As for quicksort, the worst-case execution time of quickselect is quadratic. But the expected execution time is linear and hence is a logarithmic factor faster than quicksort.

**Theorem 5.8.** The quickselect algorithm runs in expected time $O(n)$ on an input of size $n$.

**Proof.** We shall give an analysis that is simple and shows a linear expected execution time. It does not give the smallest constant possible. Let $T(n)$ denote the expected execution time of quickselect. We call a pivot *good* if neither $|a|$ nor $|c|$ is larger than $2n/3$. Let $\gamma$ denote the probability that a pivot is good; then $\gamma \leq 1/3$. We now make the conservative assumption that the problem size in the recursive call is reduced only for good pivots and that, even then, it is reduced only by a factor of $2/3$. Since the work outside the recursive call is linear in $n$, there is an appropriate constant $c$ such that

$$T(n) \leq cn + \gamma T\left(\frac{2n}{3}\right) + (1-\gamma)T(n).$$

Solving for $T(n)$ yields

$$T(n) \leq \frac{cn}{\gamma} + T\left(\frac{2n}{3}\right) \leq 3cn + T\left(\frac{2n}{3}\right) \leq 3c \left( n + \frac{2n}{3} + \frac{4n}{9} + \ldots \right) \leq 3cn \sum_{i=0}^{\infty} \left(\frac{2}{3}\right)^i = 9cn.$$

$\square$