Instance-based Classifiers

Dino Pedreschi, Riccardo Guidotti

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Slides edited from Tan, Steinbach, Kumar, Introduction to Data Mining
Nearest-Neighbor Classifier (K-NN)

Basic idea: If it walks like a duck, quacks like a duck, then it’s probably a duck.

Requires three things

1. Training set of stored records
2. Distance metric to compute distance between records
3. The value of \( k \), the number of nearest neighbors to retrieve
Nearest-Neighbor Classifier (K-NN)

Given a set of training records (memory), and a test record:

1. **Compute the distances** from the records in the training to the test.

2. **Identify the k “nearest” records**.

3. Use class labels of nearest neighbors to **determine the class label** of unknown record (e.g., by taking majority vote).
Definition of Nearest Neighbor

- $K$-nearest neighbors of a record $x$ are data points that have the $k$ smallest distance to $x$. 

(a) 1-nearest neighbor  
(b) 2-nearest neighbor  
(c) 3-nearest neighbor
Choosing the Value of K

• If k is too small, it is sensitive to noise points and it can lead to overfitting to the noise in the training set.

• If k is too large, the neighborhood may include points from other classes.

• General practice $k = \sqrt{N}$ where $N$ is the number of samples in the training dataset.
Nearest Neighbor Classification

Compute distance between two points:

- Euclidean distance \( d(p, q) = \sqrt{\sum_i (p_i - q_i)^2} \)

Determine the class from nearest neighbors

- take the majority vote of class labels among the \( k \) nearest neighbors
- weigh the vote according to distance (e.g. weight factor, \( w = 1/d^2 \))
Dimensionality and Scaling Issues

• Problem with Euclidean measure: high dimensional data can cause curse of dimensionality.
  • Solution: normalize the vectors to unit length

• Attributes may have to be scaled to prevent distance measures from being dominated by one of the attributes.

• Example:
  • height of a person may vary from 1.5m to 1.8m
  • weight of a person may vary from 10km to 200kg
  • income of a person may vary from $10K to $1M
Instance-based Classifiers

• Instead of performing explicit generalization, compare new instances with instances seen in training, which have been stored in memory.

• Sometimes called memory-based learning.

• **Advantages**
  • Adapt its model to previously unseen data by storing a new instance or throwing an old instance away.

• **Disadvantages**
  • Lazy learner: it does not build a model explicitly.
  • Classifying unknown records is relatively expensive: in the worst case, given $n$ training items, the complexity of classifying a single instance is $O(n)$. 
Parallel Exemplar-Based Learning System (PEBLS)

- PEBLS is a nearest-neighbor learning system (k=1) designed for applications where the instances have symbolic feature values.
- Works with both continuous and nominal features.
- For nominal features, the distance between two nominal values is computed using Modified Value Difference Metric (MVDM)
  \[ d(V_1, V_2) = \sum_i | \frac{n_{1i}}{n_1} - \frac{n_{2i}}{n_2} | \]
  - Where \( n_1 \) is the number of records that consists of nominal attribute value \( V_1 \) and \( n_{1i} \) is the number of records whose target label is class \( i \).
Distance Between Nominal Attribute Values

- \( d(\text{Status} = \text{Single}, \text{Status} = \text{Married}) = |2/4 - 0/4| + |2/4 - 4/4| = 1 \)
- \( d(\text{Status} = \text{Single}, \text{Status} = \text{Divorced}) = |2/4 - 1/2| + |2/4 - 1/2| = 0 \)
- \( d(\text{Status} = \text{Married}, \text{Status} = \text{Divorced}) = |0/4 - 1/2| + |4/4 - 1/2| = 1 \)
- \( d(\text{Refund} = \text{Yes}, \text{Refund} = \text{No}) = |0/3 - 3/7| + |3/3 - 4/7| = 6/7 \)

<table>
<thead>
<tr>
<th>Tid</th>
<th>Refund</th>
<th>Marital Status</th>
<th>Taxable Income</th>
<th>Cheat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Yes</td>
<td>Single</td>
<td>125K</td>
<td>No</td>
</tr>
<tr>
<td>2</td>
<td>No</td>
<td>Married</td>
<td>100K</td>
<td>No</td>
</tr>
<tr>
<td>3</td>
<td>No</td>
<td>Single</td>
<td>70K</td>
<td>No</td>
</tr>
<tr>
<td>4</td>
<td>Yes</td>
<td>Married</td>
<td>120K</td>
<td>No</td>
</tr>
<tr>
<td>5</td>
<td>No</td>
<td>Divorced</td>
<td>95K</td>
<td>Yes</td>
</tr>
<tr>
<td>6</td>
<td>No</td>
<td>Married</td>
<td>60K</td>
<td>No</td>
</tr>
<tr>
<td>7</td>
<td>Yes</td>
<td>Divorced</td>
<td>220K</td>
<td>No</td>
</tr>
<tr>
<td>8</td>
<td>No</td>
<td>Single</td>
<td>85K</td>
<td>Yes</td>
</tr>
<tr>
<td>9</td>
<td>No</td>
<td>Married</td>
<td>75K</td>
<td>No</td>
</tr>
<tr>
<td>10</td>
<td>No</td>
<td>Single</td>
<td>90K</td>
<td>Yes</td>
</tr>
</tbody>
</table>
Distance Between Records

\[ \delta(X, Y) = w_X w_Y \sum_{i=0}^{d} d(X_i, Y_i) \]

• Each record \( X \) is assigned a weight \( w_X = \frac{N_{X^{\text{predict}}}}{N_{X^{\text{correct}}}^{\text{predict}}} \), which represents its reliability

• \( N_{X^{\text{predict}}} \) is the number of times \( X \) is used for prediction

• \( N_{X^{\text{correct}}}^{\text{predict}} \) is the number of times the prediction using \( X \) is correct

• If \( w_X \approx 1 \) \( X \) makes accurate prediction most of the time

• If \( w_X > 1 \), then \( X \) is not reliable for making predictions. High \( w_X > 1 \) would result in high distance, which makes it less possible to use \( X \) to make predictions.
Characteristics of Nearest Neighbor Classifiers

• Instance-based learner: makes predictions without maintaining abstraction, i.e., building a model like decision trees.

• It is a lazy learner: classifying a test example can be expensive because need to compute the proximity values between test and training examples.

• In contrast eager learners spend time in building the model but then the classification is fast.

• Make their prediction on local information and for low $k$ they are susceptible to noise.

• Can produce wrong predictions if inappropriate distance functions and/or preprocessing steps are performed.
Given a set of training records (memory), and a test record:

1. **Compute the distances** from the records in the training to the test.
2. **Identify the k “nearest” records.**
3. Use target value of nearest neighbors to **determine the value** of unknown record (e.g., by averaging the values).
References

• Nearest Neighbor classifiers. Chapter 5.2. Introduction to Data Mining.
Exercises - kNN
b) k-NN (3 points)

Given the training set on the right, composed of elements numbered from 1 to 12, and labelled as circles and diamonds, use it to classify the remaining 3 elements (letters A, B and C) using a k-NN classifier with $k=3$. For each point to classify, list the points of the dataset that belong to its k-NN set.

Notice: A, B and C belong to the test set, not to the training set. Also, the Euclidean distance should be used.

Answer:
kNN(A) = \{3, 5, 12\} → CIRCLE
kNN(B) = \{3, 5, 7, 10\} → CIRCLE
kNN(C) = \{4, 6, 7\} → CIRCLE
Given the training set on the right, composed of elements numbered from 1 to 12, and labelled as circles and diamonds, use it to classify the remaining 3 elements (letters A, B and C) using a k-NN classifier with k=3.

For each point to classify, list the points of the dataset that belong to its k-NN set.

Notice: A, B and C belong to the test set, not to the training set. Also, the Euclidean distance should be used.
A medical expert is going to build up a case-based reasoning system for diagnosis tasks. Cases correspond to individual persons where the case problem parts are made up of a number of features describing possible symptoms and the solution parts represent the diagnosis (classification of disease). The case base contains the seven cases provided in the table below.

<table>
<thead>
<tr>
<th>Training</th>
<th>Fever</th>
<th>Vomiting</th>
<th>Diarrhea</th>
<th>Shivering</th>
<th>Classification</th>
</tr>
</thead>
<tbody>
<tr>
<td>c₁</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>healthy (H)</td>
</tr>
<tr>
<td>c₂</td>
<td>average</td>
<td>no</td>
<td>no</td>
<td>no</td>
<td>influenza (I)</td>
</tr>
<tr>
<td>c₃</td>
<td>high</td>
<td>no</td>
<td>no</td>
<td>yes</td>
<td>influenza (I)</td>
</tr>
<tr>
<td>c₄</td>
<td>high</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>salmonella poisoning (S)</td>
</tr>
<tr>
<td>c₅</td>
<td>average</td>
<td>no</td>
<td>yes</td>
<td>no</td>
<td>salmonella poisoning (S)</td>
</tr>
<tr>
<td>c₆</td>
<td>no</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>bowel inflammation (B)</td>
</tr>
<tr>
<td>c₇</td>
<td>average</td>
<td>yes</td>
<td>yes</td>
<td>no</td>
<td>bowel inflammation (B)</td>
</tr>
</tbody>
</table>

Similarity provided by an expert

<table>
<thead>
<tr>
<th>q</th>
<th>c</th>
<th>no</th>
<th>avg</th>
<th>high</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.0</td>
<td>no</td>
<td>0.7</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>avg</td>
<td>1.0</td>
<td>0.8</td>
<td></td>
</tr>
<tr>
<td>0.0</td>
<td>high</td>
<td>0.3</td>
<td>1.0</td>
<td></td>
</tr>
</tbody>
</table>

Weights

\[ w_F = 0.3 \]
\[ w_v = 0.2 \]
\[ w_D = 0.2 \]
\[ w_{Sh} = 0.3 \]
Classify the new instance \( q = (\text{high}; \text{no}; \text{no}; \text{no}) \) by applying the KNN algorithm with \( K=1,2,3 \).
Calculate the similarity between all cases from the case base and the new instance $q = (\text{high; no; no; no})$

$c1 = (\text{no; no; no; no})$:
$\text{Sim}(q; c1) = 0.3\times0.0 + 0.2 \times 1.0 + 0.2\times1.0 + 0.3\times 1.0 = 0.70$

$c2 = (\text{average; no; no; no})$:
$\text{Sim}(q; c2) = 0.3\times 0.3 + 0.2 \times 1.0 + 0.2\times1.0 + 0.3\times1.0 = 0.79$

$c3 = (\text{high; no; no; yes})$:
$\text{Sim}(q; c3) = 0.3\times1.0 + 0.2\times1.0 + 0.2\times1.0 + 0.3\times0.2 = 0.76$

$c4 = (\text{high; yes; yes; no})$:
$\text{Sim}(q; c4) = 0.3\times1.0 + 0.2\times0.2 + 0.2\times0.2 + 0.3\times1.0 = 0.68$

$c5 = (\text{average; no; yes; no})$:
$\text{Sim}(q; c5) = 0.3\times0.3 + 0.2\times1.0 + 0.2\times0.2 + 0.3\times1.0 = 0.63$

$c6 = (\text{no; yes; yes; no})$:
$\text{Sim}(q; c6) = 0.3\times0.0 + 0.2\times0.2 + 0.2\times0.2 + 0.3\times1.0 = 0.28$

$c7 = (\text{average; yes; yes; no})$:
$\text{Sim}(q; c7) = 0.3\times0.3 + 0.2\times0.2 + 0.2\times0.2 + 0.3\times1.0 = 0.47$
KNN Classification for K=1

c1 = (no; no; no; no):
\[ \text{Sim}(q; c1) = 0.3*0.0 + 0.2*1.0 + 0.2*1.0 + 0.3*1.0 = 0.70 \]

c2 = (average; no; no; no):
\[ \text{Sim}(q; c2) = 0.3*0.3 + 0.2*1.0 + 0.2*1.0 + 0.3*1.0 = 0.79 \]

c3 = (high; no; no; yes)
\[ \text{Sim}(q; c3) = 0.3*1.0 + 0.2*1.0 + 0.2*1.0 + 0.3*0.2 = 0.76 \]

c4 = (high; yes; yes; no):
\[ \text{Sim}(q; c4) = 0.3*1.0 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.68 \]

c5 = (average; no; yes; no):
\[ \text{Sim}(q; c5) = 0.3*0.3 + 0.2*1.0 + 0.2*0.2 + 0.3*1.0 = 0.63 \]

c6 = (no; yes; yes; no):
\[ \text{Sim}(q; c6) = 0.3*0.0 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.28 \]

c7 = (average; yes; yes; no):
\[ \text{Sim}(q; c7) = 0.3*0.3 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.47 \]

Class: Influenza
KNN Classification for K=2

c1 = (no; no; no; no):
Sim(q; c1) = 0.3*0.0 + 0.2 *1.0 + 0.2*1.0 + 0.3* 1.0 = 0.70

c2 = (average; no; no; no):
Sim(q; c2) = 0.3 * 0.3 + 0.2 *1.0 + 0.2*1.0 + 0.3*1.0 = 0.79

c3 = (high; no; no; yes):
Sim(q; c3) = 0.3*1.0 + 0.2*1.0 + 0.2*1.0 + 0.3*0.2 = 0.76

c4 = (high; yes; yes; no):
Sim(q; c4) = 0.3*1.0 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.68

c5 = (average; no; yes; no):
Sim(q; c5) = 0.3*0.3 + 0.2*1.0 + 0.2*0.2 + 0.3*1.0 = 0.63

c6 = (no; yes; yes; no):
Sim(q; c6) = 0.3*0.0 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.28

c7 = (average; yes; yes; no):
Sim(q; c7) = 0.3*0.3 + 0.2*0.2 + 0.2*0.2 + 0.3*1.0 = 0.47

\[
\begin{array}{cc|ccc}
q & c & \text{no} & \text{avg} & \text{high} \\
\hline
\text{no} & 1.0 & 0.7 & 0.2 \\
\text{avg} & 0.5 & 1.0 & 0.8 \\
\text{high} & 0.0 & 0.3 & 1.0 \\
\end{array}
\]

Weights
\[w_f = 0.3, \quad w_v = 0.2, \quad w_d = 0.2, \quad w_{sh} = 0.3\]

C2: Influenza
C3: Influenza
Class: Influenza
KNN Classification for K=3

\[ c_1 = \text{(no; no; no; no):} \]
\[ \text{Sim}(q; c_1) = 0.3 \times 0.0 + 0.2 \times 1.0 + 0.2 \times 1.0 + 0.3 \times 1.0 = 0.70 \]

\[ c_2 = \text{(average; no; no; no):} \]
\[ \text{Sim}(q; c_2) = 0.3 \times 0.3 + 0.2 \times 1.0 + 0.2 \times 1.0 + 0.3 \times 1.0 = 0.79 \]

\[ c_3 = \text{(high; no; no; yes):} \]
\[ \text{Sim}(q; c_3) = 0.3 \times 1.0 + 0.2 \times 1.0 + 0.2 \times 1.0 + 0.3 \times 0.2 = 0.76 \]

\[ c_4 = \text{(high; yes; yes; no):} \]
\[ \text{Sim}(q; c_4) = 0.3 \times 1.0 + 0.2 \times 0.2 + 0.2 \times 0.2 + 0.3 \times 1.0 = 0.68 \]

\[ c_5 = \text{(average; no; yes; no):} \]
\[ \text{Sim}(q; c_5) = 0.3 \times 0.3 + 0.2 \times 1.0 + 0.2 \times 0.2 + 0.3 \times 1.0 = 0.63 \]

\[ c_6 = \text{(no; yes; yes; no):} \]
\[ \text{Sim}(q; c_6) = 0.3 \times 0.0 + 0.2 \times 0.2 + 0.2 \times 0.2 + 0.3 \times 1.0 = 0.28 \]

\[ c_7 = \text{(average; yes; yes; no):} \]
\[ \text{Sim}(q; c_7) = 0.3 \times 0.3 + 0.2 \times 0.2 + 0.2 \times 0.2 + 0.3 \times 1.0 = 0.47 \]