C,D} is a frequent 4-itemset:

$$c(ABC \rightarrow D) \geq c(AB \rightarrow CD) \geq c(A \rightarrow BCD)$$

  - Confidence is anti-monotone w.r.t. number of items on the RHS of the rule

# Rule Generation for Apriori Algorithm

Lattice of rules

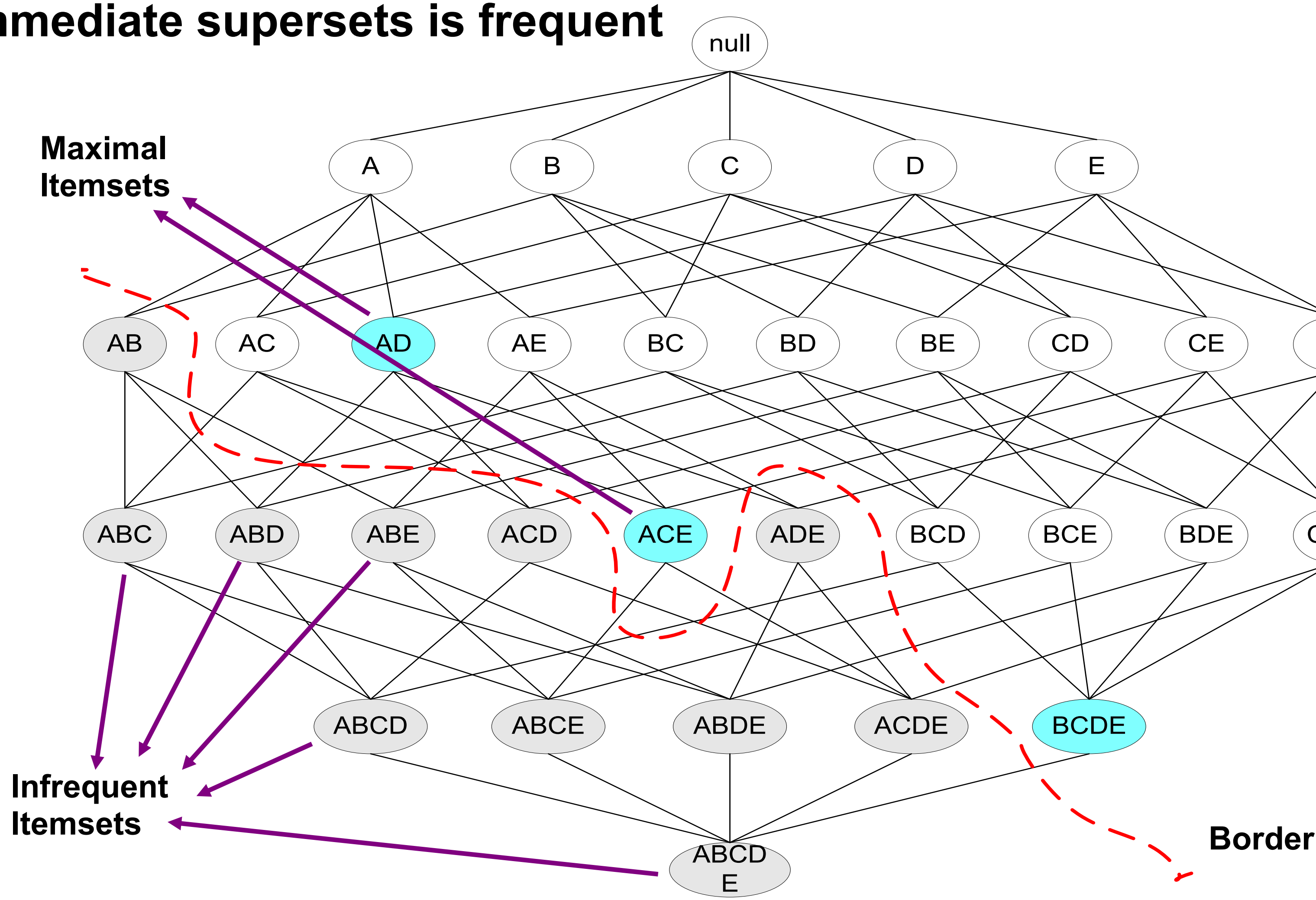


# Factors Affecting Complexity of Apriori

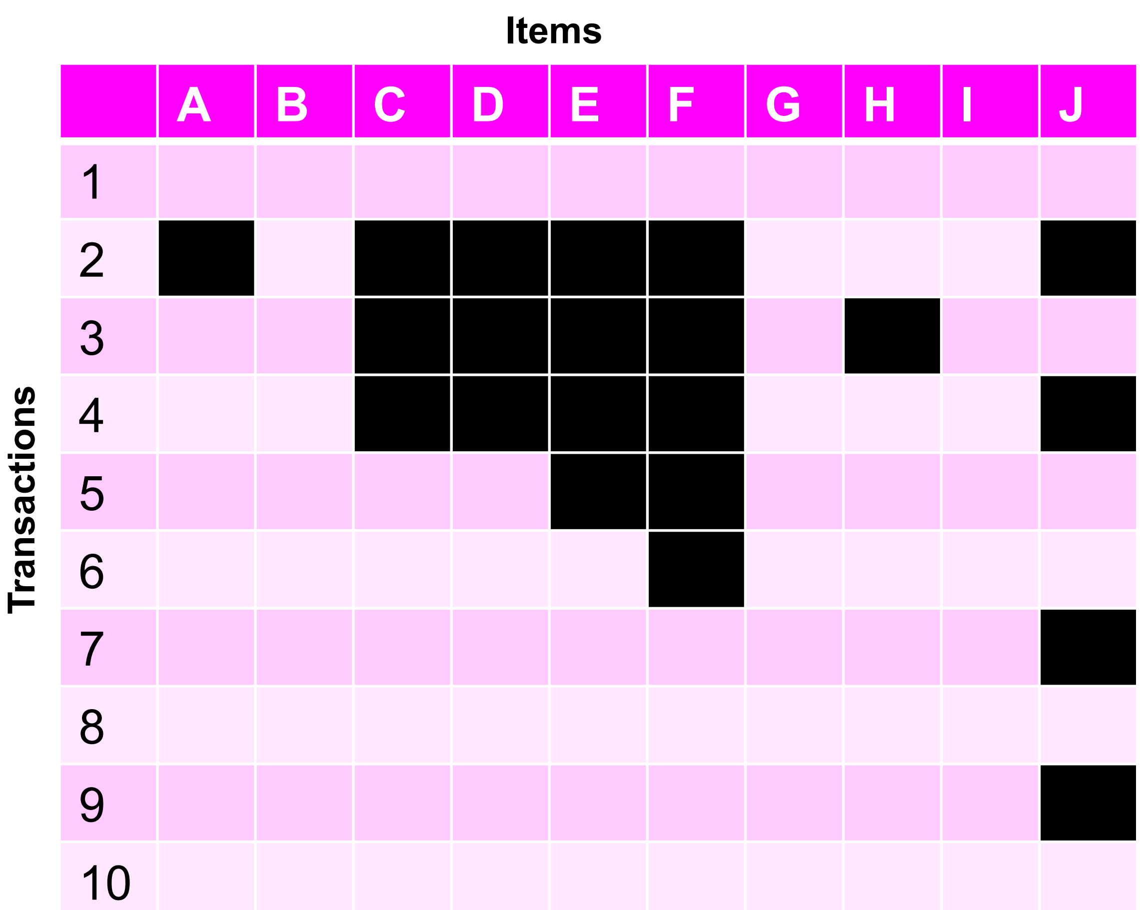
- Choice of minimum support threshold
  - lowering support threshold results in more frequent itemsets
  - this may increase number of candidates and max length of frequent itemsets
- Dimensionality (number of items) of the data set
  - More space is needed to store support count of itemsets
  - if number of frequent itemsets also increases, both computation and I/O costs may also increase
- Size of database
  - run time of algorithm increases with number of transactions
- Average transaction width
  - transaction width increases the max length of frequent itemsets
  - number of subsets in a transaction increases with its width, increasing computation time for support counting

# Maximal Frequent Itemset

**An itemset is maximal frequent if it is frequent and none of its immediate supersets is frequent**



# An illustrative example

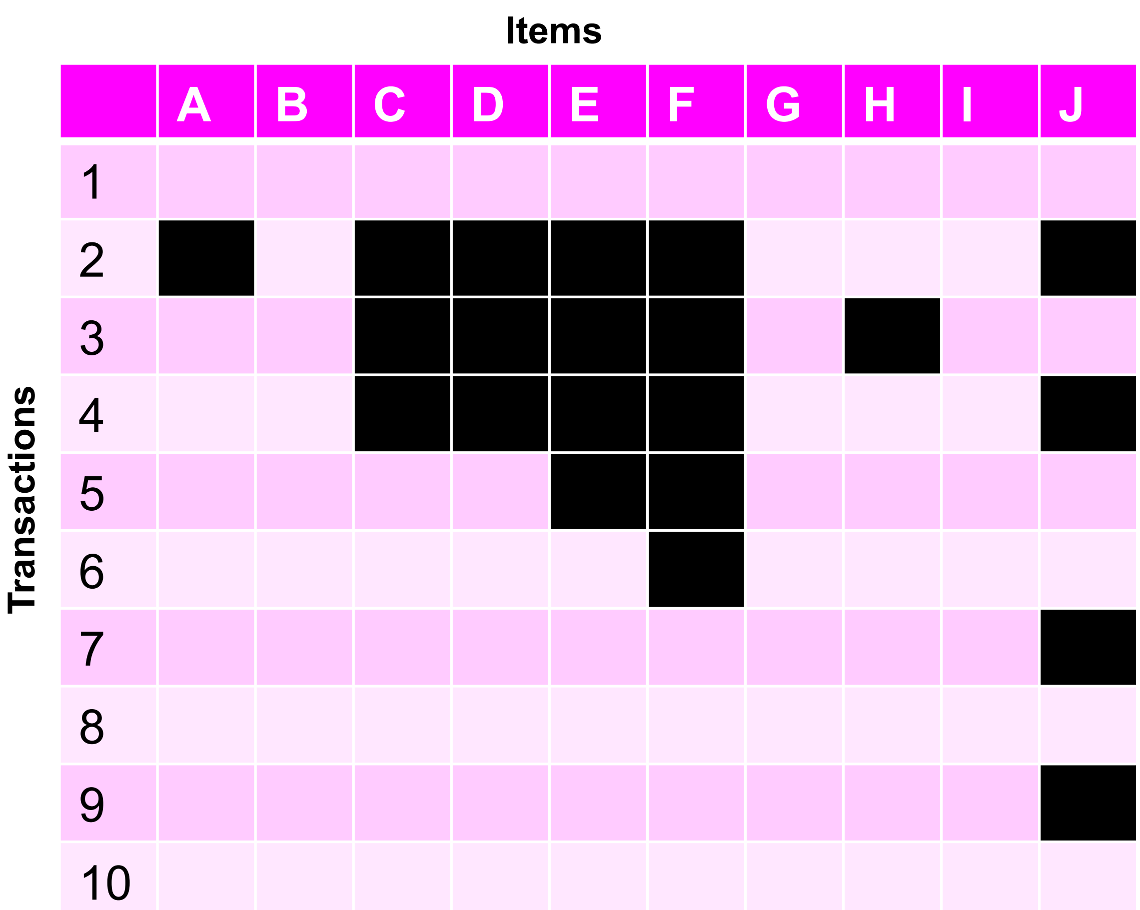



**Support threshold (by count) : 5**

**Frequent itemsets: ?**

**Maximal itemsets: ?**

# An illustrative example




**Support threshold (by count) : 5**
**Frequent itemsets: {F}**
**Maximal itemsets: {F}**

**Support threshold (by count): 4**
**Frequent itemsets: ?**
**Maximal itemsets: ?**

# An illustrative example

Items

Transactions

| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | | | | | | | | | |
| 2 | ■ | | ■ | ■ | ■ | ■ | | | | ■ |
| 3 | | | ■ | ■ | ■ | ■ | | ■ | | |
| 4 | | | ■ | ■ | ■ | ■ | | | | ■ |
| 5 | | | | | ■ | ■ | | | | |
| 6 | | | | | | ■ | | | | |
| 7 | | | | | | | | | | ■ |
| 8 | | | | | | | | | | |
| 9 | | | | | | | | | | ■ |
| 10 | | | | | | | | | | |

**Support threshold (by count) : 5**

**Frequent itemsets: {F}**

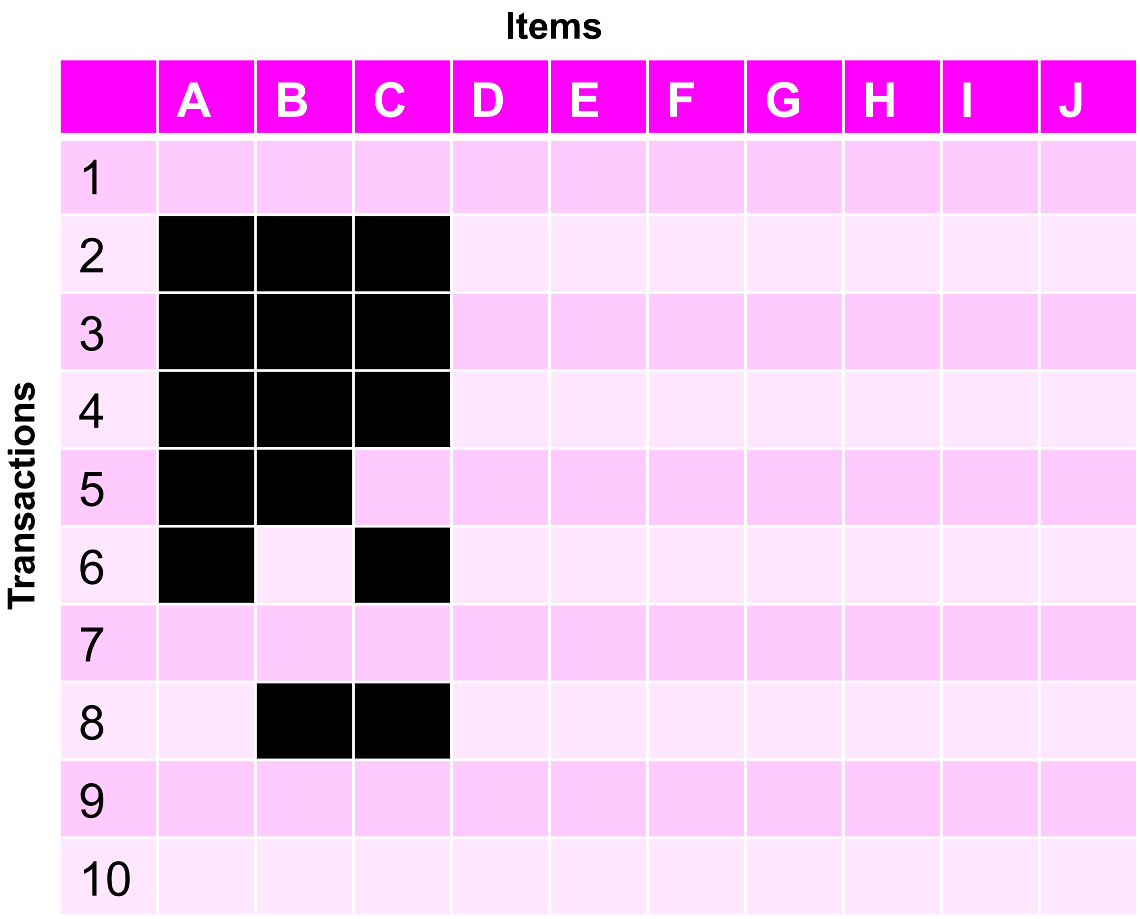
**Maximal itemsets: {F}**

**Support threshold (by count): 4**

**Frequent itemsets: {E}, {F}, {E,F}, {J}**

**Maximal itemsets: {E,F}, {J}**

# Another illustrative example




**Support threshold (by count) : 5**
**Maximal itemsets: {A}, {B}, {C}**

**Support threshold (by count): 4**
**Maximal itemsets: {A,B}, {A,C},{B,C}**

**Support threshold (by count): 3**
**Maximal itemsets: {A,B,C}**

# Closed Itemset

- An itemset X is closed if none of its immediate supersets has the same support as the itemset X.
- X is not closed if at least one of its immediate supersets has support count as X.

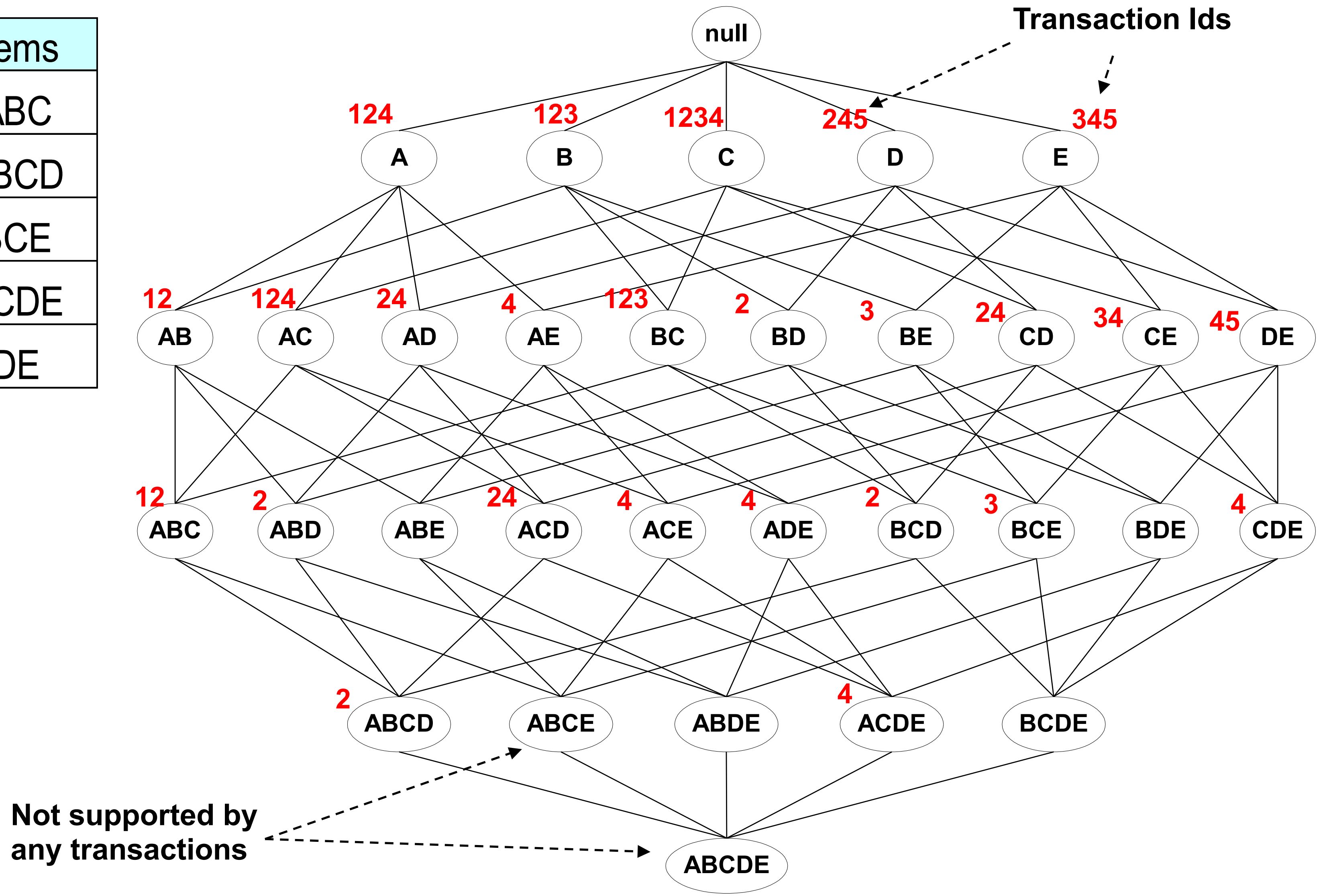
| TID | Items |
|---|---|
| 1 | {A,B} |
| 2 | {B,C,D} |
| 3 | {A,B,C,D} |
| 4 | {A,B,D} |
| 5 | {A,B,C,D} |

| Itemset | Support |
|---|---|
| {A} | 4 |
| {B} | 5 |
| {C} | 3 |
| {D} | 4 |
| {A,B} | 4 |
| {A,C} | 2 |
| {A,D} | 3 |
| {B,C} | 3 |
| {B,D} | 4 |
| {C,D} | 3 |

| Itemset | Support |
|---|---|
| {A,B,C} | 2 |
| {A,B,D} | 3 |
| {A,C,D} | 2 |
| {B,C,D} | 2 |
| {A,B,C,D} | 2 |

# Maximal vs Closed Itemsets

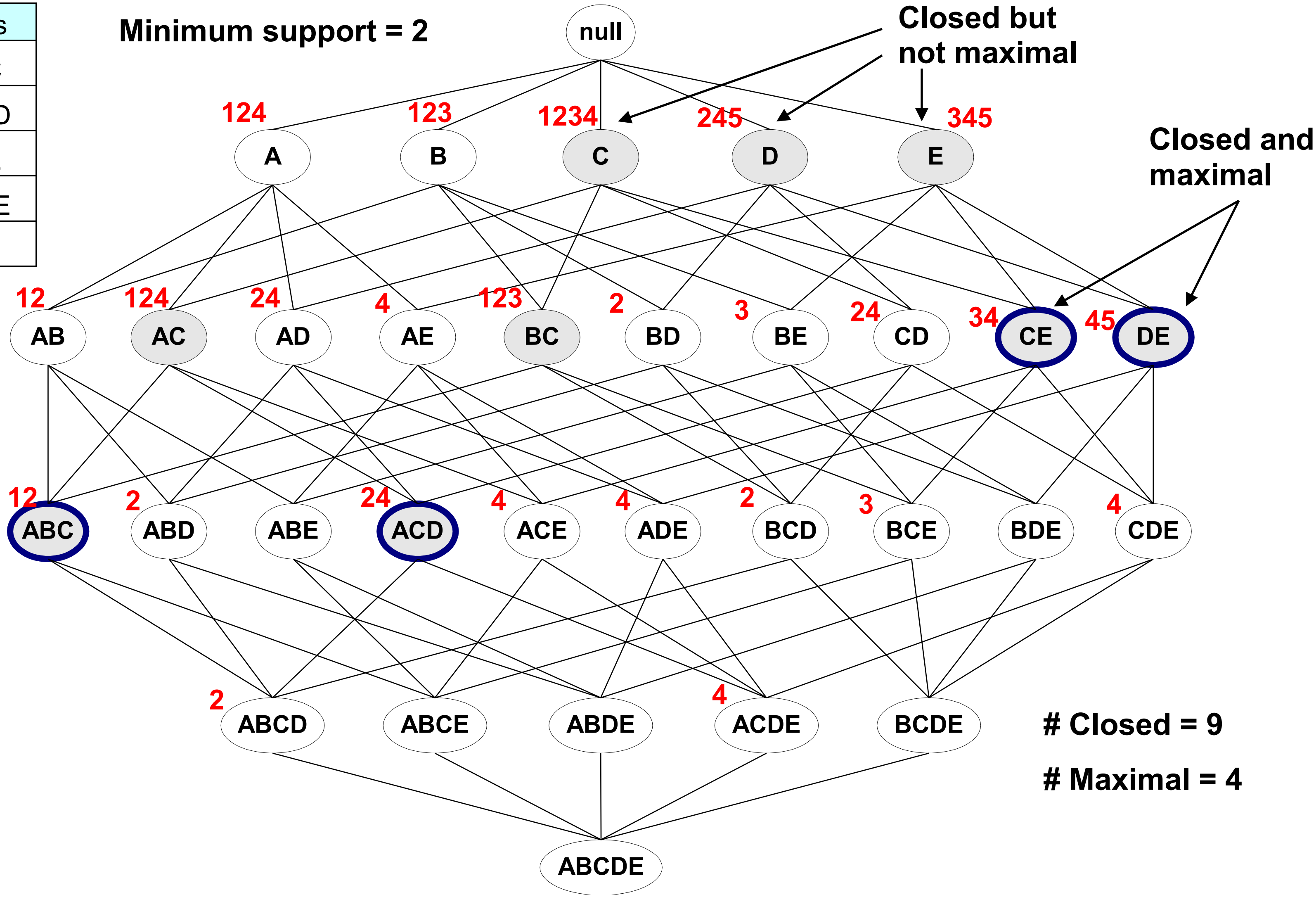
| TID | Items |
|---|---|
| 1 | ABC |
| 2 | ABCD |
| 3 | BCE |
| 4 | ACDE |
| 5 | DE |



# Maximal Frequent vs Closed Frequent Itemsets

| TID | Items |
|---|---|
| 1 | ABC |
| 2 | ABCD |
| 3 | BCE |
| 4 | ACDE |
| 5 | DE |

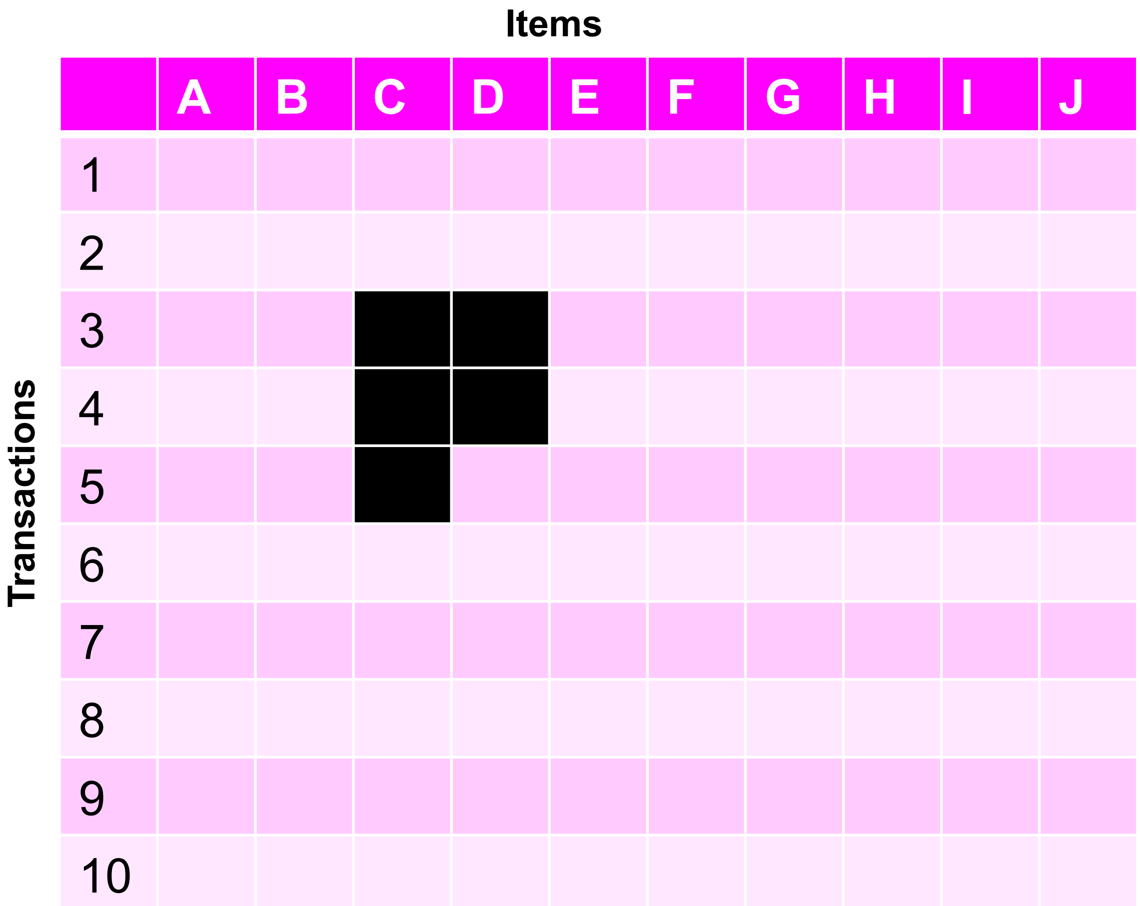
Minimum support = 2




# Closed = 9

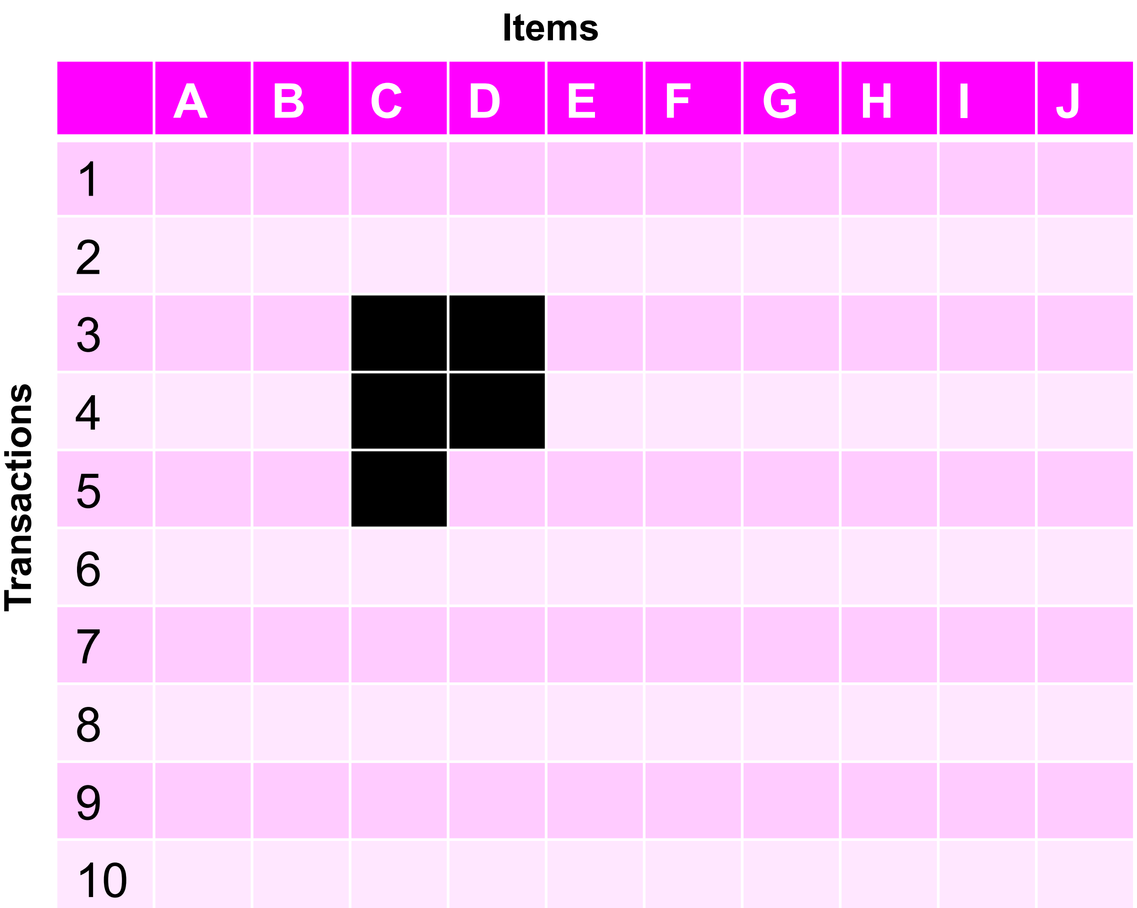
# Maximal = 4

# Example 1



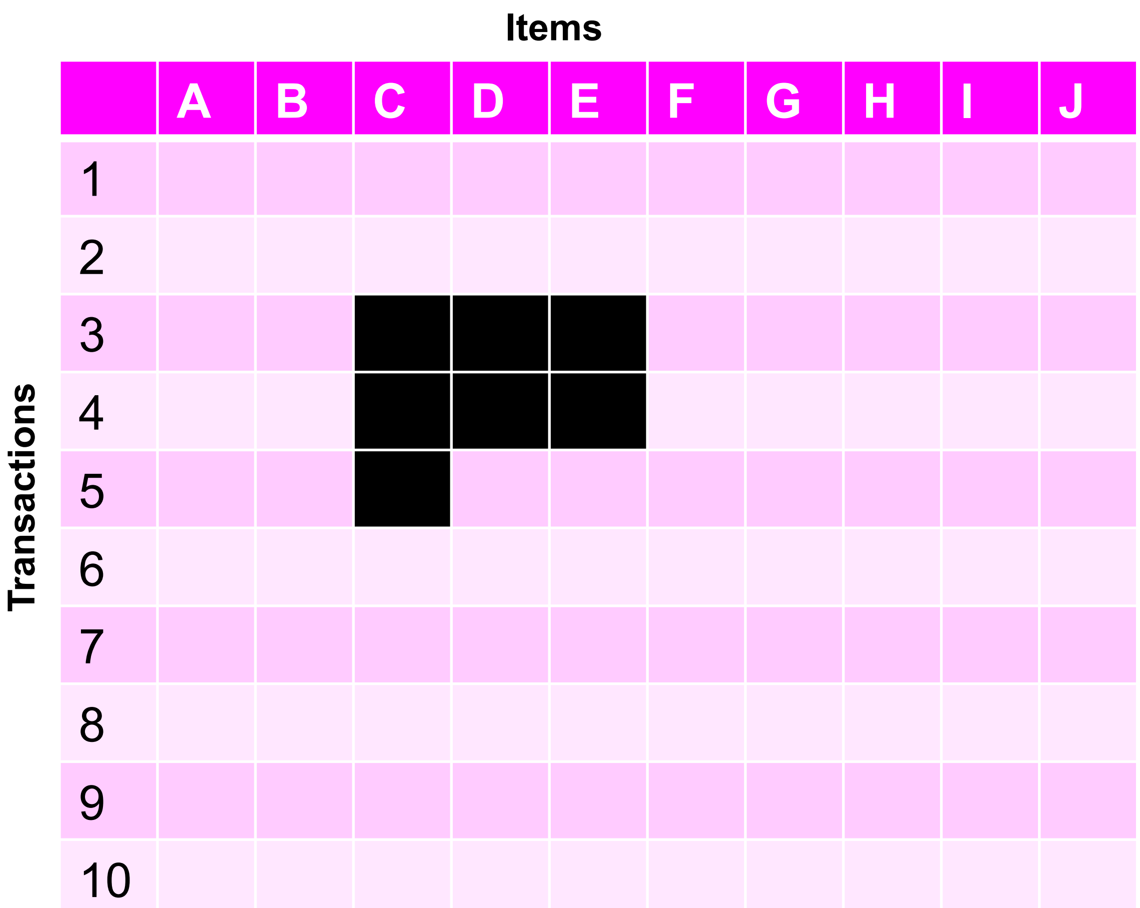

| Itemsets | Support (counts) | Closed itemsets |
|---|---|---|
| {C} | 3 | |
| {D} | 2 | |
| {C,D} | 2 | |

# Example 1



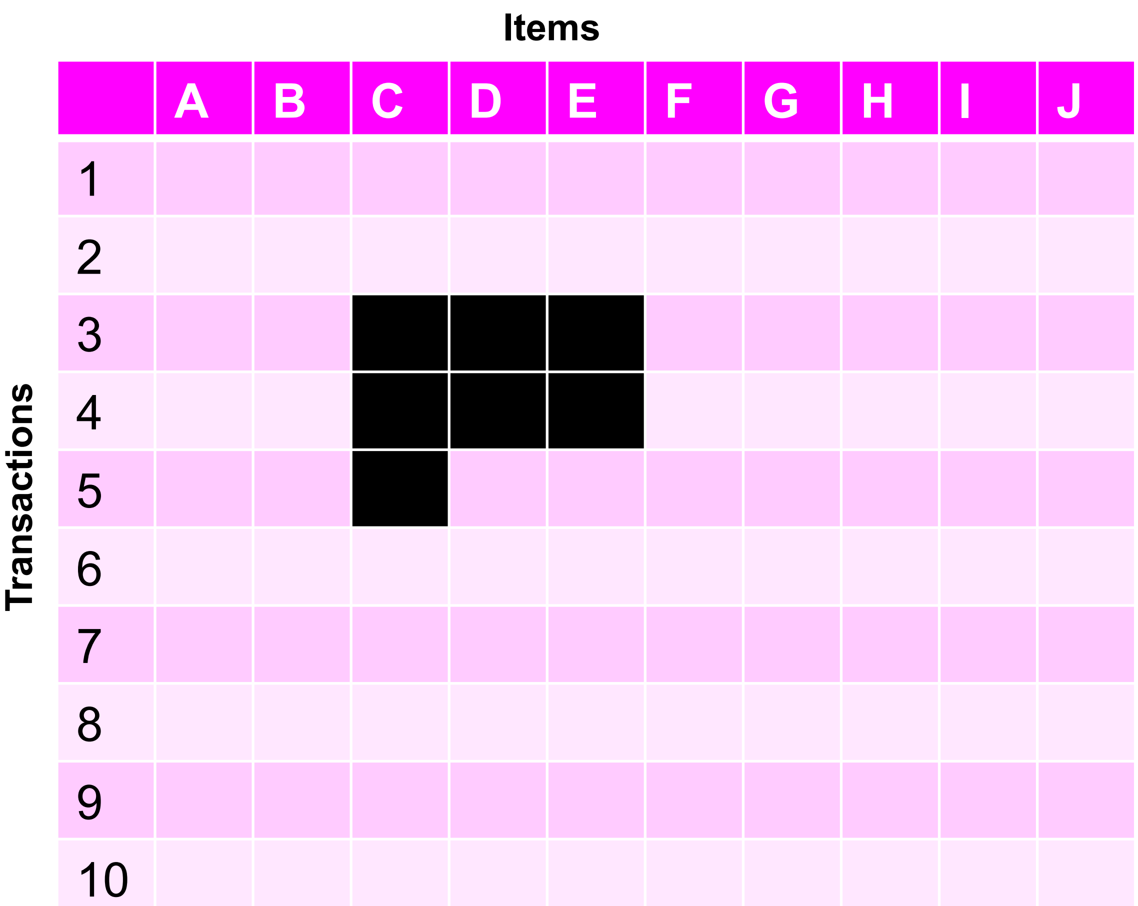

| Itemsets | Support (counts) | Closed itemsets |
|---|---|---|
| **{C}** | **3** | ✓ |
| {D} | 2 | |
| **{C,D}** | **2** | ✓ |

# Example 2




| Itemsets | Support (counts) | Closed itemsets |
|---|---|---|
| {C} | 3 | |
| {D} | 2 | |
| {E} | 2 | |
| {C,D} | 2 | |
| {C,E} | 2 | |
| {D,E} | 2 | |
| {C,D,E} | 2 | |

# Example 2




| Itemsets | Support (counts) | Closed itemsets |
|---|---|---|
| **{C}** | **3** | ✓ |
| {D} | 2 | |
| {E} | 2 | |
| {C,D} | 2 | |
| {C,E} | 2 | |
| {D,E} | 2 | |
| **{C,D,E}** | **2** | ✓ |

# Maximal vs Closed Itemsets

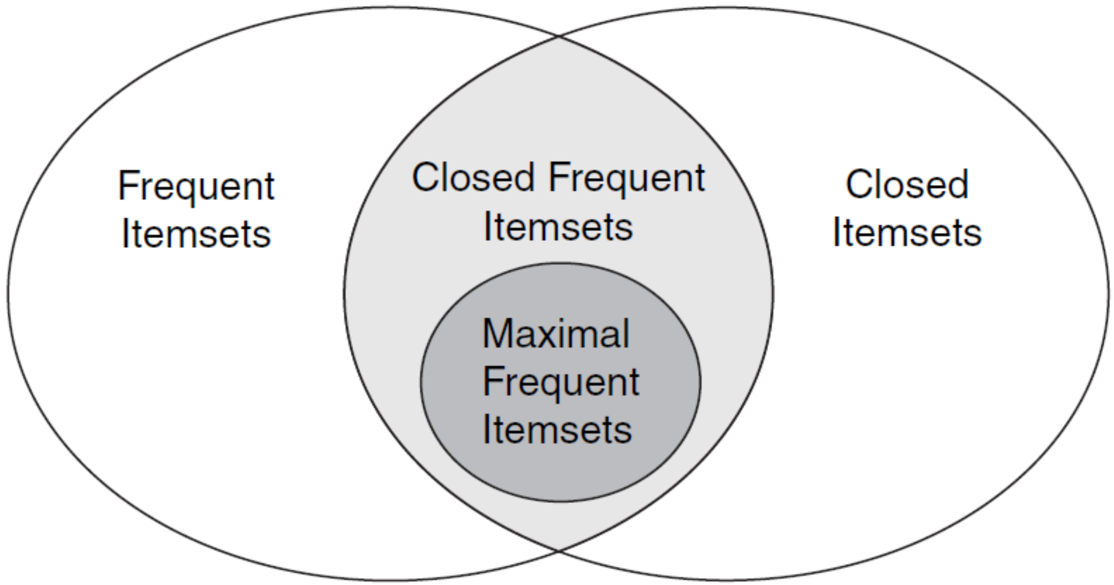



**Figure 5.18.** Relationships among frequent, closed, closed frequent, and maximal frequent itemsets.

# Pattern Evaluation

- Association rule algorithms can produce large number of rules

- Interestingness measures can be used to prune/rank the patterns
  - In the original formulation, support & confidence are the only measures used

# Computing Interestingness Measure

- Given $X \rightarrow Y$ or $\{X,Y\}$, information needed to compute interestingness can be obtained from a contingency table

Contingency table

| | $Y$ | $\overline{Y}$ | |
|---|---|---|---|
| $X$ | $f_{11}$ | $f_{10}$ | $f_{1+}$ |
| $\overline{X}$ | $f_{01}$ | $f_{00}$ | $f_{o+}$ |
| | $f_{+1}$ | $f_{+0}$ | $N$ |

$f_{11}$: support of $X$ and $Y$
$f_{10}$: support of $X$ and $\overline{Y}$
$f_{01}$: support of $\overline{X}$ and $Y$
$f_{00}$: support of $\overline{X}$ and $\overline{Y}$

Used to define various measures

- support, confidence, Gini, entropy, etc.

# Drawback of Confidence

| Customers | Tea | Coffee | … |
|---|---|---|---|
| C1 | 0 | 1 | … |
| C2 | 1 | 0 | … |
| C3 | 1 | 1 | … |
| C4 | 1 | 0 | … |
| … | | | |

| | $Coffee$ | $\overline{Coffee}$ | |
|---|---|---|---|
| $Tea$ | 150 | 50 | 200 |
| $\overline{Tea}$ | 650 | 150 | 800 |
| | 800 | 200 | 1000 |

Association Rule: Tea $\rightarrow$ Coffee

Confidence $\cong$ P(Coffee|Tea) = 150/200 = 0.75

Confidence > 50%, meaning people who drink tea are more likely to drink coffee than not drink coffee

So rule seems reasonable

# Drawback of Confidence

| | Coffee | $\overline{\text{Coffee}}$ | |
|---|---|---|---|
| Tea | 150 | 50 | 200 |
| $\overline{\text{Tea}}$ | 650 | 150 | 800 |
| | 800 | 200 | 1000 |

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 150/200 = 0.75

but P(Coffee) = 0.8, which means knowing that a person drinks tea reduces the probability that the person drinks coffee!

$\Rightarrow$ Note that P(Coffee|$\overline{\text{Tea}}$) = 650/800 = 0.8125

# Drawback of Confidence

| Customers | Tea | Honey | … |
|---|---|---|---|
| C1 | 0 | 1 | … |
| C2 | 1 | 0 | … |
| C3 | 1 | 1 | … |
| C4 | 1 | 0 | … |
| … | | | |

| | $Honey$ | $\overline{Honey}$ | |
|---|---|---|---|
| $Tea$ | 100 | 100 | 200 |
| $\overline{Tea}$ | 20 | 780 | 800 |
| | 120 | 880 | 1000 |

Association Rule: Tea $\rightarrow$ Honey

Confidence $\cong$ P(Honey|Tea) = 100/200 = 0.50

Confidence = 50%, which may mean that drinking tea has little influence whether honey is used or not

So rule seems uninteresting

But P(Honey) = 120/1000 = .12 (hence tea drinkers are far more likely to have honey

# Measure for Association Rules

- So, what kind of rules do we really want?
  - Confidence($X \rightarrow Y$) should be sufficiently high
    - To ensure that people who buy X will more likely buy Y than not buy Y
  - Confidence($X \rightarrow Y$) > support(Y)
    - Otherwise, rule will be misleading because having item X actually reduces the chance of having item Y in the same transaction
    - Is there any measure that capture this constraint?
      - Answer: Yes. There are many of them.

# Statistical Relationship between X and Y

- The criterion

  $$\text{confidence}(X \rightarrow Y) = \text{support}(Y)$$

  is equivalent to:
  - $P(Y|X) = P(Y)$
  - $P(X,Y) = P(X) \times P(Y)$ (X and Y are independent)

  If $P(X,Y) > P(X) \times P(Y)$ : X & Y are positively correlated

  If $P(X,Y) < P(X) \times P(Y)$ : X & Y are negatively correlated

# Measures that take into account statistical dependence

$$Lift = \frac{P(Y \mid X)}{P(Y)}$$

$$Interest = \frac{P(X,Y)}{P(X)P(Y)}$$

lift is used for rules while interest is used for itemsets

$$PS = P(X,Y) - P(X)P(Y)$$

$$\phi - coefficient = \frac{P(X,Y) - P(X)P(Y)}{\sqrt{P(X)[1-P(X)]P(Y)[1-P(Y)]}}$$

# Example: Lift/Interest

| | Coffee | $\overline{\text{Coffee}}$ | |
|---|---|---|---|
| Tea | 150 | 50 | 200 |
| $\overline{\text{Tea}}$ | 650 | 150 | 800 |
| | 800 | 200 | 1000 |

Association Rule: Tea $\rightarrow$ Coffee

Confidence= P(Coffee|Tea) = 0.75

but P(Coffee) = 0.8

$\Rightarrow$ Interest = 0.15 / (0.2×0.8) = 0.9375 (< 1, therefore is negatively associated)

**There are lots of measures proposed in the literature**

| Measure (Symbol) | Definition |
|---|---|
| Correlation ($\phi$) | $\frac{N f_{11} - f_{1+} f_{+1}}{\sqrt{f_{1+} f_{+1} f_{0+} f_{+0}}}$ |
| Odds ratio ($\alpha$) | $(f_{11} f_{00}) / (f_{10} f_{01})$ |
| Kappa ($\kappa$) | $\frac{N f_{11} + N f_{00} - f_{1+} f_{+1} - f_{0+} f_{+0}}{N^2 - f_{1+} f_{+1} - f_{0+} f_{+0}}$ |
| Interest ($I$) | $(N f_{11}) / (f_{1+} f_{+1})$ |
| Cosine ($IS$) | $(f_{11}) / (\sqrt{f_{1+} f_{+1}})$ |
| Piatetsky-Shapiro ($PS$) | $\frac{f_{11}}{N} - \frac{f_{1+} f_{+1}}{N^2}$ |
| Collective strength ($S$) | $\frac{f_{11} + f_{00}}{f_{1+} f_{+1} + f_{0+} f_{+0}} \times \frac{N - f_{1+} f_{+1} - f_{0+} f_{+0}}{N - f_{11} - f_{00}}$ |
| Jaccard ($\zeta$) | $f_{11} / (f_{1+} + f_{+1} - f_{11})$ |
| All-confidence ($h$) | $\min \left[ \frac{f_{11}}{f_{1+}}, \frac{f_{11}}{f_{+1}} \right]$ |

# Continuous and Categorical Attributes

**How to apply association analysis to non-asymmetric binary variables?**

| Gender | ··· | Age | Annual Income | No of hours spent online per week | No of email accounts | Privacy Concern |
|---|---|---|---|---|---|---|
| Female | ··· | 26 | 90K | 20 | 4 | Yes |
| Male | ··· | 51 | 135K | 10 | 2 | No |
| Male | ··· | 29 | 80K | 10 | 3 | Yes |
| Female | ··· | 45 | 120K | 15 | 3 | Yes |
| Female | ··· | 31 | 95K | 20 | 5 | Yes |
| Male | ··· | 25 | 55K | 25 | 5 | Yes |
| Male | ··· | 37 | 100K | 10 | 1 | No |
| Male | ··· | 41 | 65K | 8 | 2 | No |
| Female | ··· | 26 | 85K | 12 | 1 | No |
| ··· | ··· | ··· | ··· | ··· | ··· | ··· |

**Example of Association Rule:**

$\{\text{Gender=Male}, \text{Age} \in [21,30)\} \rightarrow \{\text{No of hours online} \geq 10\}$

# Handling Categorical Attributes

- Example: Internet Usage Data

| Gender | Level of Education | State | Computer at Home | Online Auction | Chat Online | Online Banking | Privacy Concerns |
|---|---|---|---|---|---|---|---|
| Female | Graduate | Illinois | Yes | Yes | Daily | Yes | Yes |
| Male | College | California | No | No | Never | No | No |
| Male | Graduate | Michigan | Yes | Yes | Monthly | Yes | Yes |
| Female | College | Virginia | No | Yes | Never | Yes | Yes |
| Female | Graduate | California | Yes | No | Never | No | Yes |
| Male | College | Minnesota | Yes | Yes | Weekly | Yes | Yes |
| Male | College | Alaska | Yes | Yes | Daily | Yes | No |
| Male | High School | Oregon | Yes | No | Never | No | No |
| Female | Graduate | Texas | No | No | Monthly | No | No |
| ... | ... | ... | ... | ... | ... | ... | ... |

{Level of Education=Graduate, Online Banking=Yes}
$\rightarrow$ {Privacy Concerns = Yes}

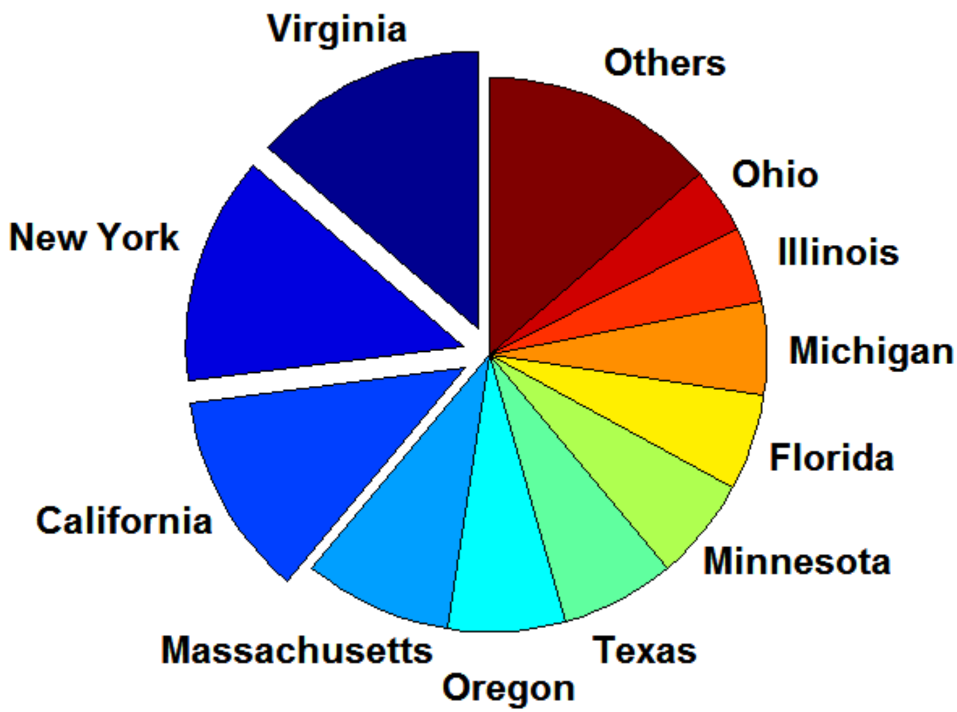
# Handling Categorical Attributes

- Introduce a new “item” for each distinct attribute-value pair

| Male | Female | Education = Graduate | Education = College | Education = High School | ... | Privacy = Yes | Privacy = No |
|---|---|---|---|---|---|---|---|
| 0 | 1 | 1 | 0 | 0 | ... | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | ... | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | ... | 1 | 0 |
| 0 | 1 | 0 | 1 | 0 | ... | 1 | 0 |
| 0 | 1 | 1 | 0 | 0 | ... | 1 | 0 |
| 1 | 0 | 0 | 1 | 0 | ... | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | ... | 0 | 1 |
| 1 | 0 | 0 | 0 | 1 | ... | 0 | 1 |
| 0 | 1 | 1 | 0 | 0 | ... | 0 | 1 |
| ... | ... | ... | ... | ... | ... | ... | ... |

# Handling Categorical Attributes

- Some attributes can have many possible values
  - Many of their attribute values have very low support
    - Potential solution: Aggregate the low-support attribute values



# Handling Categorical Attributes

- Distribution of attribute values can be highly skewed
  - Example: 85% of survey participants own a computer at home
    - Most records have Computer at home = Yes
    - Computation becomes expensive; many frequent itemsets involving the binary item (Computer at home = Yes)
    - Potential solution:
      - discard the highly frequent items
      - Use alternative measures such as h-confidence
- Computational Complexity
  - Binarizing the data increases the number of items
  - But the width of the "transactions" remain the same as the number of original (non-binarized) attributes
  - Produce more frequent itemsets but maximum size of frequent itemset is limited to the number of original attributes

# Handling Continuous Attributes

- Different methods:
  - Discretization-based
  - Statistics-based
  - Non-discretization based
    - minApriori

- Different kinds of rules can be produced:
  - {Age$\in$[21,30), No of hours online$\in$[10,20)}
    $\rightarrow$ {Chat Online =Yes}
  - {Age$\in$[21,30), Chat Online = Yes}
    $\rightarrow$ No of hours online: $\mu$=14, $\sigma$=4

# Discretization-based Methods

| Gender | ... | Age | Annual Income | No of hours spent online per week | No of email accounts | Privacy Concern |
|---|---|---|---|---|---|---|
| Female | ... | 26 | 90K | 20 | 4 | Yes |
| Male | ... | 51 | 135K | 10 | 2 | No |
| Male | ... | 29 | 80K | 10 | 3 | Yes |
| Female | ... | 45 | 120K | 15 | 3 | Yes |
| Female | ... | 31 | 95K | 20 | 5 | Yes |
| Male | ... | 25 | 55K | 25 | 5 | Yes |
| Male | ... | 37 | 100K | 10 | 1 | No |
| Male | ... | 41 | 65K | 8 | 2 | No |
| Female | ... | 26 | 85K | 12 | 1 | No |
| ... | ... | ... | ... | ... | ... | ... |

| Male | Female | ... ... | Age $< 13$ | Age $\in [13, 21)$ | Age $\in [21, 30)$ | ... ... | Privacy = Yes | Privacy = No |
|---|---|---|---|---|---|---|---|---|
| 0 | 1 | ... | 0 | 0 | 1 | ... | 1 | 0 |
| 1 | 0 | ... | 0 | 0 | 0 | ... | 0 | 1 |
| 1 | 0 | ... | 0 | 0 | 1 | ... | 1 | 0 |
| 0 | 1 | ... | 0 | 0 | 0 | ... | 1 | 0 |
| 0 | 1 | ... | 0 | 0 | 0 | ... | 1 | 0 |
| 1 | 0 | ... | 0 | 0 | 1 | ... | 1 | 0 |
| 1 | 0 | ... | 0 | 0 | 0 | ... | 0 | 1 |
| 1 | 0 | ... | 0 | 0 | 0 | ... | 0 | 1 |
| 0 | 1 | ... | 0 | 0 | 1 | ... | 0 | 1 |
| ... | ... | ... | ... | ... | ... | ... | ... | ... |

# Discretization-based Methods

- Unsupervised:
  - Equal-width binning <1 2 3> <4 5 6> <7 8 9>
  - Equal-depth binning <1 2 > <3 4 5 6 7 > < 8 9>
  - Cluster-based

- Supervised discretization

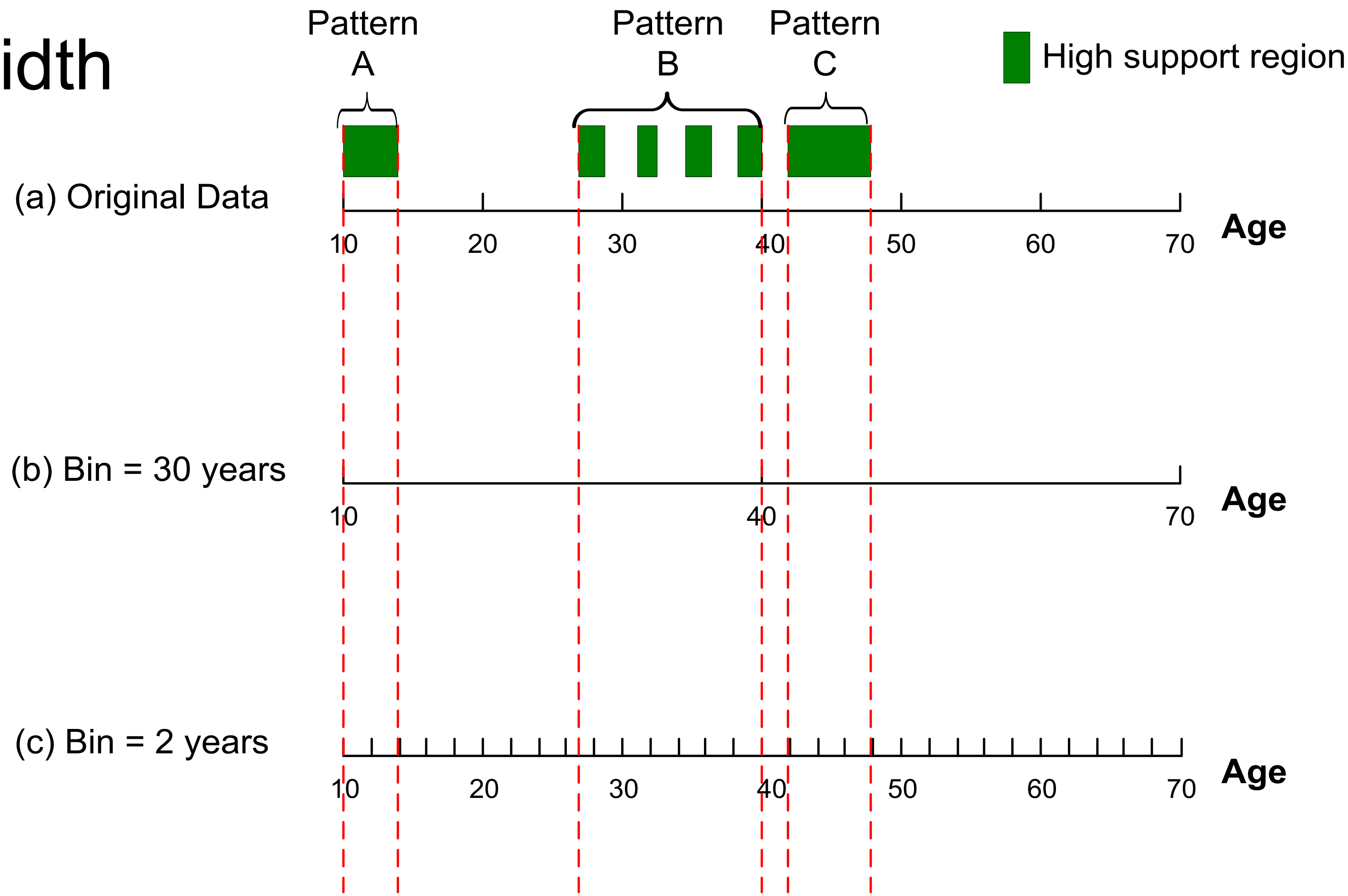
Continuous attribute, v

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|---|---|---|---|---|---|---|---|---|---|
| Chat Online = Yes | 0 | 0 | 20 | 10 | 20 | 0 | 0 | 0 | 0 |
| Chat Online = No | 150 | 100 | 0 | 0 | 0 | 100 | 100 | 150 | 100 |

bin$_1$ bin$_2$ bin$_3$

# Discretization Issues

- Interval width




Pattern A: Age $\in [10, 15) \longrightarrow$ Chat Online = Never
Pattern B: Age $\in [26, 41) \longrightarrow$ Chat Online = Never
Pattern C: Age $\in [42, 48) \longrightarrow$ Online Banking = Yes

# Discretization Issues

- Interval too wide (e.g., Bin size= 30)
  - May merge several disparate patterns
    - Patterns A and B are merged together
  - May lose some of the interesting patterns
    - Pattern C may not have enough confidence
- Interval too narrow (e.g., Bin size = 2)
  - Pattern A is broken up into two smaller patterns
    - Can recover the pattern by merging adjacent subpatterns
  - Pattern B is broken up into smaller patterns
    - Cannot recover the pattern by merging adjacent subpatterns
  - Some windows may not meet support threshold

# Statistics-based Methods

- Example:

  {Income > 100K, Online Banking=Yes} $\rightarrow$ Age: $\mu$=34

- Rule consequent consists of a continuous variable, characterized by their statistics
  - mean, median, standard deviation, etc.
- Approach:
  - Withhold the target attribute from the rest of the data
  - Extract frequent itemsets from the rest of the attributes
    - Binarized the continuous attributes (except for the target attribute)
  - For each frequent itemset, compute the corresponding descriptive statistics of the target attribute
    - Frequent itemset becomes a rule by introducing the target variable as rule consequent
  - Apply statistical test to determine interestingness of the rule

# Statistics-based Methods

| Gender | ··· | Age | Annual Income | No of hours spent online per week | No of email accounts | Privacy Concern |
|---|---|---|---|---|---|---|
| Female | ··· | 26 | 90K | 20 | 4 | Yes |
| Male | ··· | 51 | 135K | 10 | 2 | No |
| Male | ··· | 29 | 80K | 10 | 3 | Yes |
| Female | ··· | 45 | 120K | 15 | 3 | Yes |
| Female | ··· | 31 | 95K | 20 | 5 | Yes |
| Male | ··· | 25 | 55K | 25 | 5 | Yes |
| Male | ··· | 37 | 100K | 10 | 1 | No |
| Male | ··· | 41 | 65K | 8 | 2 | No |
| Female | ··· | 26 | 85K | 12 | 1 | No |
| ··· | ··· | ··· | ··· | ··· | ··· | ··· |

**Frequent Itemsets:**

**{Male, Income > 100K}**

**{Income < 30K, No hours $\in$[10,15)}**

**{Income > 100K, Online Banking = Yes}**

**….**

**Association Rules:**

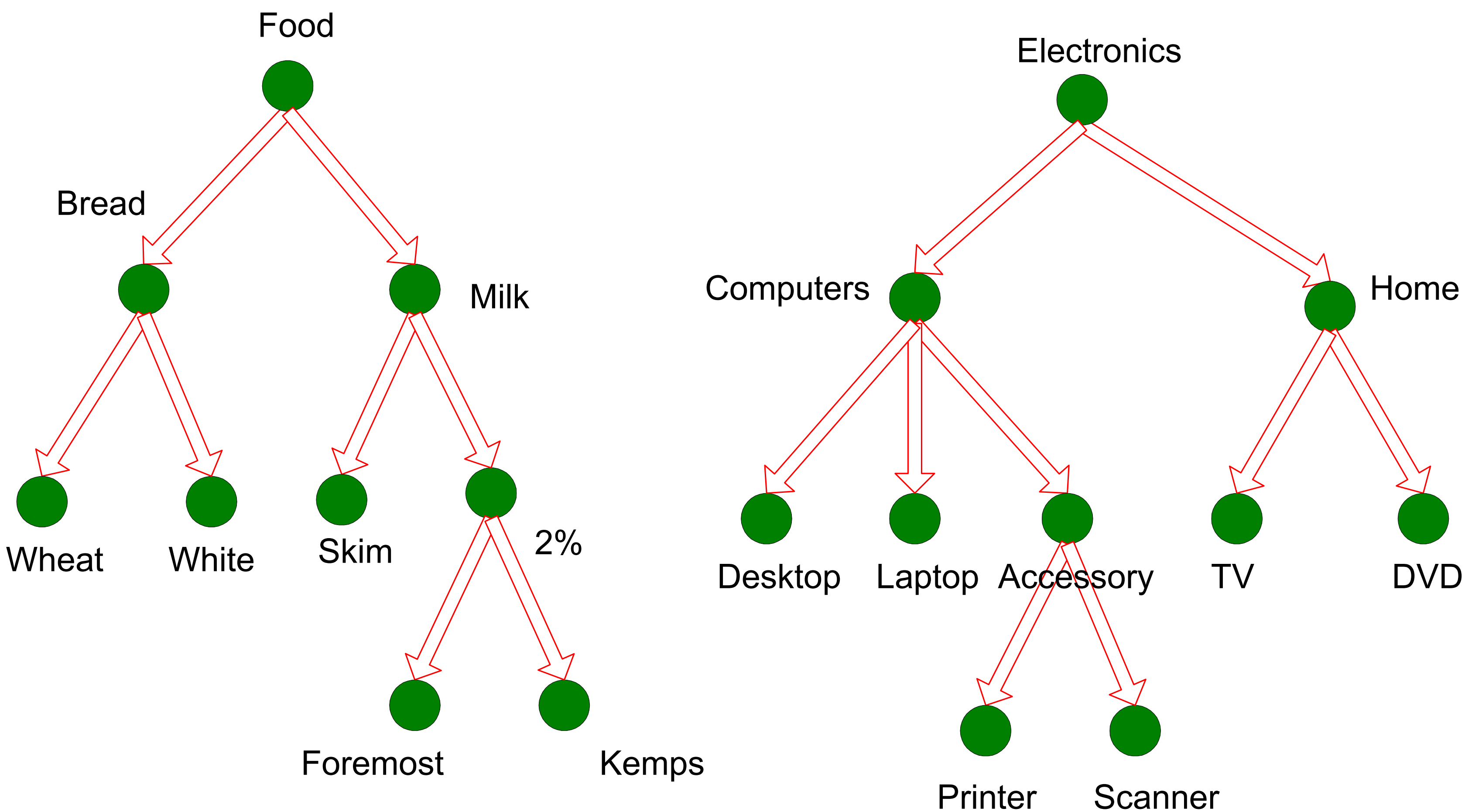
**{Male, Income > 100K} $\rightarrow$ Age: $\mu$ = 30**

**{Income < 40K, No hours $\in$[10,15)} $\rightarrow$ Age: $\mu$ = 24**

**{Income > 100K,Online Banking = Yes} $\rightarrow$ Age: $\mu$ = 34**

**….**

# Concept Hierarchies



# Multi-level Association Rules

- Why should we incorporate concept hierarchy?
  - Rules at lower levels may not have enough support to appear in any frequent itemsets
  - Rules at lower levels of the hierarchy are overly specific
    - e.g., skim milk $\rightarrow$ white bread, 2% milk $\rightarrow$ wheat bread, skim milk $\rightarrow$ wheat bread, etc.

    are indicative of association between milk and bread
  - Rules at higher level of hierarchy may be too generic

# Multi-level Association Rules

- How do support and confidence vary as we traverse the concept hierarchy?
  - If X is the parent item for both X1 and X2, then $\sigma(X) \leq \sigma(X1) + \sigma(X2)$
  - If $\sigma(X1 \cup Y1) \geq minsup$,
    and X is parent of X1, Y is parent of Y1
    then $\sigma(X \cup Y1) \geq minsup$, $\sigma(X1 \cup Y) \geq minsup$
    $\sigma(X \cup Y) \geq minsup$
  - If $conf(X1 \Rightarrow Y1) \geq minconf$,
    then $conf(X1 \Rightarrow Y) \geq minconf$

# Multi-level Association Rules

- Approach 1:
  - Extend current association rule formulation by augmenting each transaction with higher level items

Original Transaction: {skim milk, wheat bread}

Augmented Transaction:
{skim milk, wheat bread, milk, bread, food}

- Issues:
  - Items that reside at higher levels have much higher support counts
    - if support threshold is low, too many frequent patterns involving items from the higher levels
  - Increased dimensionality of the data

# Multi-level Association Rules

- Approach 2:
  - Generate frequent patterns at highest level first
  - Then, generate frequent patterns at the next highest level, and so on
- Issues:
  - I/O requirements will increase dramatically because we need to perform more passes over the data
  - May miss some potentially interesting cross-level association patterns